Multigraviton Scattering in Matrix Models

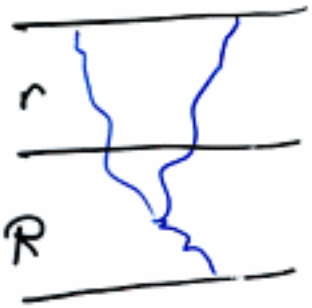
A status report
[Not a review]

M.D. + A.R. claimed discrepancy between DLCQ Supergravity/matrix models in $3 \to 3$ gravitons. Complicated explanations offered to reconcile with Sen/Seiberg.

Two aspects:

(a) Matrix model computation of S matrix.

Use various simplifications in limit $R \gg r$ to extract terms in effective action.



Then compute S-matrix

Delicate step.

[in Hamiltonian approach, operator ordering. In path integral, seemed cleaner].

(b) Compare with supergravity S matrix.

Various criticisms [K. Becker & M. Becker, Larsen et al ....
Ferretti, Fabrichessi, Iengo

W. Taylor: as $g \to 0$, graviton should
M. Raamsdonk
couple to integrated stress tensor, generator of translations in X^-
– quantized.

Decisive: Okada, Yoneya: complete, beautiful computation of effective action on both sides. Complete agreement.

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Multi-Body Interactions of D-Particles in Supergravity and Matrix Theory

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Abstract

We present detailed analyses of the 3-body interactions of D-particles from both sides of 11 dimensional supergravity and Matrix theory. In supergravity, we derive a complete expression for the classical bosonic effective action for D-particles including 2-and 3-body interaction terms. In Matrix theory, we compute 1-particle irreducible contributions to the eikonal phase shift in the two-loop approximation. The results precisely agree with the predictions from supergravity and thus provide a strong support to the discrete light-cone interpretation of the Matrix-theory conjecture as a possible nonperturbative definition of M-theory.

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where r_{bc} is the transverse distance between the D-particles : $r_{bc} = |x_b - x_c|$.

2.3 *The effective action of D-particles*

We can now derive the effective action. The contribution from the pure gravity part (2.41) is

$$S_g = -\frac{1}{4}\sum_a \frac{N_a}{R}\int d\tau\, \zeta_{a\mu\lambda} s_a^\mu s_a^\lambda - \frac{1}{3}\sum_a \frac{N_a}{R}\int d\tau\, \chi_{a\mu\lambda} s_a^\mu s_a^\lambda. \tag{2.47}$$

The source part (2.31) gives

$$S_D = \sum_a \frac{N_a}{R}\int d\tau \Big[\frac{1}{2}\Big(\frac{dx_a^i}{d\tau}\Big)^2 + \frac{1}{2}\zeta_{a\mu\nu} s_a^\mu s_a^\nu + \frac{1}{2}\chi_{a\mu\nu} s_a^\mu s_a^\nu - \frac{1}{2}\zeta_{a\mu\nu}\zeta_{a-\lambda} s_a^\mu s_a^\nu s_a^\lambda\Big]. \tag{2.48}$$

Thus the total effective action is

$$S_{eff} = \sum_a \int d\tau \frac{N_a}{R}\Big[\frac{1}{2}\Big(\frac{dx_a^i}{d\tau}\Big)^2 + \frac{1}{4}\zeta_{a\mu\nu} s_a^\mu s_a^\nu + \frac{1}{6}\chi_{a\mu\nu} s_a^\mu s_a^\nu - \frac{1}{2}\zeta_{a\mu\nu}\zeta_{a-\lambda} s_a^\mu s_a^\nu s_a^\lambda\Big]. \tag{2.49}$$

The second term gives the familiar 2-body lagrangian

$$L_2 = \sum_{a<b} \frac{15 N_a N_b}{16 R^3 M^9}\frac{v_{ab}^4}{r_{ab}^7}. \tag{2.50}$$

We have used the relation $s_a \cdot s_b = -\frac{1}{2}(v_a - v_b)^2 \equiv -\frac{1}{2}v_{ab}^2$ with $v_a = \frac{dx_a}{d\tau}$. The third and the fourth terms contain the 3-body force. Using (2.43) and (2.46), we express the 3-body term as a sum of two terms

$$L_3 = L_V + L_Y \tag{2.51}$$

Note that the V-type contribution consists of two parts corresponding to the contribution of (2.46) and the last term of (2.49)

M.D. & A.R., ...

$$\begin{aligned} L_V &= \sum_{a,b,c} \frac{(15)^2 N_a N_b N_c}{48 R^5 M^{18}}(s_b\cdot s_c)(s_a\cdot s_b)(s_a\cdot s_c)\Big(\frac{1}{r_{ab}^7}\frac{1}{r_{ac}^7} + \frac{1}{r_{ab}^7}\frac{1}{r_{bc}^7} + \frac{1}{r_{ac}^7}\frac{1}{r_{bc}^7}\Big) \\ &\quad - \sum_{a,b,c} \frac{(15)^2 N_a N_b N_c}{8 R^5 M^{18}}(s_b\cdot s_a)^2(s_c\cdot s_a)\frac{1}{r_{ab}^7}\frac{1}{r_{ca}^7}. \end{aligned} \tag{2.52}$$

cancellation

In terms of the relative velocities, the sum of two terms is rewritten as

$$L_V = -\sum_{a,b,c} \frac{(15)^2 N_a N_b N_c}{64 R^5 M^{18}} v_{ab}^2 v_{ca}^2 (v_{ca}\cdot v_{ab})\frac{1}{r_{ab}^7}\frac{1}{r_{ca}^7}. \tag{2.53}$$

form expected for matrix model effective action (MD, AR) (WT) etal.

Where was our error?

Extraction of S-matrix on matrix model side. Believe there is a concise description; in progress. [Effective action correct]

Might be useful because methods of M.Q, A.R. generalize easily to case of more gravitons.

[Ooguri: Douglas, Ooguri, Shenker = 4-graviton]

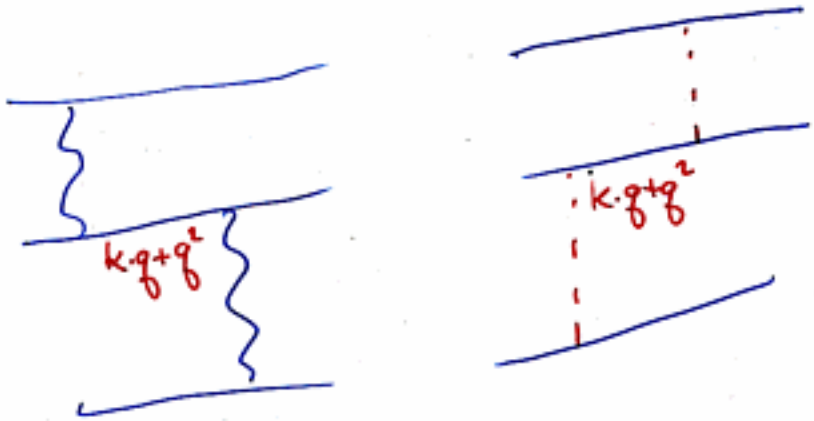
Remaining questions?

(a) Resolution of DOS?

(b) Other tests

(c) IR divergences: are they real? Important?

(d) Computing new phenomena.



$$\langle f | \frac{p^4}{r^7} | n \rangle \frac{1}{E_n - E_i} \langle n | \frac{p^4}{r^7} | i \rangle$$

Operator ordering.

If allow any ordering, can <u>almost</u> cancel denominator, giving extra term.

To quote a wiser head than mine:

"Don't Mess with Matrix Theory"

Maximal Symmetry and the Ground State of String Theory

with
Y. Nir, Y. Shadmi

The problem of moduli

A useful tool for theoretical study

But: from perspective of string theory as a theory of nature, many puzzles

- Any dynamics which selects gnd. state almost certainly strongly coupled [M.D., Seiberg] *

  Then why are couplings we observe weak + unified? Accidents?

Possible resolution: Kahler stabilization? M.D. T. Banks

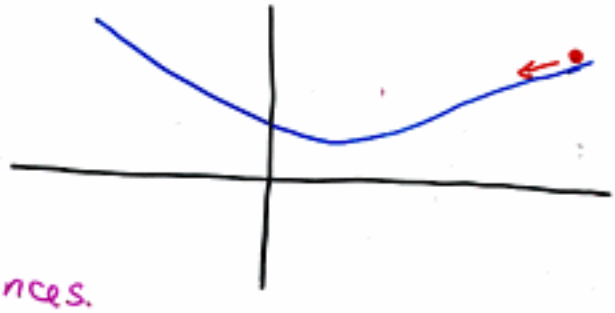
Racetrack models (Krasnikov; Dixon, Kaplunovsky, Peskin; Kaplunovsky, Louis)

* See, however, Jan Louis at this meeting

- Cosmology (Banks, Kaplan, Nelson)

If vacuum at a "random pt." in moduli space, no reason for early universe to prefer.

Too much energy stored in modulus – catastrophic consequences.



Possible resolution: <u>all</u> moduli charged under symmetries in "true vacuum" [continuous or discrete] (M.D., L. Randall, S. Thomas)

Then it is natural for universe to sit at special pt. always (finite temp; inflation)

Alternatively: <u>No moduli</u>

Such points are automatically stationary points of effective action, so candidate minima. Hope to study using <u>dualities</u>

Ex: toroidal compactification of heterotic string. Take: product of circles, each at $SU(2)$ pt. All moduli charged under $SU(2)$'s, except S.

But: $SL(2,\mathbb{Z})$. At self-dual pt, S transforms (electric-magnetic)

$\alpha = \frac{1}{2}$ [!?]

How common? Under study, but plausible that often occur in moduli space(s)

Still puzzles:

Why weak coupling, unification?

- Still accidents. But perhaps more plausible. Couplings [gauge] pure nos., perhaps determined by topological [or similar] considerations. If there are many such points, perhaps plausible that α_i^{-1} large, unified.

- Kahler potential singularities restore symmetries [toy example in M.O, Nir, Shadmi]

Adopt a set of hypotheses, explore consequences

Assume, at a high scale M, one has a theory with

(a) Maximally enhanced symmetry
<u>or</u> No moduli

(b) Approx. $N=1$ SUSY

(c) Small, unified gauge couplings.

No moduli

(a) SUSY Breaking in Low Energy Theory.

Why?

High energy string effects: $W(\Phi)$

Need a linear term for some singlet field. But not plausible that there are such light fields at strong coupling *

(b) SUSY breaking visible in low energy dynamics. E.g. SUSY-breaking "hidden sector."

Scale? If no singlets

ΦW_α^2 can't be source of gaugino mass

(ADS; Banks, Kaplan, Nelson)

$\Rightarrow$ Low energy breaking, presumably gauge mediation.

Maximal Symmetry

Again, SUSY breaking a low energy phenomenon.

$\cancel{W = \Phi}$ forbidden

Also gaugino condensation [as origin of SUSY breaking] and its generalizations forbidden

$\cancel{\Phi W_\alpha^2}$

So again SUSY breaking via a hidden sector at low energies, gauge mediation.

How are the moduli stabilized?

- Suppose all moduli charged under standard model gauge symmetries.

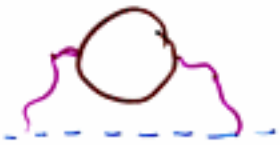
[Not crazy. MSSM has approximate flat directions in which all gauge symmetry broken. E.g. $QQQL$, i.e. $Q = \begin{pmatrix} v^{(1)} & & \\ & v^{(1)} & \\ & & v^{(2)} \end{pmatrix}$

$L = (0 \;\; v)$

12 parameters or 12 candidate moduli.

Exact flatness doesn't require string miracles; e.g. discrete R symmetries.]

Gauge mediation: pos. masses for these fields $\sim \left(\frac{\alpha}{\pi}\right)^2 \Lambda^2$



$\Rightarrow$ Local minima at symmetric pt.

- Some moduli neutral under gauge symmetries.

$$\frac{1}{M_P^2}\int d^4\theta\, \mathcal{M}^\dagger \mathcal{M}\, Z^\dagger Z$$

moduli hidden sector

$m \sim$ eV. Very light moduli but very weakly coupled to ordinary matter.

E.g.

$$\int \mathcal{M} R \sqrt{g} \quad \text{forbidden.}$$

Yukawas, etc, may be very high dimension.

- Participants in hidden sector dynamics (e.g. flat directions of IT models)

Conclusions, questions, puzzles

(a) Cosmological Constant

[Can generate a constant in W of correct order of magnitude, but...]

(b) We have seen: low energy SUSY breaking + gauge mediation are generic consequences of these hypotheses.

(c) Suggests an approach to phenomenology which does not, perhaps, require complete control of strong dynamics.

(d) Question: are thir other consequences of this viewpoint?