Bit threads, Einstein's equations and bulk locality

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It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information

June 18, 2021

Based on:

C. Agón, J. de Boer, & J.P. [1811.08879]
C. Agón, E. Cáceres, & J.P. [2007.07907]
C. Agón & J.P. [2105.08063]
J.P., A. Svesko, A. Russo & Z. Weller-Davies [2105.12735, 2106.XXXXX]

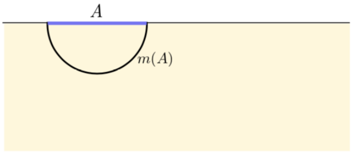
Outline

Organization of the talk:

- Motivations/advantages of bit threads
- Constructive realizations of bit threads & properties [1811.08879]
- Einstein's equations and bulk reconstruction [2007.07907]
- Generalizations:
  - $1/N$ corrections [2105.08063]
  - Lorentzian threads [2105.12735, 2106.XXXXX]

Holographic entanglement entropy

Ryu-Takayanagi (RT):

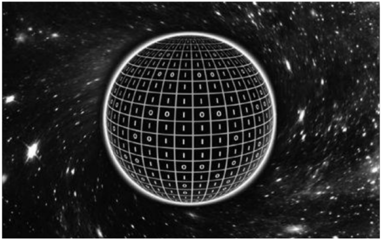


$$S(A) = \min_{m\sim A} \frac{\mathrm{area}(m(A))}{4G_N}$$

$$S(A) = -\mathrm{Tr}\hat{\rho}_A \ln \hat{\rho}_A$$
$$\hat{\rho}_A = \mathrm{Tr}_{A^c}\ \hat{\rho}$$

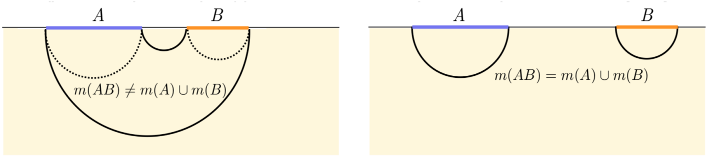
- Geometry emerges from entanglement
- Dynamics of entanglement $\Longleftrightarrow$ Dynamics of geometry
- Properties of $S(A)$ $\Longleftrightarrow$ states with holographic duals

Where are the quantum bits?



Some conceptual puzzles

[Headrick & Freedman]

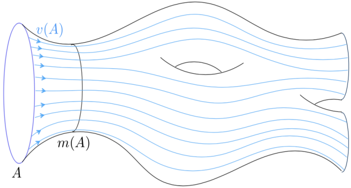


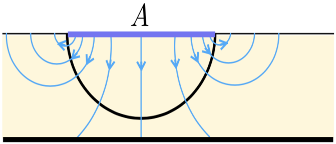
- Minimal surfaces are discontinuous
- QI meanings of quantities such as $S(A)$, $I(A:B)$ are obscure as well as their properties
- In particular SSA and MMI appear in the same footing

Bit thread re-formulation of RT

Consider a v^μ such that $|v| \leq 1/4G_N$ and $\nabla_\mu v^\mu = 0$

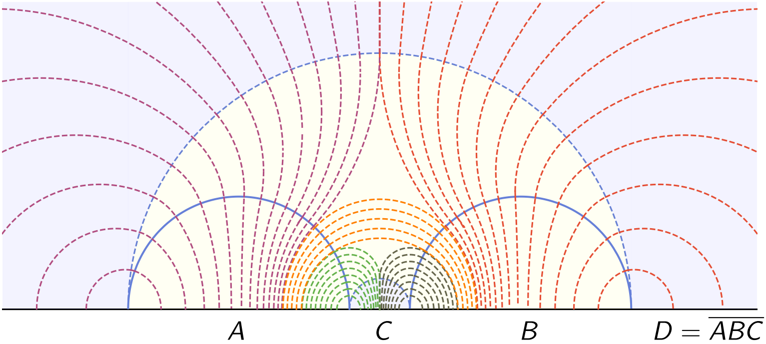
$$S(A) := \max_v \int_A \sqrt{h}\, n^\mu v_\mu = \min_{m\sim A} \frac{\text{area}(m(A))}{4G_N}$$





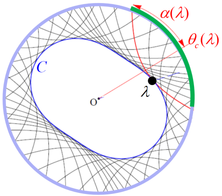
- Equivalence follows from MFMC theorem / convex optimization
  [Freedman & Headrick; Headrick & Hubeny]
- Integral lines of v, a.k.a. threads, codify local pattern of entanglement
- $m(A)$ is unique while v is highly non-unique! $\sim$ different microstates
- Oftentimes is convenient to think of threads as having finite thickness ($4G_N$)

Solution to conceptual puzzles



- Entropy $\sim$ area due to $1d$ nature of threads
- Threads and/or V^μ can be continuous
- Properties of entropy are aligned with their QI meanings
- SA and SSA comes from nesting
- MMI comes from "multicommodity "

Bulk reconstruction via RT



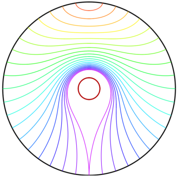
- RT surfaces probe bulk metric and can be used to reconstruct it.
- Hole-ography makes this concrete [Balasubramanian,Chowdhury,Czech,de Boer,Heller; Myers,Rao,Sugishita; Czech,Dong,Sully; Czech,Lamprou,McCandlish,Sully; Headrick,Myers,Wien; etc]
- Differential entropy E computes areas:

$$E = \oint d\lambda \frac{\partial S_A(\theta_-(\lambda)\theta_+(\bar{\lambda}))}{\partial \bar{\lambda}}\bigg|_{\bar{\lambda}=\lambda}$$

- Surfaces can be shrunk to a point. Distance between points can be computed, and ultimately $g_{\mu\nu}$ [Czech,Lamprou].

Bulk reconstruction via RT II

- Caveat 1: Shadows = regions not reached by RT surfaces.



- Caveat 2: Requires an infinite set of RT surfaces.
- BTs for **one** region probe the full bulk, including shadows.
- **Q1:** Given a vector field v (or perhaps a set of v's) is $g_{\mu\nu}$ fully determined? If so, how can we recover the metric? (not obvious, opposite problem is multivalued).
- **Q2:** What kind of thread configurations can we construct without the knowledge of the bulk metric? Is this even possible?

Max flow as a convex program

Common Techniques:

Convex Relaxation, Lagrange Duality, $\cdots$

- Proved MaxFlow-MinCut as well as a Lorentzian version MinFlow-MaxCut [Headrick,Hubeny]
- Discover and Prove the Multi-comodity of Max multiflows: Leading to the prove of MMI (Monogamy of Mutual Information) [Cui,Hayden,He, Headrick,Stoica,Walter]
- Derive an analogue of the Bit-threads, for higher curvature theories of gravity [Headrick,Harpen,Rolph].
- Metric minimization for String Field Theory [Headrick,Zwiebach]
- Derive a Bit Thread like description of membrane theory for the dynamics of holographic entanglement [Agon,Mezei]

An alternative approach /complementary

Construct and study explicit instantiations of Bit-threads

- New properties $\rightarrow$ Higher party entropic inequalities [Bao, et al.]
- Role of special constructions in studies of dynamics, bulk emergence etc..

Results:

- Two different constructions (integral curves and level set)
- Illustration of MMI
- Linearized Einstein's equations from Bit Threads
- Bulk reconstruction from Bit Threads

Integral curves method

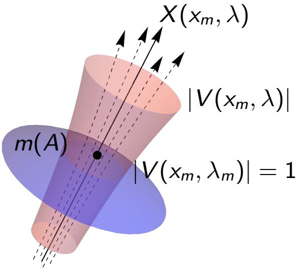
Algorithm

1. Given a connected region A with known $m(A)$
2. Proposed a set of integral curves: $V|_{m(A)} = n^\mu$ (Non-intersecting)
3. Compute $|V|$,

$$|V(x_m, \lambda)| = \frac{\sqrt{h(x_m, \lambda_m)}}{\sqrt{h(x_m, \lambda)}}$$

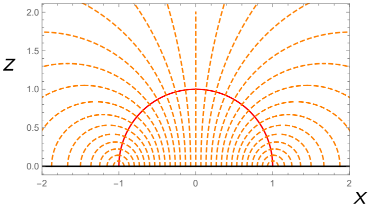
4. Check that $|V| \leq 1$ everywhere

Graphically

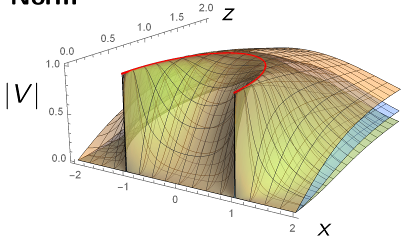


Examples for pure AdS$_{d+1}$:

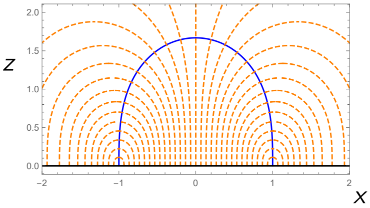
A sphere (Geodesics)



Norm



Strips (Effective Geodesics 2D)



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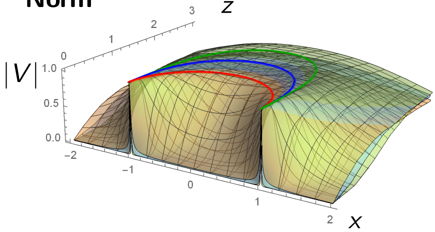

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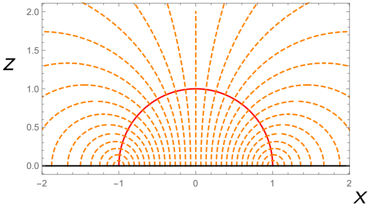

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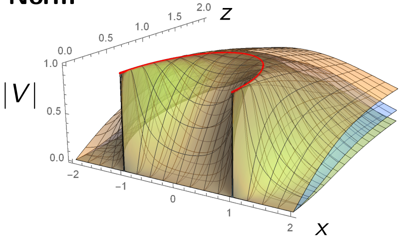


$$V^a = \left(\frac{2Rz}{\sqrt{(R^2 + r^2 + z^2)^2 - 4R^2 r^2}} \right)^d \left(\frac{rz}{R}, \frac{R^2 - r^2 + z^2}{2R} \right) .$$

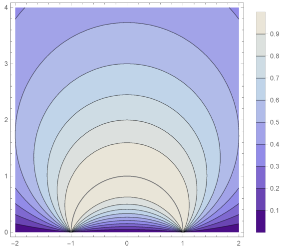
$$|V| = \left(\frac{2Rz}{\sqrt{(R^2 + r^2 + z^2)^2 - 4R^2 r^2}} \right)^{d-1}$$

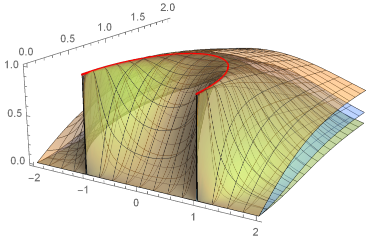
Level set method

1. They must contain the minimal surface γ_A as one of its members.
2. They must be continuous and not self-intersecting.
3. They must not include closed bulk surfaces.
4. They must be homologous to A^*.

$$v_a = \Upsilon(\varphi, g)\partial_a\varphi\,, \qquad \Upsilon^2(\varphi, g)g^{ab}\partial_a\varphi\partial_b\varphi\Big|_{\gamma_A} = 1\,,$$

$$\nabla\cdot v = 0 \quad \rightarrow \quad (\nabla\varphi)\cdot(\nabla\Upsilon) + (\nabla^2\varphi)\Upsilon = 0\,.$$





Properties of max flows

Nesting

There exist flows that simultaneously maximize the flux through a nested set of regions

Max multi flow/Max thread configuration

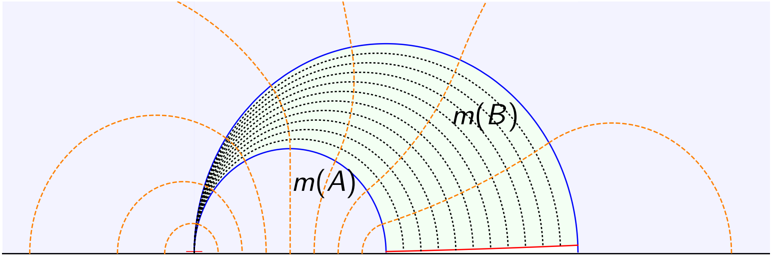
There exist a thread configuration such that for any partition of the boundary manifold: $\partial\mathcal{M} = \cup_{i=1}^{n} A_i$, the number of threads connecting each individual region is maxima.

- These properties can be used to prove SA, SSA and MMI

Properties of max flows

Nesting

There exist flows that simultaneously maximize the flux through a nested set of regions



- Green region is an example of maximally packed flows

Properties of max flows

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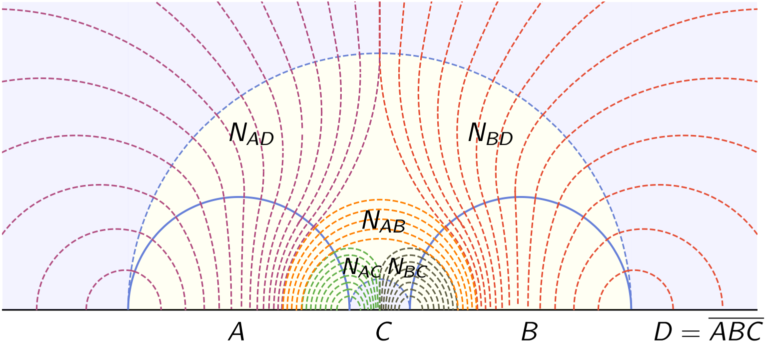


Properties of entanglement

$$S(A) = \max N_{A\bar{A}}$$

Subadditivity: $I(A, B) \equiv S(A) + S(B) - S(AB) > 0$

- For $n = 3$, regions A, B, C using $S(AB) = S(C)$

$$S(A) = N_{AB} + N_{AC}, \quad S(B) = N_{BA} + N_{BC}, \quad S(C) = N_{AC} + N_{BC}$$

$$I(A, B) = 2N_{A,B} > 0$$

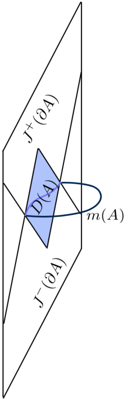
Monogamy of MI: $I(A : BC) \geq I(A : B) + I(A : C)$

- For $n = 4$ regions A, B, C, D using $S(ABC) = S(D)$
  $S(AB) \geq N_{AC} + N_{AD} + N_{BC} + N_{BD}$

$$S(AB) + S(BC) + S(AC) \geq S(A) + S(B) + S(C) + S(D)$$

Dynamical situations

Hubeny-Rangamani-Takayanagi [HRT, 07]



$$S(A) = \min_{m^*} \operatorname{ext}_{m\sim A} \frac{\text{area}(m(A))}{4G_N}$$

[Headrick, Hubeny]

- Covariant formulation

Maximin prescription

$$S(A) = \frac{1}{4G_N} \max_{\Sigma\supset\partial A} \min_{\substack{m\sim A \\ m\subset\Sigma}} \text{area}(m)$$

Maxflow version

$$S(A) = \frac{1}{4G_N} \max_{\Sigma} \max_{v\in\mathcal{F}_\Sigma} \int_A \sqrt{h}\, n_\mu v^\mu$$

$$\mathcal{F}_\Sigma \equiv \{v^\mu \in \mathfrak{X}(\Sigma) \,|\, \nabla_\mu v^\mu = 0, \; |v| \leq 1\},$$

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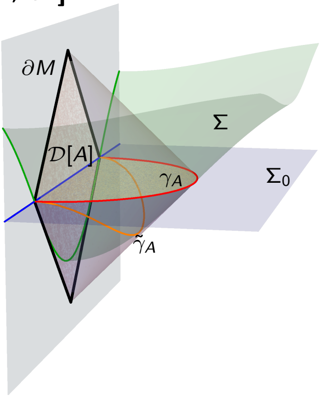

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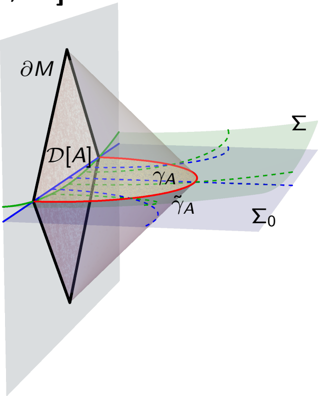

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$$|v| \leq 1\}\,,$$

Perturbations in AdS

In Fefferman Graham coordinates

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) + \delta g_{\mu\nu}(x^\sigma, z)dx^\mu dx^\nu$$

$$\delta g_{\mu\nu}(x^\sigma, z) \equiv z^{d-2}H_{\mu\nu}(x^\sigma, z)\,, \text{ and } \delta\langle T_{\mu\nu}(x^\sigma)\rangle = \frac{d}{16\pi G}H_{\mu\nu}(x^\sigma, 0)\,.$$

Q: given $\delta g_{\mu\nu}$ how can we get $v = v_0 + \delta v$? Can we invert it? Previous methods can be adapted to study perturbations of pure AdS via $v^\lambda = v + \lambda\delta v$, where v is a max flow of $g_{\mu\nu}$ and v^λ a max flow of $g^\lambda_{\mu\nu} = g_{\mu\nu} + \lambda\delta g_{\mu\nu}$

- They correctly reproduce the first law on entanglement and hence encode Einstein's equations. However, both construction are highly non-local (cannot be inverted to recover $\delta g_{\mu\nu}$).
- **Q:** can we get $v = v_0 + \delta v$ without $\delta g_{\mu\nu}$? (e.g. exploiting bulk locality?)

In the language of differential forms

$$S_A = \frac{1}{4G_N} \max_{\mathbf{w}\in\mathbf{W}} \int_A \boldsymbol{w}\,.$$

where $\boldsymbol{W}$ is the set closed $(d-1)$ forms which obeys the norm bound

$$\frac{1}{(d-1)!} g^{a_1 b_1} \cdots g^{a_{d-1} b_{d-1}} w_{a_1 \ldots a_{d-1}} w_{b_1 \ldots b_{d-1}} \leq 1$$

The map

$$v^a = g^{ab}(\star\boldsymbol{w})_b\,, \quad (\star\boldsymbol{w})_b \equiv \frac{1}{(d-1)!}\sqrt{g}\, w^{a_1\cdots a_{d-1}}\varepsilon_{a_1\ldots a_{d-1}b}\,.$$

$$d\boldsymbol{w} = (\nabla_a v^a)\epsilon, \quad \boldsymbol{w}\big|_\Gamma = (n_a v^a)\tilde{\epsilon}$$

Linear perturbations

We assume $g^{\lambda}_{ab} = g_{ab} + \lambda \delta g_{ab}$, then $\boldsymbol{w}_{\lambda} = \boldsymbol{w} + \lambda \delta \boldsymbol{w}$

$$d\left(\boldsymbol{w} + \lambda \delta \boldsymbol{w}\right) = 0 \qquad \rightarrow \qquad d\left(\delta \boldsymbol{w}\right) = 0$$

$$\left(\boldsymbol{w} + \lambda \delta \boldsymbol{w}\right)|_{\gamma_A} = \left(\tilde{\boldsymbol{\epsilon}} + \lambda \delta \tilde{\boldsymbol{\epsilon}}\right) \qquad \rightarrow \qquad \delta \boldsymbol{w}|_{\gamma_A} = \delta \tilde{\boldsymbol{\epsilon}}$$

Notice $\gamma^{\lambda}_{A} = \gamma_A$. Norm bound constraint adopts the form:

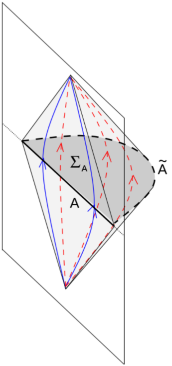
$$\langle \boldsymbol{w}, \boldsymbol{w} \rangle_g + \lambda \left[2 \langle \boldsymbol{w}, \delta \boldsymbol{w} \rangle_g + \langle \boldsymbol{w}, \boldsymbol{w} \rangle_{\delta g}\right] \leq 1\,,$$

where:

$$\langle \boldsymbol{w}, \tilde{\boldsymbol{w}} \rangle_g = \frac{1}{(d-1)!} g^{a_1 b_1} \cdots g^{a_{d-1} b_{d-1}} w_{a_1 \ldots a_{d-1}} \tilde{w}_{b_1 \ldots b_{d-1}}$$

$$\langle \boldsymbol{w}, \tilde{\boldsymbol{w}} \rangle_{\delta g} = \frac{1}{(d-1)!} \delta\left(g^{a_1 b_1} \ldots g^{a_{d-1} b_{d-1}}\right) w_{a_1 \ldots a_{d-1}} \tilde{w}_{b_1 \ldots b_{d-1}}$$

[Casini,Huerta,Myers]



Boundary conformal Killing vector (action of modular flow)

$$\xi_A = -\frac{2\pi}{R}(t-t_0)\left[(x^i - x_0^i)\partial_i\right]$$

$$+\frac{\pi}{R}[R^2 - (t-t_0)^2 - (\vec{x}-\vec{x}_0)^2]\partial_t$$

Bulk Killing vector

$$\xi = \frac{2\pi}{R}(t_0-t)\left[z\partial_z + (x^i - x_0^i)\partial_i\right]$$

$$+\frac{\pi}{R}[R^2 - z^2 - (t-t_0)^2 - (\vec{x}-\vec{x}_0)^2]\partial_t$$

Iyer-Wald formalism and Einstein's Equations [Faulkner,Guica,Hartman,Myers,Van Raamsdonk]

Associated to the killing vector ξ^A there is a conserved $(d-1)$ form $\boldsymbol{\chi}$

$$\boldsymbol{\chi} = -\frac{1}{16\pi G_N}\left[\delta(\nabla^A \xi^B \boldsymbol{\epsilon}_{AB}) + \xi^B \boldsymbol{\epsilon}_{AB}(\nabla_c \delta g^{AC} + \nabla^A \delta g^C{}_C)\right],$$

$$\text{where}\quad \boldsymbol{\epsilon}_{AB} = \frac{1}{(d-1)!}\epsilon_{ABC_3\cdots C_{d+1}} dx^{C_3} \wedge \cdots \wedge dx^{C_{d+1}},$$

which satisfies:

$$\int_{\gamma_A} \boldsymbol{\chi} = \delta S_A\,, \qquad \int_A \boldsymbol{\chi} = \delta\langle H_A\rangle\,, \qquad d\boldsymbol{\chi} = -2\xi^a \delta E^g_{ab}\, \boldsymbol{\epsilon}^b\,,$$

Taking $\tilde{\boldsymbol{\chi}} \equiv \boldsymbol{\chi}|_\Sigma$ and integrating $d\tilde{\boldsymbol{\chi}}$

$$\int_{\Sigma_A} d\tilde{\boldsymbol{\chi}} = \int_{\gamma_A}\tilde{\boldsymbol{\chi}} - \int_A \tilde{\boldsymbol{\chi}} \quad\Longleftrightarrow\quad -2\int_{\Sigma_A} \xi^t \delta E^g_{tt}\, \boldsymbol{\epsilon}^t = \delta S_A - \delta\langle H_A\rangle$$

Canonical Bit Thread from Iyer-Wald

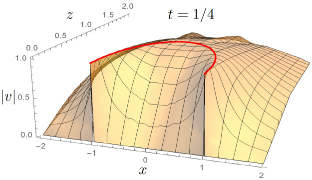
$$\tilde{\chi} = \frac{1}{4G_N}\delta \boldsymbol{w}\,.$$

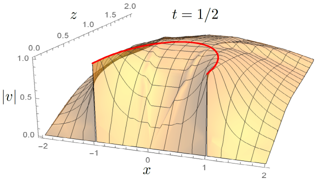
$$\delta \boldsymbol{w}|_{\gamma_A} = \delta\tilde{\epsilon} \qquad d\left(\delta \boldsymbol{w}\right) = 0 \iff Linearized - EEqs$$

Norm constraint?

$$If\ \ \boldsymbol{w} = geodesic, \rightarrow\ \langle \boldsymbol{w}_\lambda, \boldsymbol{w}_\lambda\rangle_{g_\lambda} \leq 1 + \mathcal{O}(\lambda^2)$$

$$\delta \boldsymbol{w}|_A = \frac{4\pi G_N}{R}\left(R^2 - |\vec{x} - \vec{x}_0|^2\right)\langle T_{00}\rangle\,\bar{\boldsymbol{\epsilon}}$$





Metric Reconstruction

- **Note:** $\delta \boldsymbol{w}$ can be obtained in M from boundary condition at ∂M together with the closedness condition [Wald]
- We also assume $g_{\mu\nu}^{\text{AdS}}$ (fixed by symmetries).
- Canonical tread construction provides a way to locally reconstruct bulk metrics for perturbative excited states!
- Two ways of metric reconstruction: (i) starting from a family of $\delta \boldsymbol{w}$'s associated to different regions or (ii) starting from only **one** (or a few) $\delta \boldsymbol{w}$.
- (i) Gives a set of algebraic equations that can be inverted (ii) Gives a first-order differential equation that can be inverted.

$$(\star\delta \boldsymbol{w})_a = \mathcal{F}_a^{bc}\,\delta g_{bc} \quad \rightarrow \quad \delta g_{ab} = [\mathcal{F}^{-1}]_{ab}^{c}\,(\star\delta \boldsymbol{w})_c\,.$$

- For $d=2$, $d=3$ a single $\delta \boldsymbol{w}$ suffices to solve for δg_{bc}.
- For $d \geq 4$ a finite number is needed.

$$\delta g_{bc} = \left[\mathcal{F}_{(i)}^{-1}\right]_{bc}^{a}\,(\star\delta \boldsymbol{w}^{(i)})_a$$

Metric Reconstruction (*ii*) $d = 2, 3$

- We assume $\delta \boldsymbol{w}$ is only known for one $(R, \vec{x}_0)$. For example one finds for the trace ($z_*^2 \equiv R^2 - |\vec{x} - \vec{x}_0|^2$)

$$H^i{}_i(z, \vec{x}) = 4R(z_*^2 - z^2) \int_{i\epsilon}^{1+i\epsilon} d\lambda \, \frac{\lambda^{d-1} \delta w_z(\lambda z, \vec{x})}{[z_*^2 - (\lambda z)^2]^2} \,,$$

and similarly for other components H_{ij}.

- For $d \geq 4$, can we solve the inversion with "a few" $\delta \boldsymbol{w}$
- Can also obtain the time components of the perturbation H_{tt} and H_{ti}, specializing to boosted Σ's
- At next (non-linear) orders we expect the same methodology should work, but inverting a higher order operator instead —following [Faulkner,Haehl,Hijano,Parrikar,Rabideau,Van Raamsdonk]

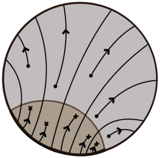
Generalizations: I. $1/N$ corrections

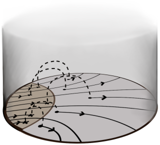
Recently, we derive a quantum corrected prescription for bit threads [Agon,JP]

$$S_A = \frac{1}{4G_N} \max_{v \in \mathcal{F}} \int_A v\,, \qquad \mathcal{F} \equiv \{v \,|\, \nabla \cdot v = -4G_N s(x),\, |v| \leq 1\}\,,$$

$$\int_{\Sigma_A} s(x) = S_{\text{bulk}}[\Sigma_A]\,.$$

- Equivalent to FLM, or rather, to QES at order $\mathcal{O}(G_N^0)$
- Derived via convex optimization and strong duality
- Interpretation of quantum corrections as distillation of bulk state





Generalizations: I. $1/N$ corrections

- Iyer-Wald works but dictionaries are modified:

$$\delta S_A^{\text{grav}} = \int_{\gamma_A} \delta \boldsymbol{w} + \int_{\Sigma_A} \xi^\mu \langle T_{\mu\nu}^{\text{bulk}}(x) \rangle \boldsymbol{\epsilon}^\nu$$

$$\delta E_A^{\text{grav}} = \int_A \delta \boldsymbol{w}$$

with

$$d(\delta \boldsymbol{w}) = -4 G_N s(x) = -4 G_N \xi^\mu \langle T_{\mu\nu}^{\text{bulk}}(x) \rangle \boldsymbol{\epsilon}^\nu$$

- Semiclassical Einstein's equations arise by consistency!

$$\delta S_A^{\text{grav}} - \delta E_A^{\text{grav}} = \int_{\gamma_A} \delta \boldsymbol{w} - \int_A \delta \boldsymbol{w} + \int_{\Sigma_A} \xi^t \langle T_{00}^{\text{bulk}}(x) \rangle \boldsymbol{\epsilon}^t = 0$$

$$-2 \int_{\Sigma_A} \xi^t \left(\delta E_{00} - \frac{1}{2} \langle T_{00}^{\text{bulk}}(x) \rangle \right) \boldsymbol{\epsilon}^t = 0$$

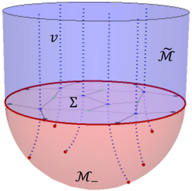
- Bulk reconstruction remains unexplored! Wald's theorem of uniqueness does not apply straightforwardly.

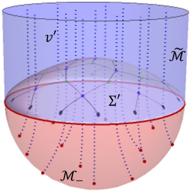
Generalizations: II. Lorentzian threads

The Lorentzian MinFlow-MaxCut theorem [Headrick,Hubeny] was recently used in the context of CV duality [JP,Svesko,Russo,Weller-Davies]

$$\mathcal{C}(A) = \frac{1}{G_N \ell} \max_{\Sigma \sim A} \mathrm{Vol}(\Sigma(A)) = \min_{v \in \mathcal{F}} \int_A v \,, \quad \mathcal{F} \equiv \left\{ v \,|\, \nabla \cdot v = 0 \,, \, |v| \geq \tfrac{1}{G_N \ell} \right\}$$

- Uncovered new properties/inequalities derived from nesting
- Tightly connected with Lorentzian AdS/CFT and state preparation
- Makes evident the role of the reference state (unclear in CV and CA)
- Interpreted in terms of 'gatelines' preparing an optimal tensor network





Generalizations: II. Lorentzian threads

- Bulk symplectic form ω_{bulk} [Belin,Lewkowycz,Sarosi] gives a canonical flow for linear perturbations over arbitrary states!
- $d\omega_{\text{bulk}} = 0$ for on-shell perturbations, so the linearized Einsten's equations can be derived covariantly from complexity!
- Lorentzian threads can probe the black hole interior, and the region near the singularity! Metric reconstruction possible?
- Gives an intuitive picture of how time emerges in quantum gravity!

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Questions?