Multigraviton Scattering in Matrix Models

A status report [Not a review]

M.D. + A.R. claimed discrepancy between DLCA Supergravity/matrix models in 3→3 gravitons. Complicated explanations offered to reconcile with Sen/Seiberg.

Two aspects:
(a) Matrix model computation of S matrix.

Use various simplifications in limit $R \gg r$ to extract terms in effective action. Then compute $S$-matrix. Delicate step.
[In Hamiltonian approach, operator ordering.
In path integral, seemed cleaner].

(b) Compare with supergravity $S$ matrix.

Various criticisms

K. Becker & M. Becker,
Larren et al....
Ferretti, Fabricchessi, Tengo

W. Taylor: as $g \to 0$, graviton should
M. Raamsdonk
couple to integrated stress tensor,
generator of translations in $x^-$
- quantized

Decisive: Okada, Yoneya: complete,
beautiful computation of effective
action on both sides. Complete
agreement.
Multi-Body Interactions of D-Particles in Supergravity and Matrix Theory

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Abstract

We present detailed analyses of the 3-body interactions of D-particles from both sides of 11 dimensional supergravity and Matrix theory. In supergravity, we derive a complete expression for the classical bosonic effective action for D-particles including 2-and 3-body interaction terms. In Matrix theory, we compute 1-particle irreducible contributions to the eikonal phase shift in the two-loop approximation. The results precisely agree with the predictions from supergravity and thus provide a strong support to the discrete light-cone interpretation of the Matrix-theory conjecture as a possible nonperturbative definition of M-theory.

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where \( r_{bc} \) is the transverse distance between the D-particles: \( r_{bc} = |x_b - x_c| \).

2.3 The effective action of D-particles

We can now derive the effective action. The contribution from the pure gravity part (2.41) is

\[
S_g = -\frac{1}{4} \sum_a \frac{N_a}{R} \int d\tau \zeta_{a\mu\nu}\sigma_\mu^a \sigma_\nu^a - \frac{1}{3} \sum_a \frac{N_a}{R} \int d\tau \chi_{a\mu\nu}\sigma_\mu^a \sigma_\nu^a. \tag{2.47}
\]

The source part (2.31) gives

\[
S_D = \sum_a \frac{N_a}{R} \int d\tau \left[ \frac{1}{2} \left( \frac{dx_a^i}{d\tau} \right)^2 + \frac{1}{2} \zeta_{a\mu\nu} s_\mu^a s_\nu^a + \frac{1}{2} \chi_{a\mu\nu} s_\mu^a s_\nu^a - \frac{1}{2} \zeta_{a\mu\nu} \zeta_{a-\lambda\nu} s_\mu^a s_\lambda^a \right]. \tag{2.48}
\]

Thus the total effective action is

\[
S_{\text{eff}} = \sum_a \int d\tau \frac{N_a}{R} \left[ \frac{1}{2} \left( \frac{dx_a^i}{d\tau} \right)^2 + \frac{1}{4} \zeta_{a\mu\nu} s_\mu^a s_\nu^a + \frac{1}{6} \chi_{a\mu\nu} s_\mu^a s_\nu^a - \frac{1}{2} \zeta_{a\mu\nu} \zeta_{a-\lambda\nu} s_\mu^a s_\lambda^a \right]. \tag{2.49}
\]

The second term gives the familiar 2-body lagrangian

\[
L_2 = \sum_{a < b} \frac{15N_a N_b v_{ab}^2}{16 R^3 M^9 r_{ab}^3}. \tag{2.50}
\]

We have used the relation \( s_a \cdot s_b = -\frac{1}{2} (v_a - v_b)^2 \equiv -\frac{1}{2} v_{ab}^2 \) with \( v_a = \frac{dx_a}{d\tau} \). The third and the fourth terms contain the 3-body force. Using (2.43) and (2.46), we express the 3-body term as a sum of two terms

\[
L_3 = L_V + L_Y \tag{2.51}
\]

Note that the V-type contribution consists of two parts corresponding to the contribution of (2.46) and the last term of (2.49)

\[
L_V = \sum_{a, b, c} \frac{(15)^2 N_a N_b N_c}{48 R^6 M^{18}} (s_b \cdot s_c)(s_a \cdot s_b)(s_a \cdot s_c) \left( \frac{1}{r_{ab}^7} \frac{1}{r_{ac}^7} + \frac{1}{r_{ab}^7} \frac{1}{r_{bc}^7} + \frac{1}{r_{ac}^7} \frac{1}{r_{bc}^7} \right)
- \sum_{a, b, c} \frac{(15)^2 N_a N_b N_c}{8 R^6 M^{18}} (s_b \cdot s_a)^2 (s_c \cdot s_a) \frac{1}{r_{ab}^7} \frac{1}{r_{ca}^7}. \tag{2.52}
\]

In terms of the relative velocities, the sum of two terms is rewritten as

\[
L_V = -\sum_{a, b, c} \frac{(15)^2 N_a N_b N_c}{64 R^6 M^{18}} v_{ab}^2 v_{ca}^2 (v_{ca} \cdot v_{ab}) \frac{1}{r_{ab}^7} \frac{1}{r_{ca}^7}. \tag{2.53}
\]
Where was our error?

Extraction of S-matrix on matrix model side. Believe there is a concise description in progress. [Effective action correct]

Might be useful because methods of M.O, A.R. generalize easily to case of more gravitons. [Ooguri: Douglas, Ooguri, Shnerke = 4-graviton]

Remaining questions?

(a) Resolution of DOS?

(b) Other tests

(c) IR divergences: are they real? Important?

(d) Computing new phenomena.
\[ \langle f \mid \frac{\mathbf{p}^4}{r^2} \mid n \rangle = \frac{1}{E_n - E_i} \langle n \mid \frac{\mathbf{p}^4}{r^2} \mid f \rangle \]

Operator ordering.

If allow any ordering, can almost cancel denominator, giving extra term.
To quote a wiser head than mine:

"Don't Mess with Matrix Theory"
Maximal Symmetry
and the
Ground State
of
String Theory

With
Y. Nir, Y. Shadmi
The problem of moduli

A useful tool for theoretical study

But: from perspective of string

theory as a theory of nature, many

puzzles

• Any dynamics which selects
gnd. state almost certainly strongly

coupled [M. D. Seiberg]*

Then why are couplings we observe

weak + unified? Accidents?

Possible resolution: Kahler stabilization? M. D. T. Banks

Racetrack models (Kraus, Kupriyanov, Dixon, Kaplunovsky, Peskin; Kaplunovsky, Louis)

* See, however, Jan Louis at this meeting
- Cosmology (Banks, Kaplan, Nelson)

If vacuum at a "random pt." in moduli space, no reason for early universe to prefer.

Too much energy stored in modulus — catastrophic consequences.

Possible resolution: all moduli charged under symmetries in "true vacuum" (continuous or discrete) (M.O., L. Randall, S. Thomas)

Then it is natural for universe to sit at special pt. always (finite temp; inflation)

Alternatively: no moduli
Such points are automatically stationary points of effective action, so candidate minima. Hope to study using dualities.

Ex: toroidal compactification of heterotic string. Take product of circles, each at SU(2) pt. All moduli charged under SU(2)'s, except S.

But: SU(2x2). At self-dual pt, S transforms (electric-magnetic).

$$\alpha = \frac{1}{\lambda} [!?]$$

How common? Under study, but plausible that often occur in moduli space(s).
Still puzzles:

Why weak coupling, unification?

• Still accidents. But perhaps more plausible. Couplings [gauge] pure nos., perhaps determined by topological [or similar] considerations. If there are many such points, perhaps plausible that $\alpha^{-1}$ large, unified.

• Kahler potential singularities restore symmetries [toy example in M.D. Nir, Shadmi]
Adopt a set of hypotheses, explore consequences.

Assume, at a high scale $M$, one has a theory with

(a) Maximally enhanced symmetry
    or No moduli;
(b) Approx. $N=1$ SUSY
(c) Small, unified gauge couplings.
No moduli

(a) SUSY Breaking in Low Energy Theory
Why?
High energy string effects: $W(\Phi)$
Need a linear term for some singlet field. But not plausible that there are such light fields at strong coupling

(b) SUSY breaking visible in low energy dynamics. E.g. SUSY-breaking "hidden sector."
Scale? If no singlets
$\tilde{\Phi} W_\alpha^2$ can't be source of gaugino mass

$\Rightarrow$ Low energy breaking, presumably gauge mediation.
Maximal Symmetry

Again, SUSY breaking a low energy phenomenon.

\[ W \neq \Phi \quad \text{Forbidden} \]

Also gaugino condensation [as origin of susy breaking] and its generalizations forbidden

\[ \Phi W^2 \neq \phi \]

So again SUSY breaking via a hidden sector at low energies, gaug-mediated.
How are the moduli stabilized?

- Suppose all moduli charged under standard model gauge symmetries. [Not crazy, MSSM has approximate flat directions in which all gauge symmetry broken. E.g. \( Q \otimes Q \otimes L \), i.e. \( Q = \begin{pmatrix} u^c \\ u^c \\ u^c \end{pmatrix} \), \( L = \begin{pmatrix} 0 \\ v \end{pmatrix} \)]

12 parameters or 12 candidate moduli. Exact flatness doesn't require string miracles; e.g. discrete \( R \) symmetries.

Gauge mediation: pos. masses for these fields \( \sim \left( \frac{\alpha}{n} \right)^2 \Lambda^2 \)

\( \Rightarrow \) Local minima at symmetric pt.
• Some moduli neutral under gauge symmetries.

\[ \frac{1}{M_p^2} \int d^4 \theta M^+ M Z^+ Z \]

moduli hidden sector

\[ m \sim \text{eV} \] Very light moduli but

very weakly coupled to ordinary matter.

E.g.

\[ \int M \tilde{Q} \tilde{f} \text{ forbidden.} \]

Yukawas, etc., may be very high dimension.

• Participants in hidden sector dynamics (e.g. flat directions of IT models)
Conclusions, questions, puzzles

(a) Cosmological Constant

[Can generate a constant in $W$ of correct order of magnitude, but...]

(b) We have seen: low energy SUSY breaking + gauge mediation are generic consequences of these hypotheses.

(c) Suggests an approach to phenomenology which does not, perhaps, require complete control of strong dynamics.

(d) Question: are there other consequences of this viewpoint?