4d gauge theory and 2d CFT from 6d point of view

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Many interesting phenomena in SUSY gauge theory can be “understood” using 6d N=(2,0) theory,

about which we don’t know much.

But somehow, assuming its existence and 6d Lorentz invariance, we can deduce a lot.

Andy’s 3rd lecture was about one aspect of this fact.

The most important relation is:

$$6=4+2$$

Today

Basics of 6d N=(2,0) theory. S-duality of 4d N=4.

Tomorrow

4d N=2 as 6d N=(2,0) compactified on C

The day after tomorrow

Relation with 2d CFT

- Non-gravitational theories can at most have 16 supercharges.

- Recall the min. dim. of spinors in each dim:

	4	5	6	7	8	9	10
$\dim_{\mathbb{C}}$	2, $\bar{2}$	4	4, $4'$	8	8, $8'$	16	16, $16'$
min. # of Q's	4	8	8	16	16	16	16

○ --- strictly real

$$\mathcal{N}=4 \leftarrow \mathcal{N}=2 \leftarrow \mathcal{N}=(1,1) \leftarrow \mathcal{N}=1 \quad \cdots$$

$$\boxed{\mathcal{N}=(2,0)}$$

N=(1,1) multiplet: R-sym is Sp(1) x Sp(1) ~ SO(4)

A_μ 4

doublet

doublet

ψ_α $\psi_{\dot\beta}$ 4 + 4

vector rep.

$\Phi_{i=1,2,3,4}$ 4

N=(2,0) multiplet: R-sym is Sp(2) ~ SO(5)

states

- $B_{[\mu\nu]} \to F_{[\mu\nu\rho]} = \partial_{[\mu} B_{\nu\rho]}$ 3

  s.t. $F_{\mu\nu\rho} = \frac{1}{6} \varepsilon_{\mu\nu\rho}{}^{\sigma\tau\upsilon} F_{\rho\tau\upsilon}$

- $\psi_{\alpha a} = J_{\alpha\beta} J_{ab} \overline{\psi}^{\beta b}$ $\frac{4 \times 4}{2}$

- $\phi_{i=1,2,3,4,5}$ self-dual tensor 5

spinor

It's weird.

It's easy to write down an action for N=(1,1) theory:

$$S = \int d^6x \frac{1}{g_{6d}^2} F^a_{\mu\nu} F^a_{\mu\nu} + \text{supersymmetrizations}.$$

parameters

- gauge group G
- g^2_{6d} : $[g^2_{6d}] = (\text{length})^2$ non-renormalizable!
- θ angle for $\pi_5(G)$

$$\int \theta \, \mathrm{tr} F \wedge F \wedge F \quad \theta \text{ is dimensionless!}$$

It's not easy to write down an action for N=(2,0) theory, even for a free theory.

Consider

$$S=\int d^6x \frac{1}{g^2} F_{\mu\nu\rho} F_{\mu\nu\rho} + \cdots$$

where $F_{\mu\nu\rho}=\partial_{[\mu}B_{\nu\rho]}$. g is dimensionless! Good!

We need to impose

$$F_{\mu\nu\rho}=\frac{1}{6}\varepsilon_{\mu\nu\rho}{}^{\sigma\tau\upsilon} F_{\sigma\tau\upsilon}$$

Then

$$F_{\mu\nu\rho}F_{\mu\nu\rho}=F_{\mu\nu\rho}\,\varepsilon^{\mu\nu\rho\sigma\tau\upsilon}F_{\sigma\tau\upsilon} = 0 .$$

The action makes no sense.

Free theories can be dealt with.

[Pasti-Sorokin-Tonin '95][Belov-Moore '06]

Recall the case of EM fields in 4d.

$$S = \int d^4x \frac{1}{e^2} F_{\mu\nu} F_{\mu\nu} .$$

The dual field is

$$G_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} \frac{\delta S}{\delta F_{\rho\sigma}} = \frac{1}{e^2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

The dual action is then

$$S_{dual} = \int d^4x \, e^2 G_{\mu\nu} G_{\mu\nu} .$$

Note that the coupling is the inverse.

$$e^2 \leftrightarrow 1/e^2$$

So, to make sense of the equality

$$\underbrace{F_{\mu\nu\rho}}_{\text{coupling } g^2} = \underbrace{\frac{1}{6}\varepsilon_{\mu\nu\rho}{}^{\sigma\tau\upsilon}F_{\sigma\tau\upsilon}}_{1/g^2}.$$

g should be **1** !

OK for a free field,
but troublesome for interacting theories.

But what interacting theories?

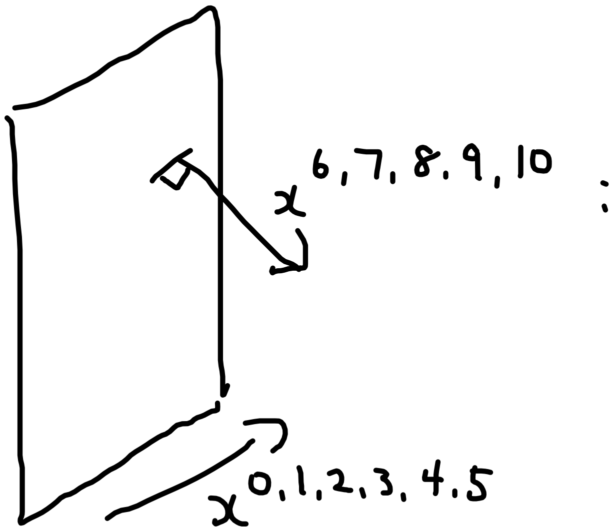
$$F_{\mu\nu}{}^{a}=\partial_{[\mu}A_{\nu]}{}^{a} + f^{a}{}_{bc}\, A_{\mu}{}^{b}A_{\nu}{}^{c}$$

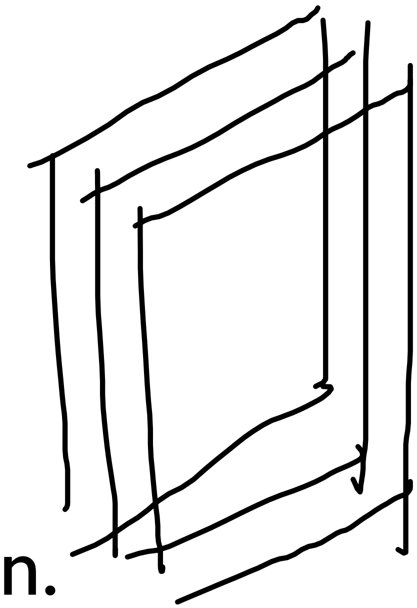
Nobody figured out how to fill the dots in

$$F_{\mu\nu\rho}{}^{a}=\partial_{[\mu}B_{\nu\rho]}{}^{a}+ \cdots$$

Why do we expect to have an interacting N=(2,0) theory anyway?

On an M5-brane, we have a free N=(2,0) multiplet.

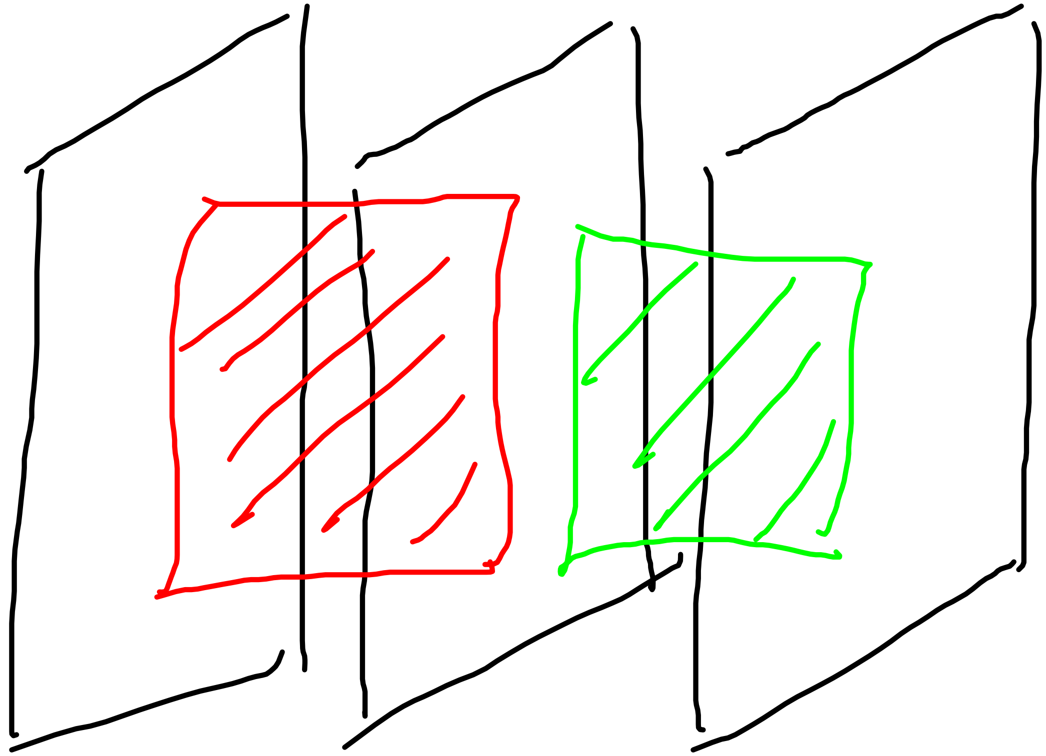




R-symmetry is the SO(5) rotation.

We should have an interacting theory on K coincident M5-branes.

Let’s study its “Coulomb phase”.



K free abelian N=(2,0) theory.

+ strings charged under them.

A particle couples to the potential as

$$q_i \int A_\mu^i \, dx^\mu$$

A string couples to the potential as

$$q_i \int B_{\mu\nu}^i \, d\sigma^{\mu\nu}$$

But we have self-duality!

$$(q_e^i, q_m^i) = (q^i, q^i)$$

In 4d, the Dirac-Schwinger-Zwanziger pairing was

$$(q_e, q_m) \circ (q_e', q_m') = q_e\, q_m' - q_m\, q_e'$$

In 6d, the pairing is

$$(q_e, q_m) \circ (q_e', q_m') = q_e\, q_m' + q_m\, q_e'$$

If you have two “self-dual” strings,

$$(q, q) \circ (q', q') = q\, q'$$

Usual Dirac quantization law requires

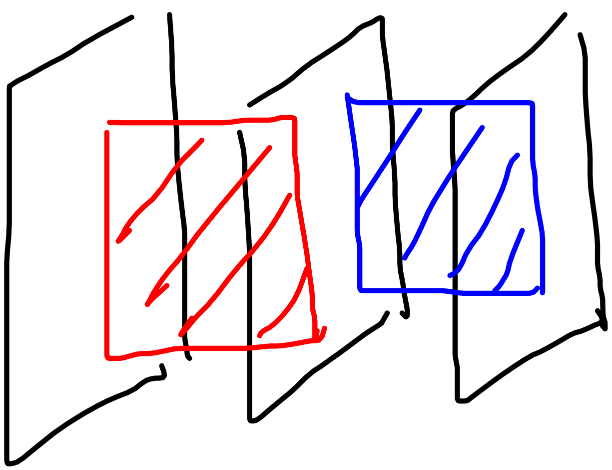
$$\vec{q} \cdot \vec{q}\,' \in \mathbb{Z}$$

[Henningson '04] also argued

$$\underbrace{\vec{q} \cdot \vec{q}}_{\text{inflow}} = \underbrace{2}_{\text{anomaly of fermion}}$$

by the anomaly cancellation on the worldsheet.

So, the charges of the strings look like simply-laced root systems: A, D, E



$$\boxed{q = (1, -1, 0)}$$

$$\boxed{q' = (0, 1, -1)}$$

All $|\alpha|^2 = 2$: Simply-laced

$$SU(N) = A_{N-1}\,,\quad SO(2N) = D_N\,,$$

$$E_6 \quad E_7 \quad E_8$$

Others : non-simply-laced

$$B_n = SO(2n+1)\,,\quad C_n = Sp(n)\,,$$

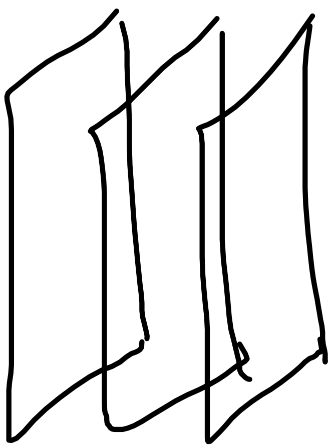
$$F_4\,,\quad G_2$$

Continuous Compact Symmetries are classified !!

How do we make them?

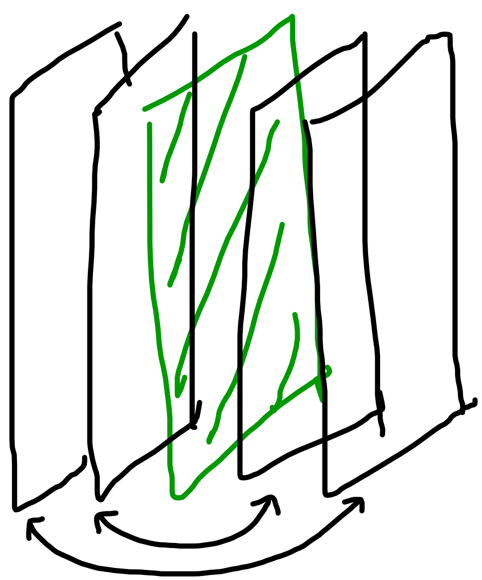
A_{K-1}

K M5s



D_K

2K M5s
+
M-orientifold



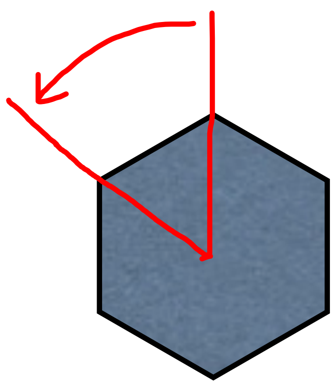
$E_{6,7,8}$

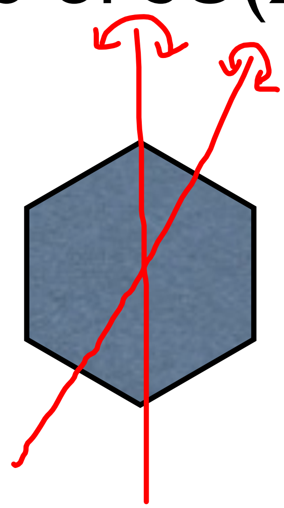
???

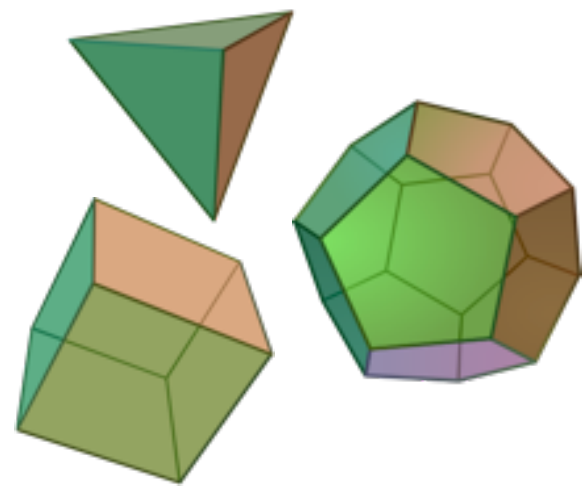
How do we make them?

	A_{K-1}	D_K	$E_{6,7,8}$
Type IIB on	$\mathbb{C}^2/\mathbb{Z}_K$	$\mathbb{C}^2/Dih_{K+1}$	$\mathbb{C}^2/$ tetra $\mathbb{C}^2/$ octa $\mathbb{C}^2/$ icosa

Note: Discrete subgroup of SU(2)~SO(3)

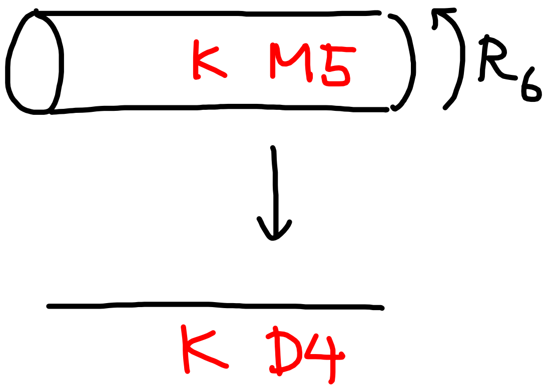






What happens when compactified on S^1 ?

6d



5d

6d $\mathcal{N}=(2,0)$ A_{K-1} th.

5d $\mathcal{N}=2$ $SU(K)$ th.

$\mu\,\nu\,\rho = 0\ 1\ 2\ 3\ 4\ ;6$

$i\,j\,k\ = 0\ 1\ 2\ 3\ 4$

$$F_{\mu\nu\rho}{}^a \to \boxed{F_{6ij}{}^a},\quad F_{\mu\nu\rho}{}^a \to \boxed{F_{ijk}{}^a}$$

remain

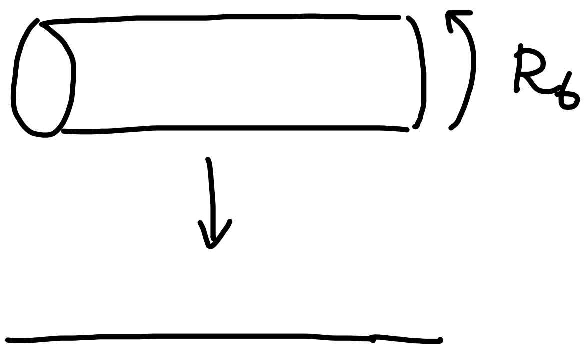
identified by

$$F_{\mu\nu\rho} = \frac{1}{6}\epsilon_{\mu\nu\rho}{}^{\sigma\tau\upsilon}F_{\sigma\tau\upsilon}$$

you can forget it

What happens when compactified on S^1 ?

6d $\qquad$ A_{K-1} th.



5d $\qquad$ $SU(K)$ gauge th.

$$S_{5d} = \int d^5x \, \frac{1}{g_{5d}^2} F_{\mu\nu}^a F_{\mu\nu}^a \quad , \quad [g_{5d}^2] = (\text{length})$$

The only scale in the setup is R_6

$$\longrightarrow \quad \frac{1}{g_{5d}^2} = \frac{1}{R_6}$$

Compactification of 6d N=(2,0) on S^1

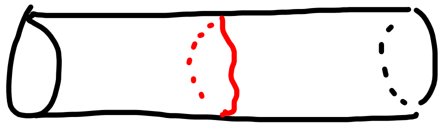
$$\int d^5x \, \frac{1}{R_6} F^a_{\mu\nu} F^a_{\mu\nu}$$

Compare this with S^1 compactification of 6d N=(1,1) theory:

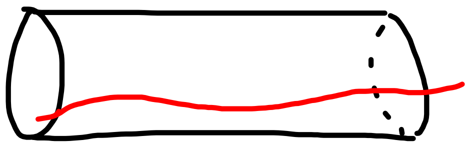
$$\int d^6x \, \frac{1}{g^2_{6d}} F^a_{\mu\nu} F^a_{\mu\nu} = \int d^5x \int dx \, \frac{1}{g^2} F^a_{\mu\nu} F^a_{\mu\nu} = \int d^5x \, \frac{R_6}{g^2_{6d}} F^a_{\mu\nu} F^a_{\mu\nu}$$

It's really weird that R_6 appears in the denominator!

What happens to the strings?



Particles in 5d ~ W-bosons



Strings in 5d ~ Monopole-strings

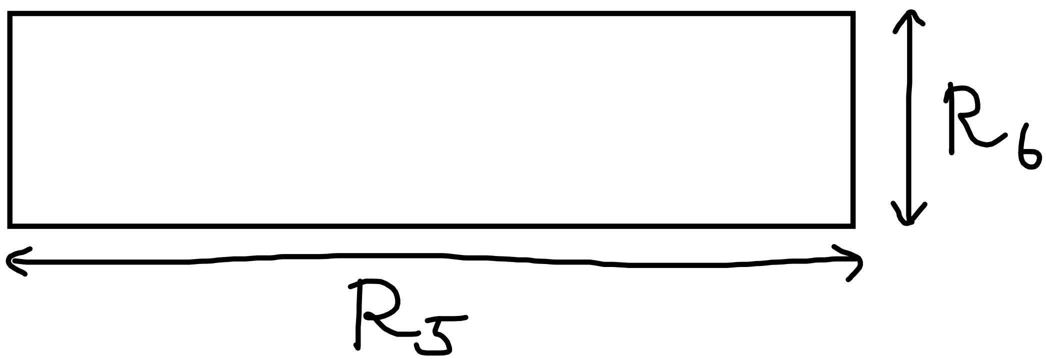
Note: take a monopole solution in 4d

$$\phi = \phi(x_1, x_2, x_3)\ , \text{ localized at } x_1 = x_2 = x_3 = 0\,.$$

and let the configuration independent of x^4

$$\phi(x_1, x_2, x_3; x_4) \equiv \phi(x_1, x_2, x_3)$$

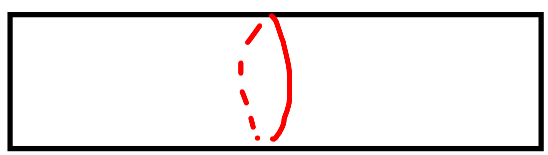
This is a string.



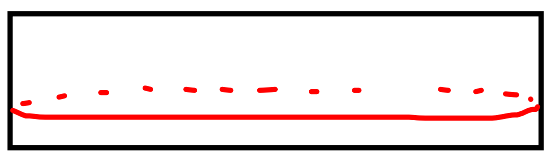
$$\int d^5x \, \frac{1}{R_6} F^a_{\mu\nu} F^a_{\mu\nu}$$

$$= \int d^4x \int dx \, \frac{1}{R_6} F^a_{\mu\nu} F^a_{\mu\nu}$$

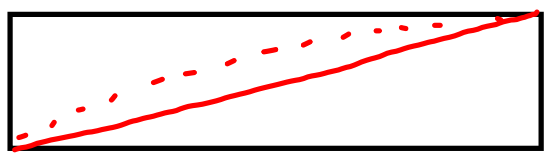
$$= \int d^4x \, \underbrace{\boxed{\frac{R_5}{R_6}}}_{= 1/g^2_{4d}} F^a_{\mu\nu} F^a_{\mu\nu}$$



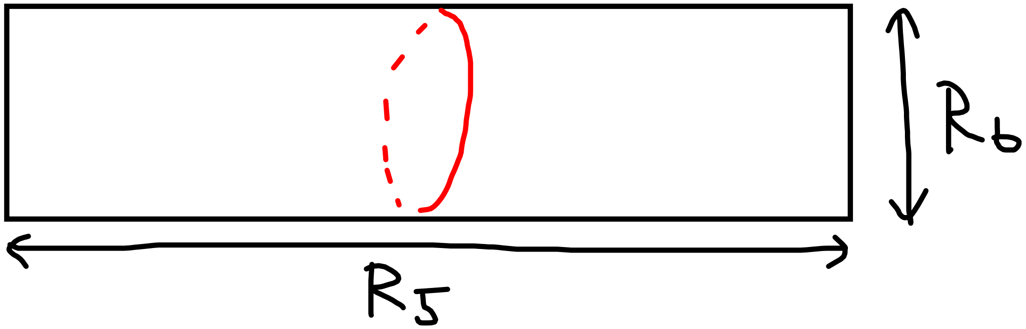
W-bosons



monopoles



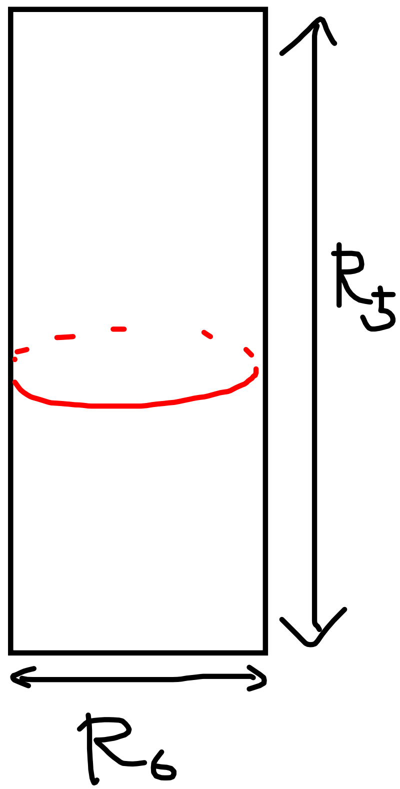
dyons



4d N=4 gauge theory with G=A,D,E

at $\frac{1}{g^2} = \frac{R_5}{R_6}$

equiv.



4d N=4 gauge theory with G=A,D,E

at $\frac{1}{g^2} = \frac{R_6}{R_5}$

For those who've heard of $\mathcal{N}=4$ S-duality for the 1st time:

$\mathcal{N}=4$ SYM has six adjoint scalars ϕ_i

$$V = \mathrm{tr}[\phi_i, \phi_j]^2$$

Set $\phi_1 = \mathrm{diag}(a_1\ a_2 \cdots a_n)$, $\phi_{2,3,4,5,6} = 0$.

W-bosons have masses from $|D_\mu \langle \phi_1 \rangle|^2$

$$\sim [A_\mu, \phi_1]^2$$

$m = |a_i - a_j|$ for the (i,j)-component.

(i, j) determines an $SU(2)$ subgroup :

$$\begin{array}{cc} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \end{array} \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Using this, we can embed standard $SU(2)$ solution into $SU(N)$.

$$m = \frac{1}{g^2} |a_i - a_j| .$$

For $SU(N)$,

W-bosons: $|a_i - a_j|$ mono: $\frac{1}{g^2}|a_i - a_j|$

For General G,

W-bosons: $|\alpha \cdot \phi|$ mono: $\frac{1}{g^2}\frac{2}{|\alpha|^2}|\alpha \cdot \phi|$

α: roots.

For $SU(N)$, $\begin{array}{rcccccccc} & & & & i & & j & & \\ \alpha_{ij} & = & (& \cdots & 1 & \cdots & -1 & \cdots & 0\,) \end{array}$

$$\phi = (a_1\, a_2 \quad \cdots \quad a_n)$$

So, if all $|\alpha|^2 = 2$, we can exchange

$$\underset{g^2}{\text{W-bosons}} \longleftrightarrow \underset{1/g^2}{\text{monopoles}}$$

All $|\alpha|^2 = 2$: Simply-laced

$$SU(N) = A_{N-1}, \quad SO(2N) = D_N,$$

$$E_6 \quad E_7 \quad E_8$$

Others : non-simply-laced

$$B_n = SO(2n+1), \quad C_n = Sp(n),$$

$$F_4, \quad G_2$$

What happens when G≠A,D,E ?

It's known that

4d N=4 gauge theory with G=$\boxed{SO(2n+1)}$ at g^2

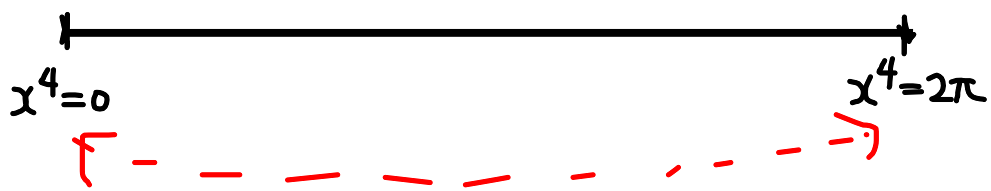
equiv.

4d N=4 gauge theory with G=$\boxed{Sp(n)}$ at $g'^2 = 1/2g^2$

So, it was good that we didn't have
6d N=(2,0) theory of type G=SO(2n+1) ...

Instead, we can use this field theory knowledge to better understand 6d N=(2,0) theory.

We can get 4d SO(2n+1) from 5d SO(2n+2):

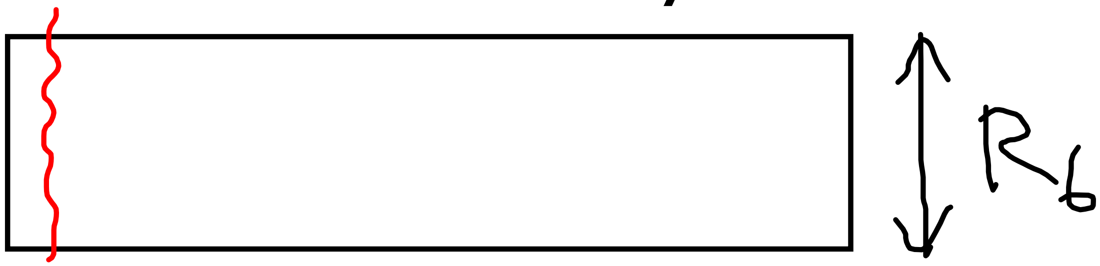


$$\phi(x^4=2\pi) = P\phi(x^4=0)P^{-1}$$

$$P = \mathrm{diag}(+1,+1,\cdots,+1,-1)$$

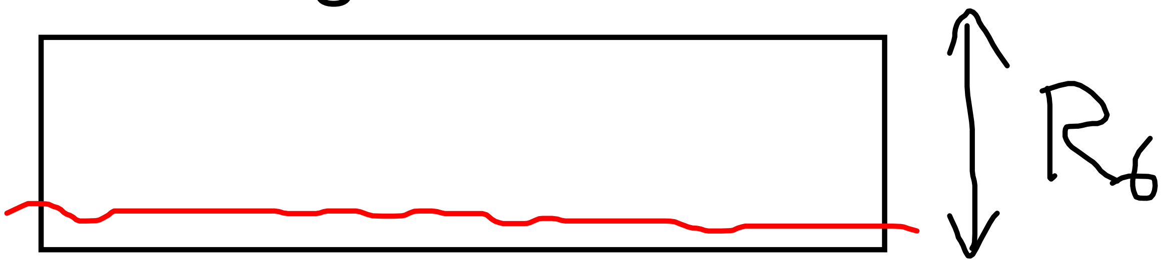
: parity of gauge group.

So, from 6d D_{n+1} theory,



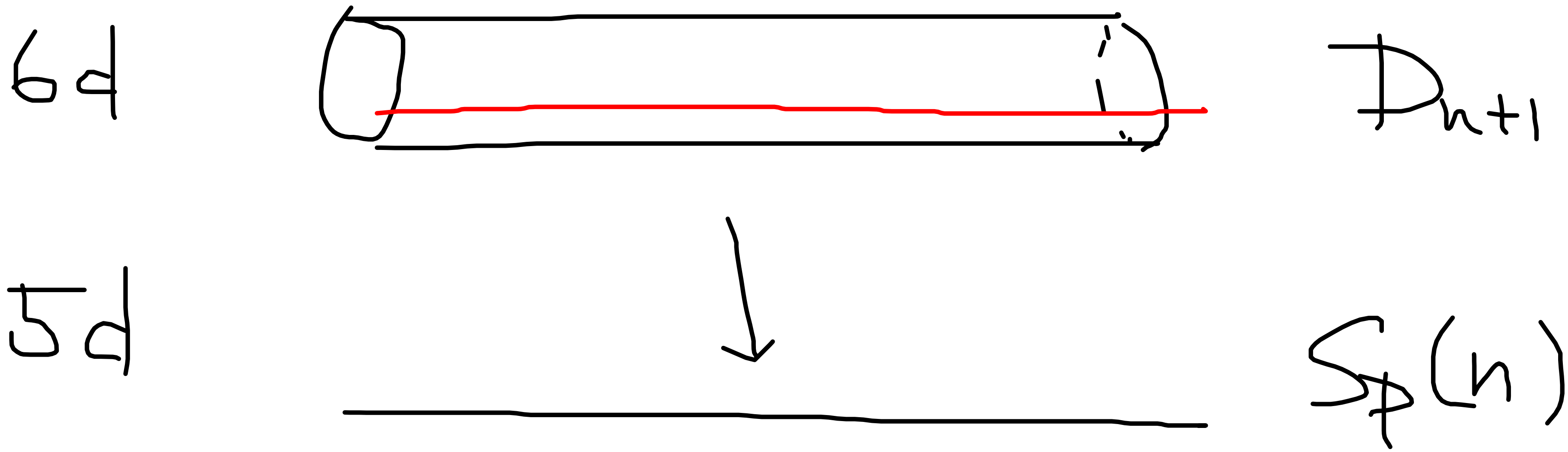
gives you 4d SO(2n+1).

S-dual configuration is this:



It should give 4d Sp(n).

Therefore, 6d N=(2,0) theory of type D_{n+1} on S^1 with the twist gives 5d Sp(n) theory.



This is crazy. Sp(n) is not a subgroup of SO(2n+2)!