

String Expansion as Large N Expansion

Based on joint work with
— Z. Kakushadze & C. Vafa
— A. Johansen

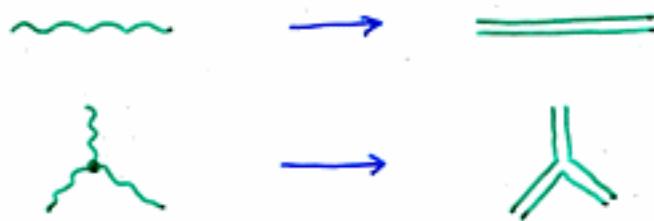
I will describe CONFORMAL field theories
(conformal in the limit $N \rightarrow \infty$) constructed
by orbifoldization of $N=4$ SUSY YM.

Moreover, all corr. functions in the reduced
theories are identical to corr. functions of
 $N=4$ (modulo gauge coupling rescaling)
in the large N limit

Historically, the construction was inspired by
AdS, but completely independent of it.

Old story of t'Hooft

In any F.T. with adjoints (in the large N limit)
 it is natural to represent Feynman diagrams
 using "fat" graphs



Any Feynman diagram can be represented as something like this:



planar

or



non planar

Each diagram contributes a factor

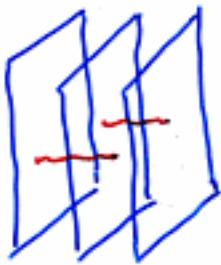
$$(N g_{\text{YM}}^2)^{2g-2+b} N^{2-2g}$$

where $b \leftarrow \#$ of boundary components

$g \leftarrow$ genus of the diagram

These "fat" diagrams can be reinterpreted from the point of view of string theory

Consider N parallel D3-branes



Collection of D3 branes has an effective description as $SU(N) \times U(1)$ SUSY gauge theory with

$$g_{YM}^2 = \lambda_s$$

Each string diagram is weighted by a factor

$$(N\lambda_s)^b \lambda_s^{2g-2} = \lambda^{2g-2+b} N^{2-2g}$$

λ_s ← IIB string coupling

$$\lambda = N\lambda_s = Ng_{YM}^2 \quad (\text{we keep } \lambda \text{ fixed as } N \rightarrow \infty)$$

b ← # of boundary components

g ← genus of the diagram

One can use D-brane/string perturbation technique to analyze gauge theory diagrams.

Efficient way to organize Feynman diagrams!

Keeping $\lambda = N g_{YM}^2$ fixed one can see that the planar diagrams dominate $N \rightarrow \infty$ limit

The YM perturbation expansion is valid for

$$\lambda < 1$$

On the other hand the case

$$\lambda > 1 \quad (\text{provided that } l_S \rightarrow 0, \dots)$$

can be effectively described by $SU(2)$ background $AdS_5 \times S_5$.

$AdS_5 \times S_5$ "sums" the t'Hooft expansion
 $\lambda^{2g-2+b} N^{2-2g}$ into the domain $\lambda > 1$.

Conformal $N=4$ SUSY YM lives on the boundary of AdS and serves as a boundary conditions for supergravity.

Fubser, Klebanov

Maldacena

Gubser, Klebanov, Polyakov
 Witten

:

What about theories with less SUSY ?

It is possible to reduce # of SUSY using
ORBIFOLD construction.

If the orbifold group acts only on S_5 keeping
 $\text{Ad}S_5$ intact, one should expect to get the
Conformal theory.

Kachru, Silverstein
Lawrence, Nekrasov, Vafa

Let us go back to string perturbative description –
– collection of D3-branes.

Transversal dir $\mathbb{R}^6 = \mathbb{C}^3$

R symmetry of $N=4$: $SU(4) = Spin(8)$

Orbifold action $\Gamma : \mathbb{C}^3 \rightarrow \mathbb{C}^3$

① $\Gamma \subset SU(2)$ $N=2$ is unbroken

② $\Gamma \subset SU(3)$ $N=1$ is unbroken

③ Γ is generic SUSY is broken

Orbifold action

$$g \in \Gamma \quad x^i \sim g^i_j x^j \quad (\vec{x} \leftarrow \text{transversal coord.})$$

$\gamma_g : \text{representation of } \Gamma \text{ into the gauge group.}$

γ_g determines the group action on Chan-Paton indices.

Massless spectrum:

① vectors $A = \gamma A \gamma^{-1}$

② Scalars $\phi_i = \gamma (g_i^j \phi_j) \gamma^{-1}$

③ fermions ...

The group action can be summarized by certain quiver diagram.

Douglas, Moore

In order to have a CONSISTENT string theory
 the representation $\gamma \rightarrow \gamma_g$ should satisfy some
 conditions.

Requirements: no tadpoles, UV finiteness.

\Rightarrow Constraints on γ_g

For example, consider vacuum amplitude

$$Z = \frac{1}{2|\Gamma|} \sum_{g \in \Gamma} \text{Tr} \left(\left. g (1 + (-1)^F) e^{-2\pi t L_0} \right|_{\text{open}} \right)$$



At every boundary one has
 to specify b.c.

The twist g correspond to $\gamma_g \otimes \gamma_g$ acting on Chan-Paton indices.

As the result each individual term is proportional
 to $(\text{tr } \gamma_g)^2 Z_g$

Now let us examine divergencies in Z_g

Let d_g be dim of locus, fixed by g action.

$$Z_g = A_g^{NS} - B_g^{RR}$$

$$A_g^{NS} = (\text{Tr } \gamma_g)^2 \int \frac{dt}{t^3} \left(\frac{1}{\gamma} \right)^{2+d_g} \chi = \\ = (\text{Tr } \gamma_g)^2 \int \frac{d\ell}{\ell^{d/2}} \sum A_n e^{-2\pi \ell \Delta_n}$$

$$B_g^{RR} = \dots = (\text{Tr } \gamma_g)^2 \int \frac{d\ell}{\ell^{d/2}} \sum B_n e^{-2\pi \ell \tilde{\Delta}_n}$$

These integrals could be divergent (as $\ell \rightarrow \infty$) due to

- i) massless tadpoles
- ii) tachyons

$\Delta_n, \tilde{\Delta}_n \leftarrow$ Spectrum of states in NS or RR closed string sectors.

Divergencies as $\ell \rightarrow \infty$

for $A(\text{or } \tilde{\Delta}) = 0 \quad \& \quad d \leq 2$
 or $\Delta(\text{or } \tilde{\Delta}) < 0$

First of All

$$\Gamma \subset \{ \text{SU}(2) \\ \text{SU}(3) \\ \text{SU}(4) \}$$

possible values of $d = 0, 2, 4, 6$.

The ground state of RR sector is massless.

i) For $d_g = 0, 2$ the B_g is divergent

$$\Rightarrow \boxed{\text{Tr } Y_g = 0}$$

ii) For $d_g = 4$ the NS sector contains tachyon

$$\Rightarrow \boxed{\text{Tr } Y_g = 0}$$

iii) $d_g = 6$ only for one element $g = I$
and in this case there are no divergencies.

Conclusion We need such representations $g \rightarrow Y_g$

that

$$\boxed{\text{Tr } Y_g = 0}$$

(Regular representations!).

Let me show the importance of this constraint.

$$\Gamma = \mathbb{Z}_3 = \{1, \omega, \omega^2\} \subset \mathrm{SU}(3)$$

one expects to get $N=1$ YM.

YM group is $\mathrm{SU}(3N)$

$$\gamma_1 = I_{3N} \quad \gamma_\omega = \begin{bmatrix} I_N & & \\ & \omega I_N & \\ & & \omega^2 I_N \end{bmatrix}$$

YM group is reduced to $G = \mathrm{SU}(N) \times \mathrm{SU}(N) \times \mathrm{SU}(N) \times \dots$

Matter: $3(N, \bar{N}, 1) + 3(1, N, \bar{N}) + 3(\bar{N}, 1, N)$.

Suppose we choose a different Rep. Let $N_1 + N_2 + N_3 = 3N$

$$\gamma_1 = I_{3N} \quad \gamma_\omega = \begin{bmatrix} I_{N_1} & & \\ & \omega I_{N_2} & \\ & & \omega^2 I_{N_3} \end{bmatrix} \quad \begin{array}{l} \text{if } N_1 \neq N_2 \neq N_3 \\ \text{Tr } \gamma \neq 0. \end{array}$$

After Reduction YM group $G = \mathrm{SU}(N_1) \times \mathrm{SU}(N_2) \times \mathrm{SU}(N_3) \times \dots$

Matter: $3(N_1, \bar{N}_2, 1) + 3(1, N_2, \bar{N}_3) + 3(\bar{N}, 1, N_3)$

→ ANOMALOUS spectrum !

Back to our original problem

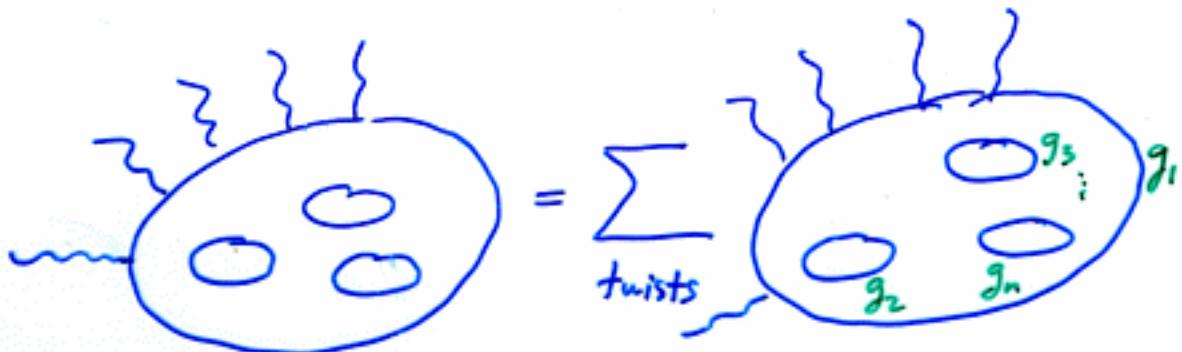
Correlation functions in $N \rightarrow \infty$ limit.

Let us use string perturbation technique to analyze large N YM diagrams

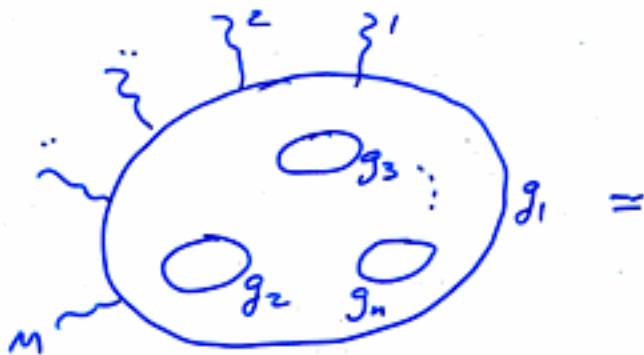
Leading diagrams

- 1) Planar / Sphere with "holes" = loops.
- 2) All external lines are attached to the same boundary

One has to sum over all possible twists



Each term



$$= \text{Tr}(\lambda_1 \lambda_2 \dots \lambda_m \delta_1) (\text{Tr} \delta_2) \dots (\text{Tr} \delta_n) Z(g_1, \dots, g_n).$$

In order the contribution to be $\neq 0$ all $\delta_2, \delta_3, \dots, \delta_n \equiv I$

Therefore $g_1 = I$ & hence this is just the old

N=4 computation.

To be a little precise, one also has to rescale the gauge coupling

$$\frac{1}{g^2} \rightarrow \frac{1}{|r|} \frac{1}{g^2}$$

Remarkable conclusion

In $N \rightarrow \infty$ limit all correlation functions in ORBIFOLD theories coincide with $N=4$ corr. functions.

$$\langle V_1 \dots V_M \rangle_{N \rightarrow \infty}^{N=2,1,0} \sim \langle V_1 \dots V_M \rangle_{N \rightarrow \infty}^{N=4}$$

For example, all momenta dependencies of colliding particles should be the same.

As a byproduct, we also proved that β -functions $\equiv 0$ (in $N \rightarrow \infty$) limit.

What about Y_N corrections?

Correlation functions clearly get corrections.

The orbifold theory is not just subsector of $N=4$

For finite N $N=1$ theory is not conformal

$$\beta_s = \mathcal{O}(\gamma_N)$$

But there are still enough parameters to fine tune to make the theory conformal.

For $N=0$ the theory is not conformal and it is unclear whether one can fine tune the parameters to make it conformal.

There are certain generalizations of this construction based on D-branes and orientifold planes

($N=4$ $SO(N)$ or $Sp(N)$ YM).

In this case $\text{Tr } \gamma_g = \mathcal{O}(1)$ Z. Kakushadze

leading to $\beta_s \simeq \mathcal{O}(\gamma_N)$

Coming back to AdS

It could be interesting to see what conformal field theories other SU(2)R solutions in the form

$$\text{AdS} \times X$$

correspond to.

A. Polyakov

In order to describe non-conformal F.T one has to understand a non-critical string theory.

Probably, one can find the examples of these non-critical strings that are exactly solvable.