

BULK GAUGE FIELDS IN ADS  
SUPERGRAVITY AND SUPERMULTIPLICITY

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Plan of the seminar:

Some aspects of the  $AdS_{d+1}/SCFT_d$

Correspondence (Maldacena, Gubser, Klebanov, Polyakov, Witten)

Specific case:  $AdS_5 \times S^5$

Horizon geometry of D3 brane, as particular case of horizon geometry of p-branes -

$AdS_{p+2} \times S_{d-p-2}$  (Gibbons, Townsend;

Kallosh, Peet; Gibbons, Kallosh, L.F.; Blauwaele, Duff ... )

Role of supersymmetry:  $SU(2, 2/4)$

O.I.R. type (Bau; Gunaydin; Horans; Howe, West)

K.K. multiplets (Klemm, Romans, van Nieuwenhuizen)

$N=4$  primaries (Ferrando, Zaffaroni, S-F.)

OPE expansions (Freedman, Mather, Mekis, Rastelli)

Supergauge on  $AdS_{d+1}$ :

Theory around  $AdS_{d+1} = \frac{O(d, 2)}{O(d, 1)}$

$$AdS: \eta^+ \eta^- - \sum_{i=1}^d \eta_i \eta^i = R^2$$

$$\eta_A = (\kappa, x_\mu, \eta^i)$$

$$(\eta_\mu = \kappa x_\mu, \eta^+ = \kappa, \eta^- = \kappa x^- \\ x^- = x^2 + \frac{\eta^2}{\kappa^2} = x^2 + \frac{R^2}{\kappa^2})$$

The boundary at infinity is obtained for

$$\kappa \rightarrow \infty, \{x_\mu, S = 2AdS_{d+1} = \tilde{H}_d = \frac{O(d, 2)}{IO(d-1, 1) \times D}$$

(Dirac, Flato, Fradkin, Fronsdal, SF; Witten)

$$\{x_\mu, S: \eta^A \eta_A = 0 \quad [\eta^A = 2q^A]$$

(Poisson identified projectively on the cone)

Conformal fields on  $\partial AdS_{d+1}$  ( $d=4$ )

$$C_I = J^{AB} J_{AB}$$

$$C_{II} = \epsilon_{ABCDEF} J^{AB} J^{CD} J^{EF}$$

$$C_{III} = J_A{}^B J_B{}^C J_C{}^D J_D{}^A$$

(Mack, Salam)

An irreducible rep is obtained by

"induced rep.", matched by  $\eta \neq 2\pi$ .

of the stability group of  $x_f = 0$

( $K_\mu, D, H_{\mu\nu}$ : Primary conformal fields)

$$[O_{\{a\}}, (\kappa=0), K_\mu] = 0$$

descendants:  $\dots \partial \dots \partial O_{\{a\}}$

(infinite dimensional representation).

$C_I, C_{II}, C_{III} \rightarrow D, H_{\mu\nu} H^{\mu\nu}, \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu} H^{\rho\sigma}$   
( $\ell, J_1, J_2$ )

For tensor reps:  $(\ell, J_1=J_2=\frac{\ell}{2})$  :

$$C_{\pm} = \ell(\ell+4) + s(s+2)$$

$$C_{\mp} = 0$$

$$C_{\mp} = [\ell(\ell+2) - s(s+2)][(\ell+2)(\ell+4) - s(s+2)]$$

Ex: Confocal scalar:  $\phi(\eta) \quad \eta^2 = 0$

$$\eta \cdot \partial \phi = \lambda \phi \quad (\lambda = -\ell)$$

Quadratic Casimir:

$$\frac{1}{2} L_{AB} L^{AB} = \eta^2 \square_d - \eta \cdot \partial (\partial + \eta \cdot \partial)$$

on the cone reduces to:  $\ell(\ell-d)$

Tensor confocal fields: (Eggers, Guille, Pani)  
S.F.

$$\Theta_{A_1 \dots A_n}(\eta): \quad \eta^{A_1} \Theta_{A_1 \dots A_n}(\eta) = 0$$
$$\partial^{A_1} \Theta_{A_1 \dots A_n}(\eta) = 0$$

Casimirs are not purely orbital.

Unitary boundary region:

$$l \geq 1 + J \quad (J_1, J_2 = 0)$$

$$l \geq 2 + J_1 + J_2 \quad \text{otherwise.}$$

Unitary thresholds:

$$l = 1 + J \quad \text{AdS}_5 \text{ miniflows} \left( \overset{\text{maximal}}{\underset{\text{on}}{\partial \text{AdS}_5}} \right)$$

$$l = 2 + J_1 + J_2 \quad \text{AdS}_5 \text{ massless fields}$$

$$\square_x \phi_{(0,J)} = 0 \quad \text{on } \partial \text{AdS}_5$$

$$\partial^{\alpha_1, \alpha_2} O_{\alpha_1, \dots, \alpha_{J_1}, \beta_1, \dots, \beta_{J_2}} = 0 \quad \text{on } \partial \text{AdS}_5$$

$$(2J_1 + 1)(2J_2 + 1) - 4J_1 J_2 = 2(J_1 + J_2) + 1$$

O contains a  $O(3)$  rep. of spin  $J = J_1 + J_2$

$O(3) \rightarrow$  little group of maximal particles in  
Poincaré limit of  $\text{AdS}_5$

$AdS_{d+1} \rightarrow SCFT_d$  connection

Highest weight VIR of  $AdS_{d+1}$

$(E_0, J_1, J_2) \rightarrow$  Primary conformal

fields  $C_I, C_{\bar{I}}, C_{\overline{\bar{I}}} (E_0, J_1, J_2) =$

$C_I, C_{\bar{I}}, C_{\overline{\bar{I}}} (\ell, J_1, J_2)$

$(In AdS_{d+1}, E_0 \rightarrow O(2), (J_1, J_2) \rightarrow O(d)_{d=4})$

$$g_{\mu\nu} \rightarrow T_{\mu\nu}$$

$$A_\mu \rightarrow J_\mu$$

$$\psi^i_{\mu a} \rightarrow J^i_{\mu a}$$

bulk

boundary

etc.

LAPLACE  
OPERATOR OF  
 $AdS_{d+1}$

$$\begin{aligned} i' L_{AB} L^{BR} \phi_{BULK} &= R^2 \square_d - \eta \partial (d + \eta \partial) = \\ &= -E_0 (d - E_0) \phi_{BULK} \end{aligned}$$

For gauge fields:  $\mathcal{O}(2 + J_1 + J_2, J_1, J_2)$

$\eta^2 \square^\epsilon \phi = 0$  admits only solutions:

$$\phi_{\mu_1 \dots \mu_p}^{\text{BULK}} \rightarrow \gamma^2 \underbrace{\partial_{\mu_1} \phi_{\mu_2 \dots \mu_p}^{\text{BULK}}}_{+ a \underbrace{\gamma_{\mu_1} \phi_{\mu_2 \dots \mu_p}^{\text{BULK}}}}$$

(Frandsen, S.F.)

More general correspondence:

$E_d(\ell)$  not integer.

(string theory  $\leftrightarrow$  Chiral dimension)

(Gukov, Klebanov, Polyakov; Witten)

Vertex (3 p. functions) and propagators  
in  $\text{EFT}_d \rightarrow$  boundary values of  $\text{AdS}_{d+1}$ ,  
giant, connected Green functions.

Supersymmetric conformal field theories:  
notion of "chiral, primary" conformal  
operators (dimensions can quantized)  
(in  $N=1$  selection between the dimension  
and the U(1) charge  $\ell = q$ ) (Wess, Zumino)

In  $N=4$  Conformal supersymmetry  
superfields can be chiral or  
"twisted", chiral

Chiral  $\phi(x, \theta^i, \bar{\theta}_i) \rightarrow \phi(x_i, \theta_i)$

higher spin  $(\frac{N}{2}, 0) \rightarrow (2, 0)$

"Twisted chiral",  $\phi(x, \theta^i, \bar{\theta}_i) \rightarrow \phi(x, \theta_a, \bar{\theta}_a)$   
higher spin  $(\frac{N}{4}, \frac{N}{4}) \rightarrow (1, 1)$

(Howe, West : Analytic superfields in  
harmonic superspace : Fokketsch, Ogiuevsky, --)

Brink-Lane superfield:

$$W_{[AB]}(x, \theta, \bar{\theta}): \quad W_{[AB]} = i \epsilon_{ABC} \bar{W}_{[C0]}$$

$$\partial_{\alpha_A} W_{[B,C]} = \partial_{\alpha_B} W_{[A,C]}$$

Y. N. nullspac.

K.K. states:

$T_2(N_R, W_2, \dots, W_{p+1})$  - terms

Multiplets with  $256 \times \frac{1}{12} p^2(p^2-1)$  states:

$\frac{1}{12} p^2(p^2-1)$ :  $\dim(0, p-2, 0) = m$ .

$SU(4)$  suggests one present for  $p \leq 4$ .

(Gunaydin, Ivanov; Kim, Romans, van Nieuwenhuizen)  
Duffin, Haarantz

Spin 2 :  $m^2 = \ell(\ell-4)$

Spin 1 :  $m^2 = (\ell-1)(\ell-3)$

Spin 1/1,0 :  $m^2 = (\ell-2)^2$

Spin 0 :  $m^2 = \ell(\ell-4)$

Spin  $1/2, 3/2$  :  $m = \ell-2$

(Ferrand, de la Houssaye)

$p=2$  supplement included (Howe, Stelle, Townsend)

(Gubser, Klebanov...; Rajaraman; Das, Thirukkanem; Meltzer...)

(Farakos, Zaffaroni, S.F.)

Andrianov et al., S.F.

Lledo, Zaffaroni, S.F.)

$$\text{Tr} (\phi_{1e, \dots, e_p}) - \text{trace} \quad (0, p, 0)$$

$$\text{Tr} (\phi_{1e, \dots, e_p, \{F_{ap}\}}) - \text{trace} \quad (0, p-1, 0)$$

$$\text{Tr} (\phi_{1e, \dots, e_{p-1}, F_{ap} F^{ep}}) - \text{trace} \quad (0, p-2, 0)$$

$$\text{Tr} (\phi_{1e, \dots, e_{p-2}, \{F_{ap} F_{ap}\}}) - \text{trace} \quad (0, p-3, 0)$$

$$\text{Tr} (\phi_{1e, \dots, e_{p-3}, \{F_{ap} F^{ep} F_{ap}\}}) - \text{trace} \quad (0, p-4, 0)$$

$$\text{Tr} (\phi_{1e, \dots, e_{p-4}, \{F_{ap} F^{ep} F_{ap} F^{ep}\}}) - \text{trace} \quad (0, p-5, 0)$$

Partial waves with  $\Delta l = 4$

1st partial wave  $(0, e_1, 0)$   $O(b)^{2ep}$ )

Singlets exist up to  $p=4$ .

Spin 4 multiplet:

$$T_2(\phi_a \phi_e) : T_2(W_{AB} W_{CD} \epsilon^{ABCD}) \Big|_{\theta=0}$$

This is a multiplet which should couple to a string state a  $AdS_5 \times S_5$

Spin 4 :  $T_2(\phi^i \tilde{\partial}_a, \tilde{\partial}_e, \tilde{D}_{a_3}, \tilde{D}_{a_4} \phi_e)$   $E=6$   
SU(4)  
a<sub>3,4</sub>

$$T_2(F_{\mu\rho} \partial_{a_1} \partial_{a_2} F_{\mu\rho})$$
$$T_2(\bar{\lambda}^i \tilde{\partial}_f \partial_{a_1} \partial_{a_2} \partial_{a_3} \lambda^i)$$

Spin 2 :  $1 + 15 + 20$   $E=4$

Spin 1 :  $1 + 15$   $E=3$

$$m^2 \sim (g_{yn}^2 N)^{1/4} \quad (\text{at strong coupling})$$

(N=4 embedding of Komati current)

We expect that the chiral primary couple to an infinite set of primary confined supermultiplets with arbitrarily high spin. (Howe, West)

In the free-field theory limit this corresponds to the statement that there are infinite many meson operators in the product of two regular spinors.

$$O_{J_L-1, J_R-1} \dots O_{J_L+1, J_R+1}$$

$$\ell = J_L + J_R \quad (\ell = 2 + J_L + J_R) \quad \ell = J_L + J_R + 4$$

$$2^8(2J_L+1)(2J_R+1) \text{ states}$$

(Guraydin, Minic, Zagermann)

$$(J_L = J_R = 1 : \text{Spin } 4)$$

These multiplets repeat the 16 states of the gravity multiplet.

Gumz N=4 neugestalten:

$$\theta_{(0,J)}, \dots, \theta_{(0,J-1)}$$
$$l=1+J \qquad \qquad l=J$$

Since any sign which is contained  
in the product of 2 tangleon signs.

i) mention in AdS<sub>5</sub>, in free dual  
theory the spin 4 (as well as spin 3 and  
spin 4) would be "coupled" to  
conformal fields.. This can be  
understood because in free  $N=4$   
Maxwell theory there is an additional  
 $(U(1))$  current and two  $SU(4)$   
currents made up of scalars and spinors.

From general results on OPE in  
N=1 supersymmetric field theories

(Aharony, Freedman, Grisaru, Johansen)

it is known that the Komatsu-current  
appears in the OPE of the stress tensor  
supermultiplet

$$(J_{\alpha\dot{\alpha}}(x, \theta, \bar{\theta}): J^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = 0, \sum_{\theta=0} = T_h \phi \bar{\epsilon}^V \phi^{\bar{V}}))$$

$$J(z) J(z') = \frac{c}{(z - z')^3} + \frac{\Sigma(z')}{(z - z')^{2-h/2}} + \dots$$

$$\Sigma(z) \Sigma(z') = \frac{c'}{(z - z')^{2+h}} + \frac{\Sigma(z')}{(z - z')^{1+h/2}}$$

$$c = \frac{1}{24} (3N_V + N_C) \quad c' = N_X$$

$$z = (x - x' + i\theta\gamma\bar{\theta}')$$

In  $N=4$  Yang-Mills theory: (Ameduri  
Zappalà, SF.)

$$C_{\text{quarium}} = C$$

$$C'_{\text{quarium}} = C' \quad (\text{up to two loops})$$

$$h \approx g_{yn}^2 N$$

$$\text{At strong coupling } h \sim (g_{yn}^2 N)^{1/4}$$

$$\langle J J \Sigma \rangle \propto c' \langle \dots \rangle$$

$l_J = \text{constant}$   
 $l_\Sigma = 2 + h$

Consider the conformal primary

$$[\zeta(x), \partial_x - \partial_x \Sigma(x)]$$

We can compute

$$\langle J J J J \rangle = \text{Diagram with four external lines labeled } \Sigma, \partial \Sigma, \partial \bar{\Sigma}, \dots$$

in terms of hypergeometric function

$$h = \frac{3}{16\pi^2} \frac{\Psi_{ijk}\Psi^{ijk}}{N_x}$$

$$c = \frac{1}{24} (3N_u + N_c + N_v \frac{\beta(g)}{g} - \gamma_i^i)$$

$$\gamma_j^i = \frac{1}{16\pi^2} \left[ \frac{1}{2} \Psi_{jkm} \Psi^{i k m} - 2g^2 G(\mu) \gamma_j^i \right]$$

OPE on  $\partial \text{Ad } S_5$  (Gatto, Gukov, Petrini  
D.F. 1994)  
 $A(\eta), B(\eta)$  N.C. 194, 667 (74)  
 N.P. B49, 72 (72)

$$A(\eta) B(\eta') = \sum_{n=0}^{\infty} E_n(\eta \cdot \eta') D^{nA_1 \dots nA_n}(\eta, \eta') O_{A_1 \dots A_n}(\eta')$$

pseudodifferential operator.

$$E_n(\eta \cdot \eta') = (\eta \cdot \eta')^{-\frac{1}{2}} (\ell_A + \ell_B - \ell_n + n)$$

$$D^{nA_1 \dots nA_n}(\eta, \eta') = \eta^{A_1} \dots \eta^{A_n} D^{nA_1 \dots nA_n}(\eta, \eta')^{-\frac{1}{2}(\ell_A - \ell_B + \ell_n + n)}$$

well defined at  $\eta^2 = \eta'^2 = \eta \cdot \eta' = 0$

$$D(\eta, \eta') = \eta \cdot \eta' \square'_6 - 2\eta \cdot \partial' (1 + \eta \cdot \partial')$$

$$D^h(\eta, \eta') = (L-1)^h \sum_{J=0}^{\infty} \binom{h}{J} \frac{(\eta \cdot \eta')^J (2\eta \cdot \partial')^{h-J} \square'_6^J}{(L-1)^J}$$

$$D(\eta, \eta') = 2 \frac{k}{k'} \left[ (L - (x-x')\partial') (1-L) - \left(\frac{x-x'}{2}\right)^2 \square'_6 \right]$$

$$\zeta = -k' \frac{\partial}{\partial k'}$$

$$A(r)B(0) = \left(\frac{1}{x^2}\right)^{\frac{1}{2}(\ell_1 + \ell_B - \ell)} \int_0^1 du u^{\ell_2(p_A - q_B + \ell) - 1}$$

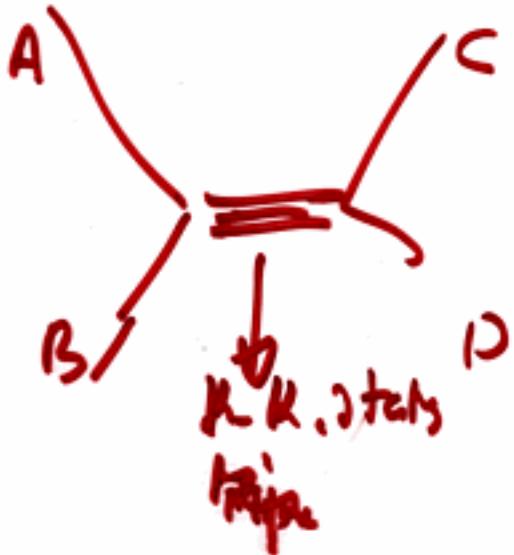
$$(1-u)^{\frac{1}{2}(p_B - p_A + \ell) - 1} F_1\left(\ell - 1; -\frac{x^2}{4} \square u(u)\right) O(ux).$$

↓

$$O(d, z) \longrightarrow \ell + 1 - \frac{D}{2}$$

$$\left( \exp[-\frac{i}{2}v\pi] J_\nu (2\exp[i\frac{\pi}{2}]) = \frac{\left(\frac{v}{2}\right)^\nu}{P(vn)} F_1\left(2n+1, \frac{1}{4}z^2\right) \right)$$

$$\left( F_{0,1}(\nu, z) = \sum_{h=0}^{\infty} \frac{1}{h!} \frac{\Gamma'(h)}{P(v+h)} z^h \right)$$



Banks Green  
 Lee, Minwalla,  
 Rajaguru, Seiberg

Derivation from 3-point function:

$$\langle 0 | C(x) A(y) B(z) | 0 \rangle = \left( \frac{1}{y-x} \right)^{\frac{1}{2}(l_c + l_A - l_B)}$$

$$\left( \frac{1}{x^2} \right)^{\frac{1}{2}(l_A + l_B - l_c)} \left( \frac{1}{y^2} \right)^{\frac{1}{2}(l_c + l_B - l_A)} \Rightarrow$$

$$\left( \frac{1}{y^2} \right)^{\frac{1}{2}(l_c + l_B - l_A)} \frac{1}{(y-x)^{\frac{1}{2}(l_c + l_A - l_B)}} =$$

$$= \int_0^1 du u^{\frac{1}{2}(l_c + l_A - l_B)-1} \frac{1}{(1-u)^{\frac{1}{2}(l_c + l_B - l_A)}} =$$

$$\left[ (y-ux)^2 \right]^{-l_c} \left( 1 + \frac{x^2 G(1-u)}{(y-ux)^2} \right)^{-l_A} =$$

$$= \int_0^1 du u^{\frac{1}{2}(l_c + l_A - l_B)-1} \frac{1}{(1-u)^{\frac{1}{2}(l_c + l_B - l_A)-1}}$$

$$\sum_{h=0}^{\infty} \frac{1}{h!} \frac{P(l_c+h)}{P(l_c)} (-x^2)^h (u(1-u))^h \left( \frac{1}{y-ux} \right)^{l_c+h}$$

Riemann-Liouville fractional integral.

Using the fact that

$$\left[ \frac{1}{(y-ux)^2} \right]^{e_0+h} = \left( \frac{\pi}{4} \right)^h \frac{\Gamma(e_0) \Gamma(e_0-1)}{\Gamma(e_0+h) \Gamma(e_0-1+h)} \text{ (12)}$$

We see that the above expression comes from

$$\int_0^1 du u^{\frac{1}{2}(e_0+e_0-1)} (1-u)^{\frac{1}{2}(e_0+e_0-1-1)} \sum_{h=0}^{\infty} \frac{1}{h!} \frac{\Gamma(e_0-1)}{\Gamma(e_0-1+h)} \left( -\frac{x^2}{4} u(1-u) \right)^h O(ux)$$

$F_{0,1}(e_0-1; -\frac{x^2}{4} u(1-u))$

$$\langle A_1(\eta_1) \dots A_n(\eta_n) \rangle \quad \eta_1^2 \dots \eta_n^2 = 0$$

$$\frac{1}{2}n(n-3), \quad nD - \frac{(D+2)(D+1)}{2}$$

$$n=4, \quad 2 \text{ variables: } p = \frac{(x-t)^2(z-y)^2}{(x-y)^2(z-t)^2}$$

$$\eta = \frac{(x-z)^2(y-t)^2}{(x-y)^2(z-t)^2}$$

$$\langle A(x_1)B(x_2)C(x_3)D(x_4) \rangle = \text{cross } f(p, \eta)$$

$$f(p, \eta) = \eta^{\frac{1}{2}(l_A - l_B + l_C - l_D)} p^{-\frac{1}{2}(l + l_C - l_D)}$$

$$\left\{ \left[ \frac{\Gamma(-\frac{1}{2}(l_A - l_B + l_C - l_D))}{\Gamma(l - l_A + l_B)} \right] \Gamma(l + l_C - l_D) \right.$$

$$F_4 \left( \frac{1}{2}(l + l_C - l_D); \frac{1}{2}(l + l_A - l_B); l + 1 - \frac{P}{2}; \right.$$

$$\left. 1 + \frac{1}{2}(l_A - l_B + l_C - l_D); \frac{1}{2}p, \eta_p \right)$$

$$+ \left( \frac{p}{q} \right)^{\frac{1}{2}(l_A + l_B + l_C - l_D)} / (\Delta_{mn} - \Delta_{pq}, A \in D - Q_A)$$