QUANTITATIVE TESTS OF MALDACENA'S CONJECTURE

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Based partly on:

OA, Oz, Yin hep-th/9803051
OA, Fayyazuddin, Maldacena hep-th/9806159
work in progress with Kachru, Silverstein

and many other papers...

Discussion will be limited to:

* Field theory aspects
* Quantitative tests
* 2+1 dimensions or more; CFTs

Maldacena's conjecture:

IIB string theory on $AdS_5 \times S^5$ $\simeq$ $\mathcal{N}=4$ $d=4$ $U(N)$ SYM

$R \sim (g_{YM}^2 N)^{1/4} \sqrt{\alpha'} \sim N^{1/4} \ell_p$

$g_{st} \sim g_{YM}^2 \quad (\tau_{IIB} = \tau_{YM})$

M on $AdS_7 \times S^4 \simeq (2,0)$ $d=6$ A_{N-1} SCFT

$R \sim N^{1/3} \ell_p$

M on $AdS_4 \times S^7 \simeq \mathcal{N}=8$ $d=3$ A_{N-1} SCFT
(IR limit of U(N) SYM)

$R \sim N^{1/6} \ell_p$

Derivation: start from p-branes as

solitonic solutions of SUGRA (string/M theory) $\simeq$ D-branes: open strings coupled to closed bulk strings

and take $\ell_s(\ell_p) \to 0$.

Works for any N, g_s but more convincing for large N, $g_s N$ (left hand side = SUGRA).

Possibilities:

1) Equivalence for all N, g_s (strong)

2) Equivalence for $N \to \infty$ with fixed $g_s N$, 't Hooft limit (weak)

3) Equivalence only for large N, $g_s N$.

Would be nice to have direct tests, but unfortunately almost all tests check only 3).

To compare field theories need AdS → CFT:

Boundary values of fields on AdS ↔ couplings of CFT operators

Gubser, Klebanov, Polyakov
Witten

$$\langle e^{\mathcal{L}_{CFT} + \int \phi_0^I \mathcal{O}^I} \rangle = \langle e^{W} \Big|_{\phi^I \to \phi_0^I U^{\lambda_I}} \rangle$$

$$\left(ds^2 \simeq \frac{dU^2}{U^2} + U^2 dx^2\right)$$

λ_I is determined by $m(\phi^I)$:

$$\lambda_I = -\frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m_I^2} \leftrightarrow \Delta(\mathcal{O}^I) = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m_I^2}$$

Mass spectrum on AdS ↔ spectrum of dimensions in CFT.

Q: Are all operators and correlation functions the same?

Problem: different regimes of computability for most quantities.

Can test things that do not depend on $g_s N$, but usually (on string/M theory side) only for large N (SUGRA limit): test only very weak conjecture.

Examples:

1) Symmetries: (Maldacena)

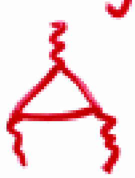
$d=4$ $\quad SO(2,4)\times SO(6) \subset SU(2,2|4)$

$\qquad\quad SL(2,\mathbb{Z})$

$d=3$ $\quad SO(2,3)\times SO(8) \subset OSp(8|4,R)$

$d=6$ $\quad SO(2,6)\times SO(5) \subset OSp(8^*|4)$

2) Anomaly matching: (Witten; Freedman et al)

$d=4$ $\quad SO(6)^3$ $\quad$  $\quad \leftrightarrow$ CS term in SUGRA

$\qquad\quad \langle JJJ \rangle$

$$N^2(N^2-1) \quad \simeq \quad N^2$$

$d=6$ $\quad SO(5)^4$ $\quad ?$

Similarly, can compare other 2- and 3-point functions that do not depend on $g_s N$.

3) Large N matching of chiral operators (described below).

$d=4$: Horowitz + Ooguri; Witten; Ferrara, Fronsdal, Zaffaroni; Andrianopoli, Ferrara; Witten

$d=3,6$: OA, Oz, Yin; Minwalla; Leigh, Rozali; Halyo

All test very weak conjecture, and seem to be guaranteed to work from derivation (e.g. chiral operators = couplings to bulk fields around the brane).

Matching of chiral operators $(d=3,4,6)$:

Field theory: (from DLCQ in $d=6$,
from UV SYM in $d=3$)

Chiral operators are small multiplets with spin 0–2 (up to $Q^4\bar{Q}^4$ and not $Q^8\bar{Q}^8$).

Lowest component = real scalar in $\overbrace{\square\square\square\cdots\square}^{k}$ representation of $SO(8,6,5)$ R-symmetry, $k=(1),2,3,\cdots,N$.

For $d=4$ it is $tr(X^{I_1}\cdots X^{I_k})$ where X = adjoint scalar in vector multiplet, contractions = non-chiral commutators = descendants.

All other chiral fields are descendants of this, including spin 2 field in same reps.

IIB string theory / M theory: identified with low-energy SUGRA modes, spin 0–2.

Example: spin 2 fields on AdS come from graviton, $h_{\mu\nu}(x,y) = \sum_{k=0}^{\infty} \underset{AdS}{h^k_{\mu\nu}(x)}\, \underset{S}{Y^k(y)}$,

Y^k are in $\overbrace{\square\square\square\cdots\square}^{k}$ reps $\rightarrow$ spin 2 fields match $\xrightarrow{SUSY}$ all fields match.

* Field theory spectrum truncates at $k=N$, $\Delta=N \rightarrow m^2 \sim N^2/R^2 >> M_p^2 \rightarrow$ cannot be understood from SUGRA, but required for finite N matching. Similar issues for AdS_3.

* Matching of most operators for orbifolds/orientifolds is similar, with same truncations on both sides.

New states also exist for orientifolds: for dual of $SO(2N)$ Witten

Pfaffian $\epsilon^{\alpha_1 \cdots \alpha_{2N}} X^{i_1}_{\alpha_1 \alpha_2} \cdots X^{i_N}_{\alpha_{2N-1} \alpha_{2N}} \leftrightarrow$ 3-brane wrapped on 3-cycle in S^5/Z_2.

Operator with $\Delta=N$: test of finite N duality!

Similar wrapped membrane (fivebrane) exists for dual of $d=6$ $(d=3)$ D_N theory.

Generalization to $\mathcal{N}=2$: Fayyazuddin + Spalinski
OA, Maldacena, Fayyazuddin

Similar test for $\mathcal{N}=2$ SCFTs of 3-branes at 7-brane singularities:

more elaborate (susy less restrictive) but still weak test.

Conformal theories arise for 3-branes at $G = H_0, H_1, H_2, D_4, E_6, E_7, E_8$ 7-branes (singularities in F-theory on K3). Sen
Banks + Douglas + Seiberg
Dasgupta + Mukhi

7-brane metric:

$$ds^2 = \frac{dz^2}{|z|^\alpha} + dx_i dx_i \quad (\alpha = \tfrac{1}{3}, \tfrac{1}{2}, \tfrac{2}{3}, 1, \tfrac{4}{3}, \tfrac{3}{2}, \tfrac{5}{3})$$

Metric transverse to 3-brane probe can be written as $ds^2 = dr^2 + r^2 d\tilde{\Omega}_5^2$

$$d\tilde{\Omega}_5^2 = d\theta^2 + \sin^2(\theta)\, d\phi^2 + \cos^2(\theta)\, d\Omega_3^2$$

where ϕ has periodicity $2\pi(1-\frac{\alpha}{2})$, incorporating deficit angle (also monodromies for some fields in ϕ).

Can compute metric for N 3-branes and take near-horizon limit:

find IIB string theory on $AdS_5 \times \tilde{S}^5$, a background with a G-7-brane wrapped on $S^3 \times AdS_5$.

Conjecture: this is equivalent to the corresponding $\mathcal{N}=2$ SCFTs.

Compare chiral operators:

Low-energy states now come both from reduction of SUGRA fields on $\tilde{S}^5$, and from reduction of 7-brane fields on S^3.

Originally lowest components of SUGRA multiplets had $\Delta = K$: changed periodicity leads to operators with $\Delta = K/(1-\frac{\alpha}{2\pi})$; $K = 1, 2, \ldots$ which can be identified with the Coulomb branch coordinates of the corresponding theories.

In D_4 case can compare all operators to $\mathcal{N}=2$ $USp(2N)$ + antisymmetric 4 + 4 fundamental hypermultiplets q. OA, Sonnenschein, Theisen, Yankielowicz; Douglas, Lowe, Schwarz

For instance, 7-brane fields have spin 0-1 : all in small multiplets of $\mathcal{N}=2$ SCA. Can identify their KK modes on S^3 with field theory chiral multiplets, whose lowest component is $q_i Y^k q_{j}$; $k=0,1,2,\ldots$ (for $k=0$ the $Q\bar{Q}$ component is the global symmetry current).

Get another large N test + predictions for large N spectrum of many theories ($\mathcal{N}=0,1,2$) arising from 3-branes at 7-brane singularities, most of which have no Lagrangian description.

A possible test whose success does not seem to be guaranteed from the brane derivation: comparison of exactly marginal deformations of $\mathcal{N}=4, d=4$ SYM.
(Work in progress with Kachru, Silverstein)

Field theory: can prove (Leigh+Strassler) that the chiral deformation

$$W = h_1 \, \mathrm{tr}(\phi_1^3 + \phi_2^3 + \phi_3^3) + h_2 \, \mathrm{tr}(\phi_1 \phi_2 \phi_3 + \phi_1 \phi_3 \phi_2)$$

is exactly marginal $\rightarrow$ 3 dimensional surface of $\mathcal{N}=1$ SCFTs parametrized by g, h_1, h_2.

String theory (SUGRA):

Marginal deformation = solution with non-zero boundary values for some of the massless fields (B_{ab} in this case).

Exactly marginal = solution which doesn't depend on AdS coordinates (preserves $SO(2,4)$).

Field theory argument here is for finite N, so this would test at least the finite N duality. Result – TBA.