Late time physics of holographic quantum chaos

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East Asian String Webinar

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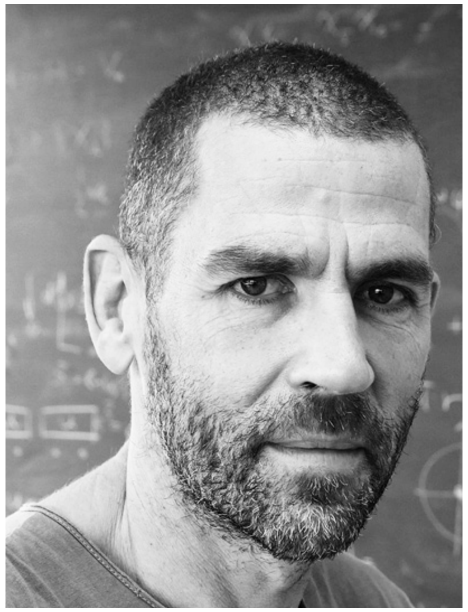
work with



Alexander Altland (Cologne)

[1603.04856], [1707.08013], [1903.00478],
[1903.03143], [1907.10061], [2008.02271]

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Contents

Quantum chaos and holography

AdS/CFT relates gravity (often in AdS) to unitary field theory (often CFT)

Goal: use the quantum dynamics of field theory to shed light on the paradoxical behaviour of black holes.

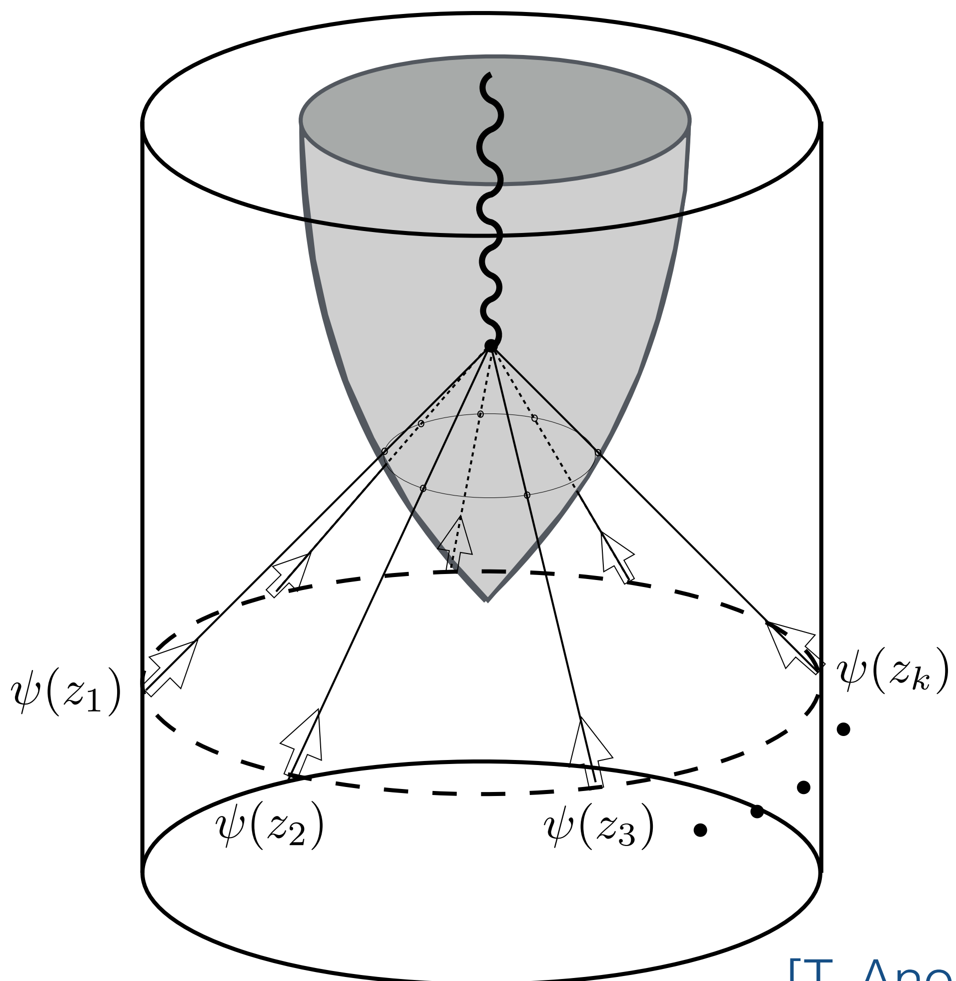
Puzzle: are you saying black holes are just lumps of coal? Where does all the intrigue hide? Where does the universality of black-hole physics come from?

Resolution: the long-time dynamics of complex Hamiltonians is chaotic and at the same time universal. This replaces the (naive) semiclassical late time physics of bulk gravity.

The premise: Thermalization ➛ BH formation (& evaporation)

boundary:

Quantum quench



bulk:

Black-hole collapse

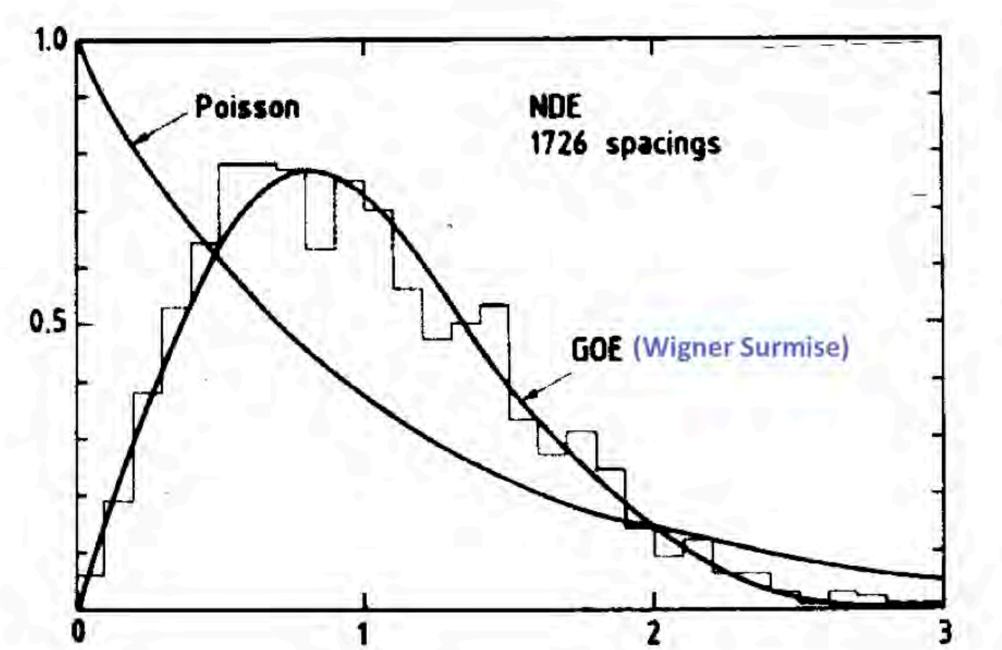
[T. Anous, T. Hartman, A. Rovai, J. Sonner]

The strategy: Study quantum thermalisation at all relevant timescales

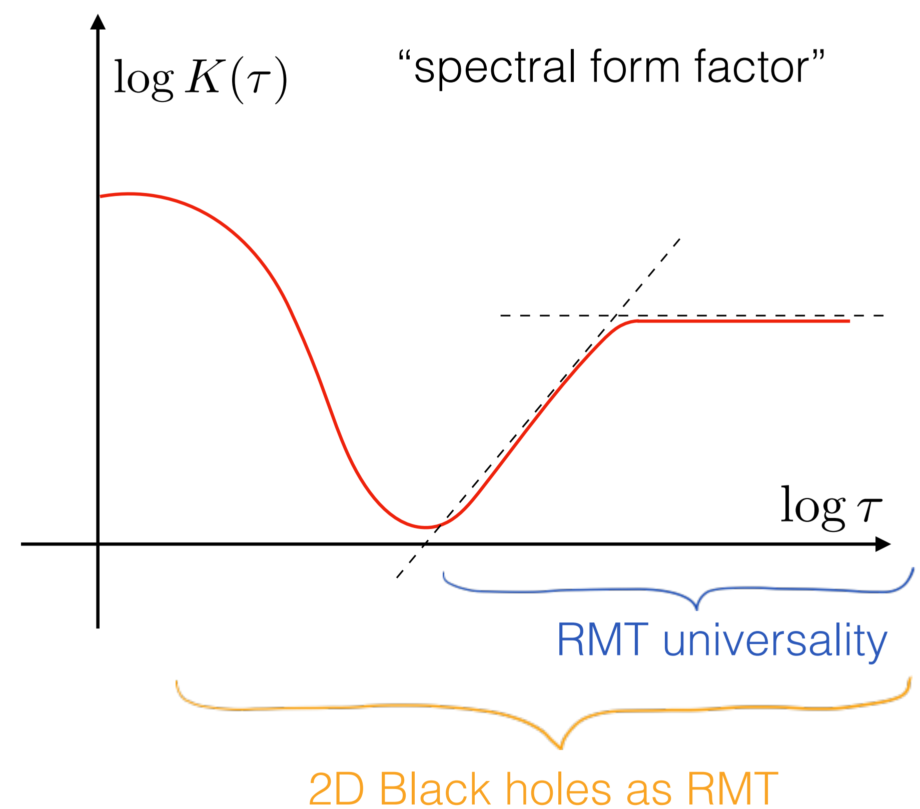
➛ Quantum chaos, quantum ergodicity

The Big Picture

Bohigas-Giannoni-Schmidt: (paraphrased)
A classically chaotic system behaves* quantum mechanically like a random matrix ensemble (at late times)



[from Bohigas et al.]



Renaissance in recent years: 2D BHs are **exactly** described by random-matrix ensembles [Kitaev, Stanford et al., Maldacena et al.]

Weird feature of lower dimensions or important general lesson?

Our Result

Boundary theories in holographic duality* exhibit late-time RMT behaviour in accordance with BGS.

The main technical insight:

RMT behaviour is universal because it follows from a simple symmetry breaking pattern and the associated Goldstone EFT!

Causal symmetry breaking

The EFT of quantum chaos

In particular, this is true for **individual** quantum systems!

Causal symmetry and its breaking: an aperçu

Key observable: spectral correlations

$$R_2(\omega) = \Delta^2 \left\langle \rho\left(E + \frac{\omega}{2}\right) \rho\left(E - \frac{\omega}{2}\right)\right\rangle$$

Define density via the discontinuity

$$\Delta^{-1} := \rho(E) = G^+(E + i\epsilon) - G^-(E - i\epsilon)$$

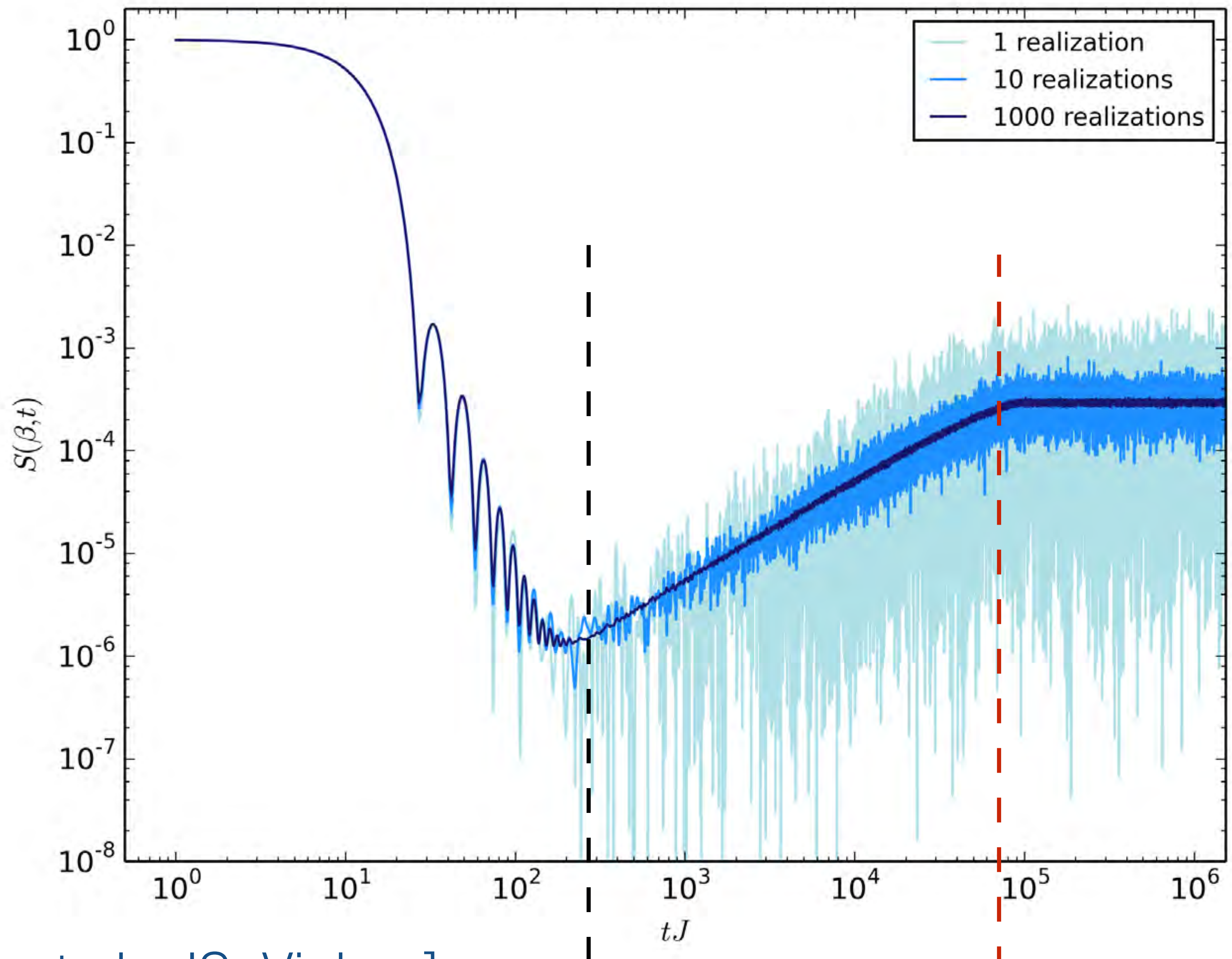
If opposite causality propagators are equal, clearly **ρ** = 0

⇒ there is a "causal symmetry" $G^+ \leftrightarrow G^-$ (actually a GL(n|m) symmetry)

$\rho(E) \neq 0 \quad \Longleftrightarrow \quad$ ~~Causal GL(n|m)~~

Goldstones of causal symmetry fix R_2 ⇒ RMT Universality

(Doubly) non-perturbative spectroscopy



SYK spectral Form factor

[Cotler et al.; JS, Vielma]

$$t \sim e^{S} \Rightarrow \omega \sim e^{-S}$$

Need to control effects of $\mathcal{O}\left(e^{-e^{S}}\right)$ in bulk & boundary!

Contents

Spectral determinants

Point of departure: ratios of determinants, e.g.

$$\mathcal{Z}_{(4)}(\hat{z}) = \frac{\det(z_1 - H)\det(z_2 - H)}{\det(z_3 - H)\det(z_4 - H)}$$

For example, we can recover the spectral correlation:

Advanced

Retarded

$$\langle \rho(E)\rho(E')\rangle = \frac{\mathrm{Re}}{\pi^2 L^2}\frac{\partial^2}{\partial z_3 \partial z_4}\left\langle \mathcal{Z}_{(4)}(z_1, z_2, z_3^+, z_4^-)\right\rangle\Big|_{\substack{z_1 = z_3^+ = E \\ z_2 = z_4^- = E'}}$$

Remark: there is a “Weyl symmetry”: $z_1 \leftrightarrow z_2$

First we exponentiate

We can trivially write a determinant as a (functional) integral

$$\det\left(E - H\right) = \int d(\bar{\eta}, \eta) e^{\bar{\eta}(E-H)\eta}$$

We introduced 2L Grassmann numbers η

(Inverse determinants are introduced by boson integrals)

$$\mathcal{Z}_{(4)}\left(\hat{z}\right) = \int e^{-i\bar{\psi}(\hat{z}-H)\psi} d(\psi, \bar{\psi})$$

ψ is now a graded object of dimension 4L and the action invariant under GL(2L|2L)

Then we integrate

An example to motivate us: actual RMT

$$\langle \mathcal{Z}(\hat{z}) \rangle = \int dH \int d(\bar{\psi}, \psi)\, e^{-V(H) + i\bar{\psi}(\hat{z} - H)\psi}$$

Due to integral only H-singlets can survive: GL(2L|2L) ➛ GL(2|2)

Results in an action over only a (2|2) graded matrix with causal symmetry GL(2|2)!

The presence of a non-zero density of states ρ spontaneously breaks
GL(2|2) ➛ GL(1|1) x GL(1|1)

IR theory (i.e. $\omega \sim e^{-S}$) is given by Goldstones on coset

$$\mathcal{M} = \mathrm{GL}(2|2)/\left(\mathrm{GL}(1|1) \times \mathrm{GL}(1|1)\right)$$

Constructing the EFT

Used RMT as a motivating example, but mechanism is general: large GL(2L|2L) projects on singlets GL(2|2) $\Longrightarrow$ broken to causal sectors

Can use well-known coset method [CCWZ] to construct EFT

$$S[Q] = -iv \int \mathrm{str}\left(\hat{z}Q\right) dV + F^2 \int \mathrm{str}\left(\nabla_i Q \nabla^i Q\right) dV + \cdots$$

[Wegner, Efetov]

Where Q is analogous to the pion field in the chiral Lagrangian:

$$Q = T\tau_3 T^{-1}\,, \qquad T \in \mathrm{U}(2|2)$$

symmetry breaking vacuum

Well, technically the pion field really is W, where

$$T = e^W$$

Comments

Showed “single-particle version”, more relevant is many-body version, which lives instead on many-body Hilbert space (harder)

We used RMT average to motivate singlet projection. General quantum system: take late time limit (+ perhaps ‘mild’ average)

“Altland-Zirnbauer” classification of 10 RMT ensembles becomes Cartan classification of 10 symmetric spaces (Weyl symmetry)

Generalises easily to ratios of more spectral determinants

We used graded (‘susy’) approach. Also common to use replicas.

The universal content of RMT

Wait long enough so that all non-ergodic modes have decayed (define Thouless time)

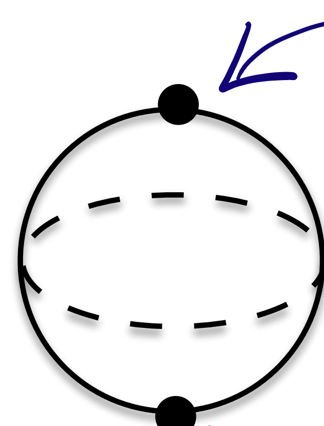
$$S(Q) = -i\frac{\pi}{2\Delta}\mathrm{str}(Q\hat{z})$$

Remember: wanted to evaluate spectral determinants. This is now easily done in terms of the “pions”

$$T = \exp(W), \qquad W = \begin{pmatrix} & B \\ \tilde{B} & \end{pmatrix}$$

Key point: there is more than one causal symmetry breaking saddle

$$\mathcal{M} = H_2 \times S^2$$



“standard” saddle $S_{\mathrm{SS}} = 0$

“Andreev-Altshuler” saddle $S_{\mathrm{AA}} \sim e^{-S}$

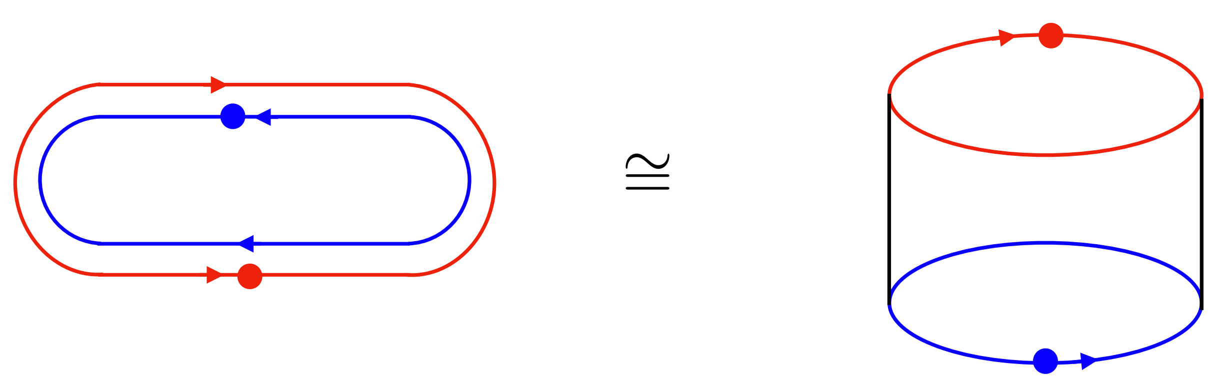
Topological expansion of EFT

Let us exhibit some of the leading contributions to $\mathcal{Z}_{(4)}(\hat{z})$

$$\langle \dots \rangle \equiv \int d(B,\tilde{B})\, e^{-2is\,\mathrm{str}(B\tilde{B})}(\dots)\,, \qquad \left(s := \frac{\omega}{\pi\Delta} \sim e^{S}\right)$$

Simplest diagram: “wagon wheel” (topology of cylinder / annulus)

$$\left\langle \mathrm{str}(B\tilde{B}{\color{red}P^{\mathrm{f}}})\mathrm{str}(\tilde{B}B{\color{blue}P^{\mathrm{f}}}) \right\rangle$$



$\sim \quad s^{-2}$

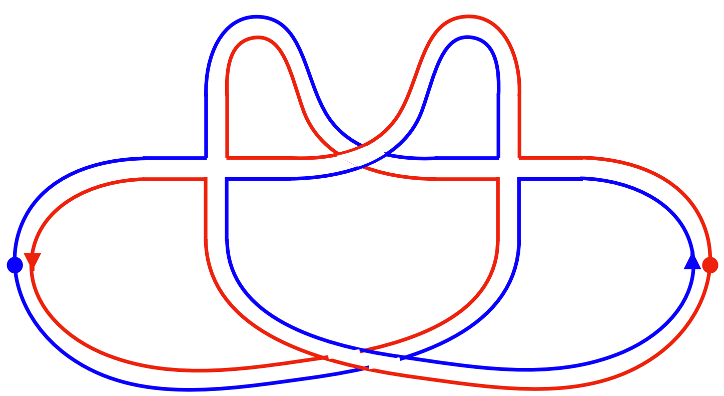
Coefficient of diagram is fixed!
gives linear ramp

$$\mathrm{FT}(s^{-2}) \sim t$$

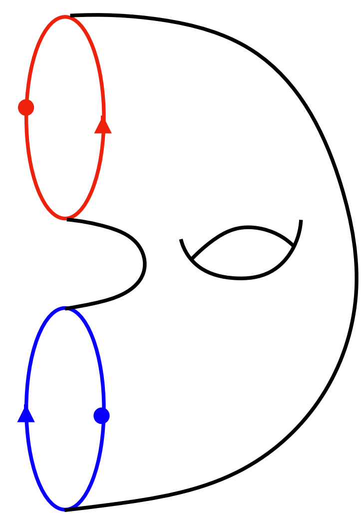
Higher Topologies

First correction: “Sieber-Richter pair” (topology of cylinder with one handle)

$$\left\langle \mathrm{str}(B\tilde{B}{\color{red}P^{\mathrm{f}}})\mathrm{str}(\tilde{B}B{\color{blue}P^{\mathrm{f}}})\mathrm{str}((B\tilde{B})^4)\right\rangle$$



$\cong$



$\sim s^{-4}$

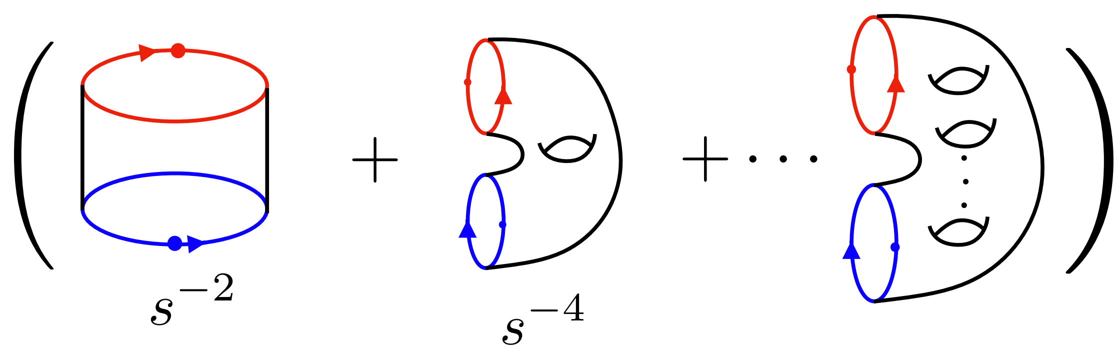
The coefficient of this term depends on symmetry class

In unitary class coefficient is zero. In fact all corrections vanish (can show this using Duistermaat-Heckmann)

Things get (doubly) non-perturbative

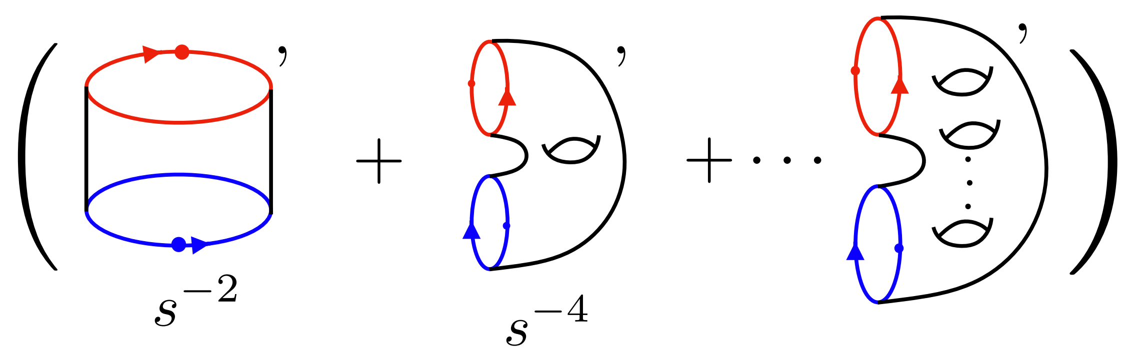
So far only diagrams around “standard saddle”. Full picture:

$R_2(s) \;=\; e^{s\times 0}$



“standard” saddle

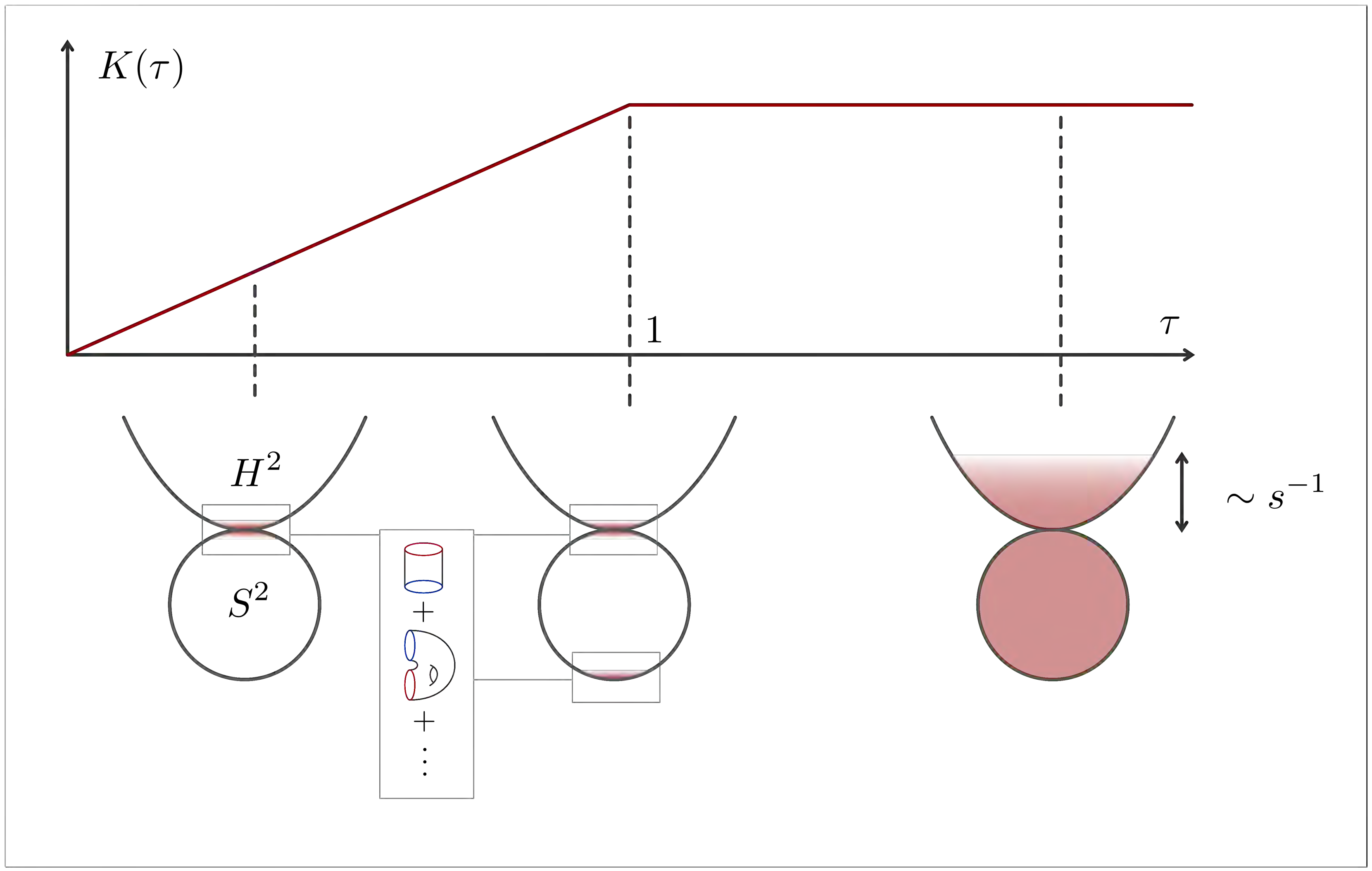
“Andreev-Altshuler” saddle $+e^{2is}$



This results in the famous sine kernel [F. Dyson (1970)]

$$R_2(s) = -\mathrm{Re}\,\frac{1}{2s^2}(1-e^{-2is}) = -\frac{\sin^2 s}{s^2}.$$

To summarise



next: the terms in the topological expansion are arranged into the same topologies as the expansion of the JT matrix model in Euclidean wormholes and baby universes. This is not a coincidence…

Contents

A bulk picture

Let us write the determinant in terms of the loop operator

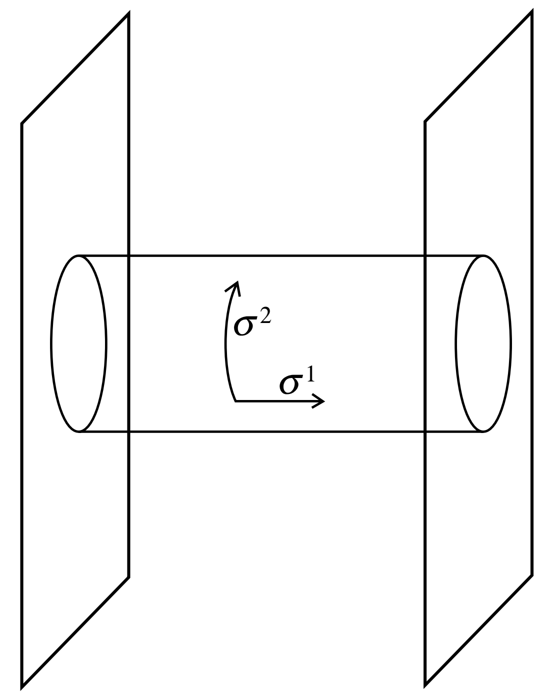
$$\det(E^{\pm} - H) = e^{\mathrm{Tr}\log(E^{\pm} - H)}$$

Such a determinant corresponds to a brane in string theory.
Each insertion of

$$\mathcal{W}(E) = \mathrm{Tr}\log\left(E - H\right)$$

adds a world sheet boundary labelled by the fixed energy E.

Textbook example with two boundary insertions



[source: Polchinski Volume I]

Fig. 8.5. Exchange of a closed string between two D-branes. Equivalently, a vacuum loop of an open string with one end on each D-brane.

A physical role for the psi's

In this way the spectral determinants are represented by "spectral branes"

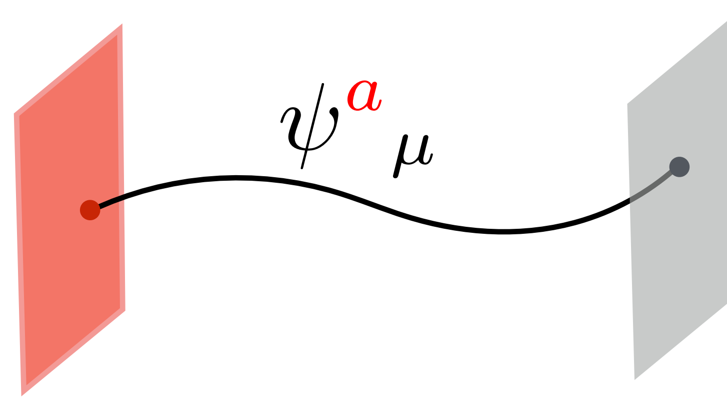
$$\det(E^{\pm} - H)$$

O(1) number of "spectral branes"

"sea stack" of L >> 1 branes (think of L D3 branes e.g.)

We have "advanced" and "retarded" spectral branes, defined by the analytic continuation in energy

The auxiliary objects ψ now are physical: they are open strings stretching between sea branes and spectral branes



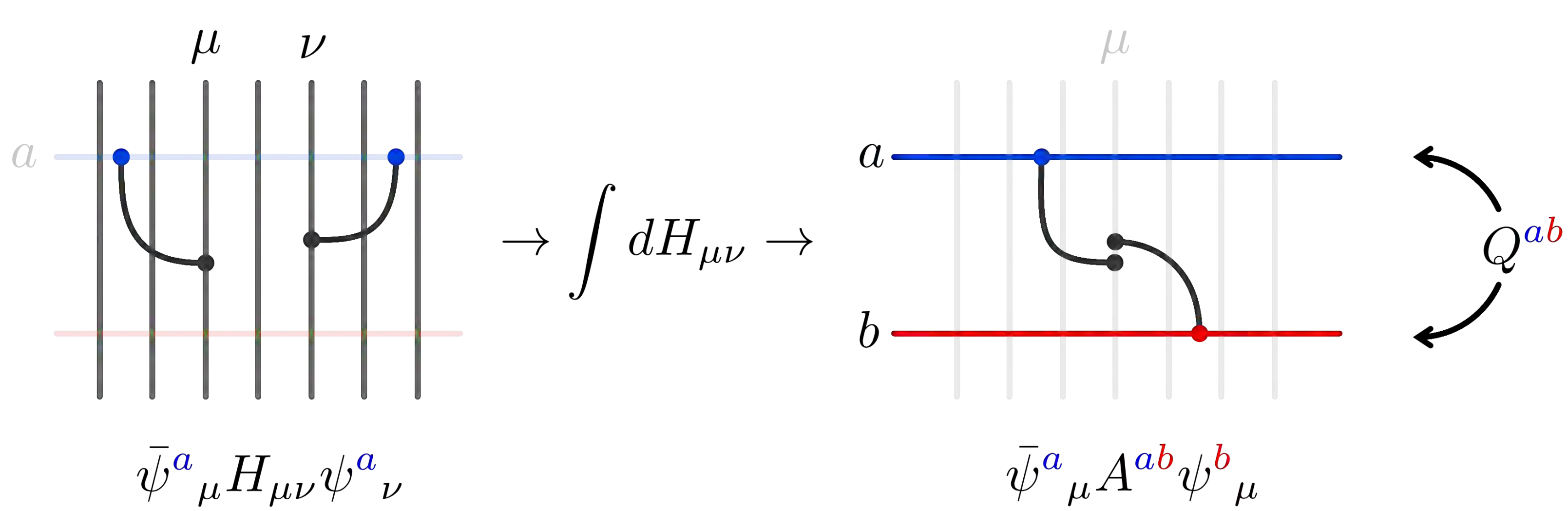
Bulk causal symmetry breaking

Causal symmetry is the non-abelian symmetry of coincident spectral branes

Broken spontaneously by a condensate of string bound-states

$$\Pi^{ab} = \sum_{\mu=1}^{L} \psi_\mu^a \bar{\psi}_\mu^b \qquad \text{with} \qquad \langle \Pi^{ab} \rangle \propto \tau_3$$

Goldstones are mesonic fluctuations around this vev (c.f. holographic QCD)



Making this fully concrete: MMST

This picture can be made fully explicit in minimal string theory:

Take Liouville (as a world sheet theory) and couple to (q,p) minimal model matter CFT

Minimal string theory has D-branes of the kind we need:

Sea brane ↔ ZZ brane

Spectral brane ↔ FZZT brane

There exists a direct map between matrix model and worldsheet formulation ➛ can explicitly prove every assertion we made above

Matrix model picture

To connect bulk (= worldsheet) and boundary (= matrix model) focus again on determinant operators

In the double-scaled limit (at the spectral edge) these are computed by a graded variant of Kontsevich's cubic matrix model (q,p) = (2,1)

$$\langle \mathcal{Z}(\hat{z}) \rangle \simeq \int da \, e^{-e^{S_0} \mathrm{str}\left(\frac{a^3}{3} + \hat{\zeta} a\right)}$$

For example: spectral density is given by Airy function, with asymptotic behavior

$$\begin{aligned} \rho(\zeta) &= e^{2S_0/3} \left(-\mathrm{Ai}(x)^2 + x \mathrm{Ai}'(x)^2 \right) \\ &\sim \frac{e^{S_0}}{\pi} \sqrt{\zeta} \,, \end{aligned}$$

Open / closed duality

Kontsevich's matrix model is interpreted as the effective theory of the spectral brane, having integrated out sea branes

Individual contributions to the determinant operators can be calculated directly in the world-sheet approach. For example:

$$\langle \Psi(\zeta_1)\Psi(\zeta_2)\rangle \simeq e^{\mathrm{Disk}(\zeta_1)+\mathrm{Disk}(\zeta_2)+\mathrm{ann}(\zeta_1,\zeta_2)+\frac{1}{2}(\mathrm{ann}(\zeta_1,\zeta_1)+\mathrm{ann}(\zeta_2,\zeta_2))}$$

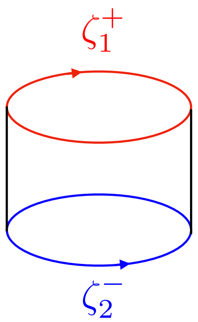
Disk(ζ_1): world sheet disk amplitude with one FZZT boundary

ann(ζ_1,ζ_2): world sheet cylinder amplitude with two FZZT boundaries

Leading singularity

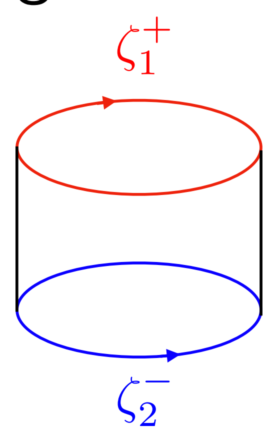
Some years ago Martinec computed the c < 1 annular partition function in full generality. We only need:

$$Z_{(2,p)}(\zeta_1, \zeta_2) = \int_0^\infty dP \frac{\cos(4\pi s_1 P)\cos(4\pi s_2 P)}{2\pi P \sinh 2\pi \frac{P}{b} \cosh 2\pi \frac{P}{b}} =$$



leading singular diagram contributing to the ratio $\mathcal{Z}_{(4)}$

$$\left\langle \mathrm{Tr}\frac{1}{\zeta_1^+ - H}\,\mathrm{Tr}\frac{1}{\zeta_2^- - H}\right\rangle^{\text{annulus}}_{(2,p)} = \partial^2_{\zeta_1,\zeta_2}$$



$$\sim s^{-2}$$

With factors:

$$R_2(\zeta_1 - \zeta_2) = -\frac{\Delta^2}{2\pi^2(\zeta_1 - \zeta_2)^2} = -\frac{1}{2s^2}$$

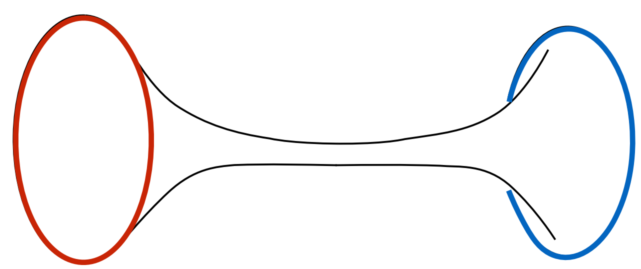
EFT prediction ✓

Remark: can also do this in 3D, thanks to $\mathbb{T}^2 \times I$ wormhole of Cotler and Jensen

Wormholes and baby universes

Taking Liouville as our theory of 2D gravity, the annulus is a Euclidean wormhole connecting two boundaries (JT “double trumpet”)

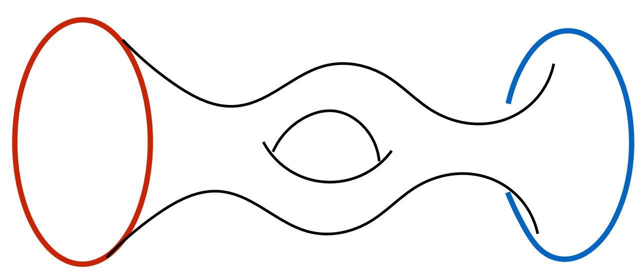
Boundary 1



Boundary 2

Working very hard one might be able to compute the same process with the exchange of a baby universe in the intermediate state

Boundary 1



Boundary 2

However, the Kontsevich model easily generates all these, as well as arbitrarily many multi boundaries. It is thus a “Universe Field Theory”

Conclusions

Conclusions

Chaos in the RMT sense is ubiquitous. The reason is that its physical content follows from a universal principle, the breaking of causal symmetry

In this way we understand why and how generic quantum chaotic systems at late times look like random matrix theories

Bulk picture of causal symmetry breaking in terms of spectral banes

Gives meaning to bulk Euclidean wormholes and baby universes even in theories which are not defined via ensembles (e.g. in higher dimensions)

Outlook

Connection to replica wormholes and Page curve?

Refactorization? Erratic fluctuations?

Explicit bulk picture in higher dimensions? Maldacena-Maoz wormholes…

Plug: results on operator ergodicity coming up with Nayak and Vielma…

thank you for your attention