M is a spin manifold of dimension 4, with $\mathbb{C}^2 \to S \to M$ the spin bundle. The gauge fields come from the principal bundle

$$SU(3) \times SU(2) \times U(1) \to P \to M,$$

and the matters are the sections of the associated bundle

$$\underline{V_{\text{boson}}} \oplus (\underline{V_{\text{fermions}}} \otimes S) \to M,$$

$$V_{\text{boson}} = \mathbf{2} \otimes V_{+1/2}, \qquad V_{\text{fermion}} = \mathbb{C}^3 \otimes V_{\text{single generation}}$$

where $V_{\text{single generation}}$ is

$$\begin{array}{cccccll}
 & \mathbf{3} & \otimes & \mathbf{2} & \otimes & V_{+1/6} & (\text{the quark doublet} \quad Q_L) \\
\oplus & \bar{\mathbf{3}} & & & \otimes & V_{-2/3} & (\text{the up-type antiquark} \quad \bar{u}_R) \\
\oplus & \bar{\mathbf{3}} & & & \otimes & V_{+1/3} & (\text{the down-type antiquark} \quad \bar{d}_R) \\
\oplus & & & \mathbf{2} & \otimes & V_{-1/2} & (\text{the lepton doublet} \quad \ell_L) \\
\oplus & & & & \otimes & V_{+1} & (\text{the charged anti-lepton} \quad \bar{e}_R) \\
\oplus & & & & & \mathbb{C} & (\text{the right-handed neutrino} \quad \bar{\nu}_R).
\end{array}$$

where $\mathbf{3}$, $\mathbf{2}$ are the standard representations of $SU(3)$ and $SU(2)$, and $U(1) \curvearrowright V_q \simeq \mathbb{C}$ is via $z \cdot v = z^{6q} v$ for $z \in U(1)$, $v \in V_q$.

Table 1: The bundle content of the Standard Model

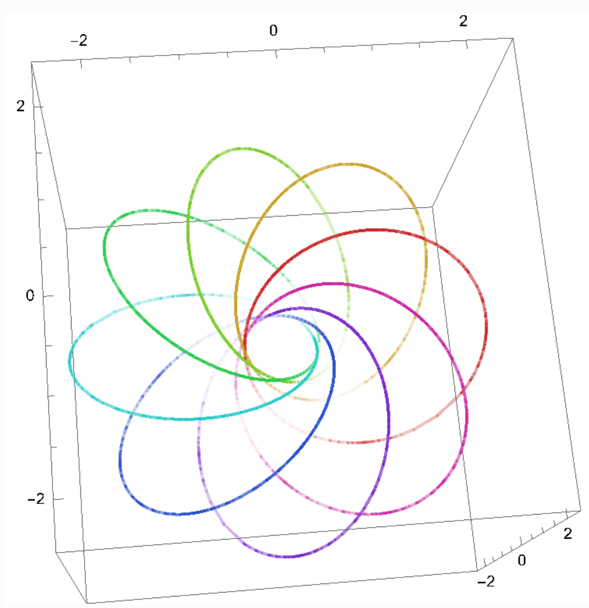

Figure 1: Fibers of the Hopf fibration $S^3 \to S^2$, drawn after a stereographic projection from $(S^3 \setminus \text{north pole}) \to \mathbb{R}^3$. Here the fibers over $(\cos 2\pi k/8, \sin 2\pi k/8, 0)$ for $k = 0, \ldots, 7$ are shown.

k	0	1	2	3	4	5	6	7	8	9
S^1	$\mathbb{Z}$	0	0	0	0	0	0	0	0	0
S^2	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{12}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$
S^3	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{12}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$	$(\mathbb{Z}_2)^2$
S^4	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_{12}$	$(\mathbb{Z}_2)^2$	$(\mathbb{Z}_2)^2$	$\mathbb{Z}_{24} \times \mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$	$(\mathbb{Z}_2)^3$
S^5	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{30}$	$\mathbb{Z}_2$	$(\mathbb{Z}_2)^3$
S^6	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_{60}$	$\mathbb{Z}_{24} \times \mathbb{Z}_2$	$(\mathbb{Z}_2)^3$
S^7	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_{120}$	$(\mathbb{Z}_2)^3$	$(\mathbb{Z}_2)^4$
S^8	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_{120}$	$(\mathbb{Z}_2)^4$	$(\mathbb{Z}_2)^5$
S^9	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_{240}$	$(\mathbb{Z}_2)^3$	$(\mathbb{Z}_2)^4$
S^{10}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_{240}$	$(\mathbb{Z}_2)^2$	$\mathbb{Z} \times (\mathbb{Z}_2)^3$
S^{11}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_{240}$	$(\mathbb{Z}_2)^2$	$(\mathbb{Z}_2)^3$
S^{12}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_{240}$	$(\mathbb{Z}_2)^2$	$(\mathbb{Z}_2)^3$

Table 2: Table of $\pi_{n+k}(S^n)$

$G \setminus d$	2	3	4	5	6	7	8	9	10	11
$Sp(1)$	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{12}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$
$Sp(2)$	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	$\mathbb{Z}_{120}$	$\mathbb{Z}_2$
$Sp(3)$	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
$SU(3)$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\mathbb{Z}_6$	0	$\mathbb{Z}_{12}$	$\mathbb{Z}_3$	$\mathbb{Z}_{30}$	$\mathbb{Z}_4$
$SU(4)$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\mathbb{Z}_{24}$	$\mathbb{Z}_2$	$\mathbb{Z}_{120} \times \mathbb{Z}_2$	$\mathbb{Z}_4$
$SU(5)$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\mathbb{Z}_{120}$	0
$SU(6)$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
$Spin(7)$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$(\mathbb{Z}_2)^2$	$(\mathbb{Z}_2)^2$	$\mathbb{Z}_8$	$\mathbb{Z} \times \mathbb{Z}_2$
$Spin(8)$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}^2$	$(\mathbb{Z}_2)^3$	$(\mathbb{Z}_2)^3$	$\mathbb{Z}_{24} \times \mathbb{Z}_8$	$\mathbb{Z} \times \mathbb{Z}_2$
$Spin(9)$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$(\mathbb{Z}_2)^2$	$(\mathbb{Z}_2)^2$	$\mathbb{Z}_8$	$\mathbb{Z} \times \mathbb{Z}_2$
$Spin(10)$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}$
$Spin(11)$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$Spin(12)$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z} \times \mathbb{Z}$
$Spin(13)$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$
G_2	0	$\mathbb{Z}$	0	0	$\mathbb{Z}_3$	0	$\mathbb{Z}_2$	$\mathbb{Z}_6$	0	$\mathbb{Z} \times \mathbb{Z}_2$
F_4	0	$\mathbb{Z}$	0	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z} \times \mathbb{Z}_2$
E_6	0	$\mathbb{Z}$	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$
E_7	0	$\mathbb{Z}$	0	0	0	0	0	0	0	$\mathbb{Z}$
E_8	0	$\mathbb{Z}$	0	0	0	0	0	0	0	0

Table 3: Homotopy groups of simply-connected simple Lie groups $\pi_d(G)$, $2 \leq d \leq 11$.