6d SUPERGRAVITY ON $AdS_3 \times S^3$ AND 2d CFT

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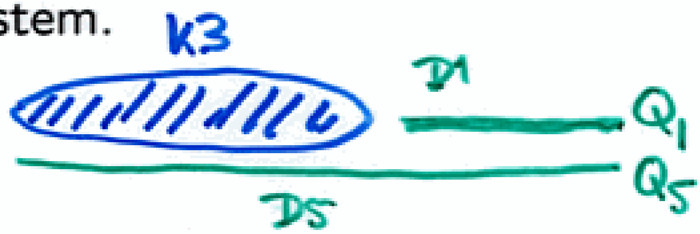
Motivation

- Test $AdS \leftrightarrow$CFT conjecture, how does supergravity account for the complete CFT spectrum, which aspects of black holes can be seen in the supergravity description?

- Which CFT's are obtained by putting some arbitrary 6d supergravity on $S^3 \times AdS_3$?

First look at IIB on $K3 \times S^3 \times AdS_3$. This is the near horizon geometry of a D1-D5 brane system.



$$
\begin{aligned}
\frac{ds^2}{\alpha'} &= \frac{U^2}{\ell^2}(-dt^2 + (dx^5)^2) + \frac{\ell^2}{U^2}dU^2 \\
&\quad + \ell^2 d\Omega_3^2 \\
&\quad + \sqrt{\frac{Q_1}{Q_5}} ds_{K3}^2
\end{aligned}
$$

where $\ell^2 = g_6\sqrt{N}$, $N = Q_1 Q_5$.

Supergravity limit: $g_6 \to 0$, $g_6 Q_i$ large and fixed.

KK-states: $m^2\alpha' \sim 1/\ell^2$.

Massive string states: $m^2\alpha' \sim 1$.

Note: vol($K3$) $\ll$ vol(S^3), vol(AdS_3) $\Longrightarrow$ can use 6d supergravity.

Maldacena, Strominger, hep-th/9804085; Martinec, hep-th/9804111

To compute KK spectrum of supergravity use representation theory. The three-sphere S^3 is invariant under the $SO(4)_I$ isometry group. First find the set of $SO(4)_I$ quantum numbers, then deduce the CFT spectrum from this information.

$$\phi_{d=6} = \sum_{r,R} \psi_r^R(x) \chi_r^R(y)$$

where $x \in S^3$, $y \in AdS_3$, r is a representation of $SO(3)$, the local Lorentz group of S^3, and R is a representation of $SO(4)_R$.

Two basic facts:

- Only those R appear that yield r when decomposing under $SO(4)_I \supset SO(3)$.

  Salam, Strathdee

- If ϕ transforms in the representation S of the little group $SO(4)_l$, the sum is over those r that appear in the decomposition of S under $SO(4)_l \supset SO(3)$.

Example: vector field in 6d transforms as $\mathbf{4}$ under $SO(4)_l$. Then $r = \mathbf{1}$ or $r = \mathbf{3}$.

$r = \mathbf{1} \Longrightarrow R = (\mathbf{m}, \mathbf{m})$, $m \geq 1$.

$r = \mathbf{3} \Longrightarrow R = (\mathbf{m}, \mathbf{m})$, $m \geq 2$, $R = (\mathbf{m}, \mathbf{m+2})$, $m \geq 1$, or $R = (\mathbf{m+2}, \mathbf{m})$, $m \geq 1$.

IIB on $K3$ yields 6d $(2,0)$ supergravity with $n_T = 21$ tensormultiplets. Get

$$
\begin{aligned}
&(5n_T + 1)(\mathbf{1},\mathbf{1}) \\
&+(4n_T + 4)((\mathbf{1},\mathbf{2}) + (\mathbf{2},\mathbf{1})) \\
&+(n_T + 6)((\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})) \\
&+(6n_T + 7)(\mathbf{2},\mathbf{2}) \\
&+\oplus_{m\geq 1}((\mathbf{m},\mathbf{m+4}) + (\mathbf{m+4},\mathbf{m})) \\
&+\oplus_{m\geq 1} 4((\mathbf{m},\mathbf{m+3}) + (\mathbf{m+3},\mathbf{m})) \\
&+\oplus_{m\geq 1}(n_T + 7)((\mathbf{m},\mathbf{m+2}) + (\mathbf{m+2},\mathbf{m})) \\
&+\oplus_{m\geq 2}(4n_T + 8)((\mathbf{m},\mathbf{m+1}) + (\mathbf{m+1},\mathbf{m})) \\
&+\oplus_{m\geq 3}(6n_T + 8)(\mathbf{m},\mathbf{m}))
\end{aligned}
$$

Organize representations in representation of the AdS supergroup $SU(1,1|2)_L \times SU(1,1|2)_R$. $SU(1,1|2) \supset SU(2) \times SL(2,\mathbf{R})$, and is generated by $\{L_{\pm 1}, L_0, G^i_{\pm 1/2}, J^i_0\}$.

Short representation $(\mathbf{m})_S$ of $SU(1,1|2)$ contains one chiral primary

$SU(2)$	L_0
$\mathbf{m}$	$(m-1)/2$
$\mathbf{m-1}$	$m/2$
$\mathbf{m-1}$	$m/2$
$\mathbf{m-2}$	$(m+1)/2$

Rewrite KK spectrum in terms of short representations $(\mathbf{m},\mathbf{m'})_S$ of $SU(1,1|2)_L \times SU(1,1|2)_R$.

Result:

$(\mathbf{1},\mathbf{3})_S$, $(\mathbf{2},\mathbf{4})_S, \ldots$
$n_T(\mathbf{2},\mathbf{2})_S$, $(n_T+1)(\mathbf{3},\mathbf{3})_S$, $(n_T+1)(\mathbf{4},\mathbf{4})_S$, $\ldots$
$(\mathbf{3},\mathbf{1})_S$, $(\mathbf{4},\mathbf{2})_S, \ldots$

The representations $(\mathbf{1},\mathbf{3})_S$ and $(\mathbf{3},\mathbf{1})_S$ contain the generators of the $N=(4,4)$ algebra.

They are "singletons", appearing in the boundary condition of the gauge field of the $SU(1,1|2)_L\times$ $SU(1,1|2)_R$ Chern-Simons theory on AdS_3.

Brown,Henneaux

Chiral algebra on boundary is a Hamiltonian reduction of the current algebra based on the anti-de Sitter group $G_L \times G_R$.

Conformal field theory is a deformation of $K3^N/S_N$ orbifold theory. Vafa, hep-th/9511088; Strominger, Vafa, hep-th/9601029

Chiral primaries are in one-to-one correspondence with the cohomology of $K3^N/S_N$. For $N \to \infty$, cohomology ring is freely generated by classes of degree

(0, 2), (1, 3), ...
21(1, 1), 22(2, 2), 22(3, 3), ...
(2, 0), (3, 1), ...

Perfect agreement: multiparticle states in supergravity yield all chiral primaries.

Much more detailed check: elliptic genus, invariant under marginal deformations.

$$Z(\tau, z) = \mathrm{Tr}_{RR}(-1)^F q^{L_0 - c/24} \bar{q}^{\bar{L}_0 - c/24} y^{J_0}$$

For $K3$,

$$\begin{aligned} Z(\tau, z) &\equiv \sum_{m,l} c(m,l) q^m y^l \\ &= 24 \left(\frac{\theta_3(\tau,z)}{\theta_3(\tau,0)}\right)^2 - 2\frac{\theta_4(\tau,0)^4 - \theta_2(\tau,0)^4}{\eta(\tau)^4} \left(\frac{\theta_1(\tau,z)}{\eta(\tau)}\right)^2 . \end{aligned}$$

Kawai, Yamada, Yang, hep-th/9306096

For $K3^N/S_N$,

$$\sum_{N \geq 0} p^N Z(K3^N/S_N; \tau, z) = \prod_{n>0, m\geq 0, l} \frac{1}{(1 - p^n q^m y^l)^{c(nm,l)}} .$$

Dijkgraaf, Moore, Verlinde, Verlinde, hep-th/9608096

Elliptic genus contains information about non-chiral primaries. How to get this from supergravity? Vafa, hep-th/9804172

Supergravity yields single particle states that are representations of $SU(1,1|2)_L \times SU(1,1|2)_R$. Multiparticle states are obtained by taking tensor products of these representations. Tensor products do contain non-chiral primaries.

Example: A, B are chiral primary, so is AB. Then $\partial AB + A\partial B = \partial(AB)$ is a descendant of a chiral primary, but

$$\frac{1}{h_A}\partial AB - \frac{1}{h_B}A\partial B$$

is a non-chiral primary.

One can compute a "supergravity" elliptic genus, by taking a trace over all multiparticle states obtained this way, and compare to the CFT elliptic genus.

$\to (m, m')_S$

- CHIRAL PRIMARY OF WEIGHT $\left(\frac{m-1}{2}, \frac{m'-1}{2}\right)$
- COHOMOLOGY CLASS IN $H^{\frac{m-1}{2}, \frac{m'-1}{2}}(K3^N/S_N)$
- APPEARS FIRST ON $K3^d/S_d$

$\Rightarrow$ ASSIGN "DEGREE" d TO $(m, m')_S$

"EXCLUSION" PRINCIPLE:

TAKE ONLY $\bigotimes_i (m_i, m_i')_S$ WITH $\sum d_i \leq N$

(weighted) degeneracies of states in NS sector

$\lvert h,q\rangle$	coefficient
$\lvert 0,0\rangle$	$N+1$
$\lvert \frac{1}{2},1\rangle$	$22N-2$
$\lvert 1,2\rangle$	$277N-323$
$\lvert 1,0\rangle$	$464N-592$
$\lvert \frac{3}{2},3\rangle$	$2576N-5752$
$\lvert \frac{3}{2},1\rangle$	$5652N-13716$
$\lvert 2,4\rangle$	$19574N-64474$
$\lvert 2,2\rangle$	$51088N-181904$
$\lvert 2,0\rangle$	$67131N-244053$
$\lvert \frac{5}{2},5\rangle$	$128156N-557524$
$\lvert \frac{5}{2},3\rangle$	$378554N-1772518$
$\lvert \frac{5}{2},1\rangle$	$593928N-2862336$
$\lvert 3,6\rangle$	$746858N-4035502$
$\lvert 3,4\rangle$	$2422800N-14040288$
$\lvert 3,2\rangle$	$4319145N-25852311$
$\lvert 3,0\rangle$	$5160016N-31199792$

Complete agreement for $h \leq \frac{N+1}{4}$.

Remarks/Conclusions

- Can do other cases too; IIA on $K3$ gives also $K3^N/S_N$, IIA/IIB on T^4 gives $(T^4)^N/S_N$.

- F-theory on M^6 yields $(0,4)^N/S_N$. $N=(0,4)$ CFT is presumably what enters in dual type I/heterotic description. Spectrum organized in terms of $H^*(M^6)$.

- M-theory on $M^6 \times S^2 \times AdS_3$ yields $(0,4)^N/S_N$? Spectrum organized in terms of $H^*(M^6)$, relation to 4d black holes?

- What is the role of similar multiparticle states in $N=4$, $d=4$ with $G=SU(2,2|4)$?

- Prove elliptic genus works. Understand transition from supergravity regime to string theory regime. Precise meaning of "stringy exclusion principle". Implications for entropy?