

Comments on String Theory on AdS_3

D. Kutasov, N. Seiberg, A.G.

9806194

Amit Giveon

Strings 98

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- string theory $\backslash w \Lambda < 0$
"defined" through AdS/CFT
- studied extensively in SUGRA limit

THIS TALK IS ABOUT
STRING THEORY $\backslash w \Lambda < 0$.
on AdS_3 :

- Standard worldsheet construction
Produces direct relation to
bdry CFT: operators,
bdry coor., dimensions
- string theory automatically
computes the interesting quantities
like correlation funs. in bdry CFT

Applications to some problems of recent interest:

- Virasoro alg. and affine G in spacetime are identified with ∞ # of holomorphic vertex operators

$$\oint dz \, f(z)$$

- BTZ black holes
Fundamental string states

- Establish conjectured

$$AdS_3 \leftrightarrow CFT$$

- States contributing to entropy of black objects with AdS_3 near horizon geometry are identified
- The boundary coor. of AdS_3 are identified explicitly in string theo. and may lead to better understanding of holography

2. Bosonic Strings on

$$\text{AdS}_3 \times \mathcal{N}$$

$$\hat{\text{SL}}(2)_k \times \hat{\text{SL}}(2)_k$$

$$J^A(z), \bar{J}^A(\bar{z})$$

target space
for a CFT

$$C = 26 - \frac{3k}{k-2}$$

$$J^A(z) J^B(w) = \frac{k \eta^{AB}}{(z-w)^2} + \frac{i \epsilon^{AB} c J^c}{z-w}$$

$$A, B = 1, 2, 3$$

$$\text{sign}(\eta) = (+ + -)$$

$$C = \frac{3k}{k-2}$$

Remarks

- Tachyon \rightarrow will disappear in SUSY case
- Bosonic case technically simpler – generalize to superstring later

Geometry

Euclidean $\text{AdS}_3 \equiv H_3^+$:

r, τ, θ

$$ds^2 = \left(1 + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + \ell^2 \left(1 + \frac{r^2}{\ell^2}\right) d\tau^2 + r^2 d\theta^2$$

$\phi, \gamma, \bar{\gamma}$

$$ds^2 = \ell^2 (d\phi^2 + e^{2\phi} d\gamma d\bar{\gamma})$$

$$\gamma = \frac{r}{\sqrt{\ell^2 + r^2}} e^{-\tau + i\theta}$$

$$\phi = \tau + \frac{1}{2} \log \left(1 + \frac{r^2}{\ell^2}\right)$$

Boundary of AdS_3 :

$$r \rightarrow \infty$$

$$\phi \rightarrow \infty$$

$$(\tau, \theta)$$

$$\gamma \approx e^{-\tau + i\theta}$$

cylinder

$(\gamma, \bar{\gamma})$ Sphere

Free field rep. of $SL(2)$

$$\mathcal{L} = \frac{2\ell^2}{l_s^2} (\partial\phi\bar{\partial}\phi + e^{2\phi}\bar{\partial}\gamma\partial\bar{\phi})$$



$$\partial\phi\bar{\partial}\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\phi} - e^{-2\phi}\bar{\beta}\bar{\beta}$$

↓
exp. renormalize
and dilaton turned on,
like in Liouville

$$\partial\phi\bar{\partial}\phi - \frac{2}{\alpha_+} R^{(2)}\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\phi} - \beta\bar{\beta}e^{-\frac{2\phi}{\alpha_+}}$$

$$\alpha_+^2 = 2k - 4$$

$$\ell^2 = l_s^2 k$$

$$J^3 = \beta\gamma + \frac{\alpha_+}{2} \partial\phi$$

$$J^+ = \beta\gamma^2 + \alpha_+\gamma\partial\phi + k\partial\gamma$$

$$J^- = \beta$$

$$V_{jm\bar{m}} = \gamma^{j+m} \bar{\gamma}^{j+\bar{m}} e^{\frac{2\phi}{\alpha_+}}$$

Spacetime Properties of Strings on AdS_3

- Worldsheet affine $SL(2)$



3 conserved charges in the spacetime theory:

$$L_0 = -\oint dz J^3(z)$$

$$L_{\pm 1} = -\oint dz J^{\pm}(z)$$

which satisfy

$$[L_n, L_m] = (n-m)L_{n+m} \quad n, m = 0, \pm 1$$

- This extends to ∞ -d Virasoro

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3-n)\delta_{n+m,0}$$

- Our next task: find Virasoro and compute c

- Warmup exercise:

Worldsheet CFT on \mathcal{N} contain affine G

$$K^a(z) K^b(w) = \frac{k' \delta^{ab}/2}{(z-w)^2} + i f^{ab}_c K^c$$

$$a, b, c = 1, \dots, \dim G$$



Normally, this leads to $\dim G$ conserved currents in spacetime:

$$T_0^a = \oint dz K^a(z)$$

$$[T_0^a, T_0^b] = i f^{ab}_c T_0^c$$

However, in our case the spacetime theory is 2-d CFT and we find

Affine G in spacetime:

$$[T_n^a, T_m^b] = i f^{ab} \underset{c}{\cancel{T}}{}_{n+m}^c + \frac{\tilde{k}}{2} n \delta^{ab} \delta_{n+m,0}$$

$$[L_m, T_n^a] = -n T_{n+m}^a$$

$$T_n^a = \oint dz K^a(z) \gamma^n(z)$$

$$V_{j=0,m} \equiv \gamma^m$$

$$[T_n^a, T_m^b] = \oint dw \oint dz K^a \gamma^n(z) K^b \gamma^m(w)$$



$$K^a(z) K^b(w) = \frac{if^{ab}{}_c K^c}{z-w} + \frac{k' \delta^{ab}/2}{(z-w)^2}$$

↓

$$\text{if } f^{ab}{}_c \oint dw \oint dz \frac{K^c \gamma^{n+m}(w)}{z-w}$$

$$\text{if } f^{ab}{}_c \oint dw K^c \gamma^{n+m}$$

$$\text{if } f^{ab}{}_c T_{n+m}^c$$

$$\frac{k' \delta^{ab}}{2} \oint dw \oint dz \frac{\gamma^n(z) \gamma^m(w)}{(z-w)^2}$$

||

$$\frac{k' \delta^{ab}}{2} \oint dw (\partial_w \gamma^n) \gamma^m$$

||

$$\frac{n k' \delta^{ab}}{2} \oint dw \gamma^{n+m-1} \partial_w \gamma$$



central

commutes with T_n^a

(and all phys. vertex operators)

$$\oint dw \gamma^{n+m-1} \partial_w \gamma = P \delta_{n+m,0}$$

$$P = \oint dz \frac{\partial_z \gamma}{\gamma}$$

↑

charge carried by the vacuum
counting winding # of γ around 0
when z winds once

- Spacetime interpretation:

- $P = \#$ of times string worldsheet wraps Θ
- target-space simply connected \Rightarrow
no winding of perturbative string states

$$[P, V_{\text{phys}}] = 0 \quad \downarrow$$

Interpret string theory on AdS_3
as having P stretched fundamental
strings at $r \rightarrow \infty$

summary:

- Affine G on worldsheet is lifted to Affine G in spacetime with level

$$k_{\text{spacetime}} = P k'$$

- string vacua with different P correspond to
 - II- Sectors of the theory

We are now ready to turn to the original problem of finding the spacetime Virasoro algebra:
2.9

$$L_n = \oint dz [n J^{-} \gamma^{n+1} - (n+1) J^3 \gamma^n]$$

$$[L_n, L_m] = \oint dw \oint dz [\quad](z) [\quad](\omega)$$

$$= (n-m)L_{n+m} + \frac{C_{\text{spacetime}}}{12} (n^3 - n) \delta_{n+m, 0}$$

$$C_{\text{spacetime}} = 6 k P$$

level of $SL(2)$

of fun.
strings in the
 $SL(2)$ backg.

Remark

Vir. alg. acts as holomorphic reparametrization symm. on γ :

$$[L_n, \gamma(z)] = -\gamma^{n+1}(z)$$



$$L_n \equiv -\gamma^{n+1} \frac{\partial}{\partial \gamma}$$

Quantization of g_s :

(2+1)-d gravity with $\Lambda < 0$

$$S = \frac{1}{16\pi G_N} \int d^3x \sqrt{-g} \left(R + \frac{2}{\ell^2} \right) + S_m$$



$$C_{\text{spacetime}} = \frac{3\ell}{2G_N} \quad \begin{array}{l} \leftarrow \text{radius of} \\ \text{AdS}_3 \\ \Lambda = -1/\ell^2 \end{array}$$

$\ell_p = \text{Newton's const.}$

In string theory:

$$\left. \begin{array}{l} \ell = \sqrt{R} l_s \\ \frac{1}{\ell_p} = \frac{\sqrt{r}}{g_s^2 l_s^{24}} \\ C_{\text{spacetime}} = 6kp \end{array} \right\} \Rightarrow g^2 \sim \frac{1}{P}$$

physical states, for example

$$\sqrt{\text{phys}}^{(S)}(\mathfrak{J}, m, \bar{m}) = W_{\Delta_L, \Delta_R}(\mathfrak{J}) \gamma^{\Delta_R - \Delta_L} \sqrt{g_{mm}}$$

primary of the
2-d CFT on \mathcal{N}
with $\text{dim} = (\Delta_L, \Delta_R)$
and spin $S = \Delta_R - \Delta_L$

coupled to

AdS_3
sector

$$\sqrt{g_{mm}} = \gamma^{\mathfrak{J}+m} \bar{\gamma}^{\mathfrak{J}+\bar{m}} e^{\frac{2\mathfrak{J}}{\alpha_+} \phi}$$

In spacetime

$$\sqrt{\text{phys}}^{(S)}(\mathfrak{J}, m, \bar{m}) \leftrightarrow A_{m, \bar{m}}^{h, \bar{h}}$$

mode m, \bar{m} of primary

in spacetime with

Scaling dim = (h, \bar{h}) ; $\bar{h} = \mathfrak{J} + 1$

and

Spin = $S = \bar{h} - h$; $h = \mathfrak{J} + 1 - S$

to see how it works, restrict to holomorphic part,
for simplicity: example

$$V_{\text{phys}}(j, m) = W_N V_{jm}$$

Primary of
CFT on \mathcal{N}
 $\dim(W_N) = N$

$\xrightarrow{\quad}$
 $\begin{array}{c} \text{AdS}_3 \text{ "dressing"} \\ \downarrow \\ J^3 \text{ quantum \#} \end{array}$

$$\dim(V) = N - \frac{j(j+1)}{k-2} = 1$$

$$[L_n, V_{\text{phys}}(j, m)] = (n_j - m) V_{\text{phys}}(j, m+n)$$



$V_{\text{phys}}(j, m) \leftrightarrow$ mode m of
primary operator
in the spacetime
CFT with scaling
dim.:

$$h = j + 1$$

explanation: 2.13

In 2-d CFT: primary $A^{(h)}(z)$
 h =scaling dim. has mode expand.:

$$A^{(h)}(z) = \sum_m A_m^{(h)} z^{-m-h}$$

$A_m^{(h)}$ satisfy:

$$[L_n, A_m^{(h)}] = [n(h-1)-m] A_{n+m}^{(h)}$$



$$V_{\text{phys}}(j, m) \leftrightarrow A_m^{(h)} ; \quad h=j+1$$

properties of phys. states under
spacetime \hat{G} : generators of G
in rep. R

worldsheet \downarrow

$$K^a(z) W_N(\omega) = \frac{t^a(R)}{z - \omega} W_N(\omega)$$

spacetime \downarrow

$$[T_n^a, V_{\text{phys}}(j, m)] = t^a(R) V_{\text{phys}}(j, m+n)$$

i.e., V_{phys} transforms as a
primary in rep. R
of spacetime \hat{G}

summary

Worldsheet \leftrightarrow Spacetime
correspondence:

$SL(2)_k \times SL(2)_k \leftrightarrow \text{Vir.} \times \text{Vir. } c=6kp$

$$\hat{G}_k \leftrightarrow \hat{G}_{kp}$$

$$j, s \leftrightarrow h, \bar{h} = h + s$$

$$R \text{ of } G \leftrightarrow R \text{ of } G$$

$$\text{Left/Right moving} \leftrightarrow \text{L/R chirality}$$

$\langle V_{\text{phys}} \dots V_{\text{phys}} \rangle$ satisfy

Ward identities of 2-d CFT

AdS₃ / 2-d CFT

• Fermionic strings on AdS_3 :

$$N=0 \rightarrow N=1$$

• Superstrings on $M = AdS_3 \times S^3 \times T^4$:

perform chiral "GSO projection"

- find 8 mutually local BRST invariant spin fields $S \Rightarrow$ supercharges:

$$Q = \oint dz S$$

- together with $C = 6kp$ Vir. alg.
and $\hat{SU}(2)_{kp}$



$N=(4,4)$ SCFT in spacetime

- Physical states, Ward identities,...
- NS/R sectors

...

4. Applications

- Relation to the Theory of the NS Fivebrane
- Relation to the D1/D5 System
- Comparison to σ -model on
 T^{4RP}/S_{kp}
- BTZ black holes
and
Fundamental string states

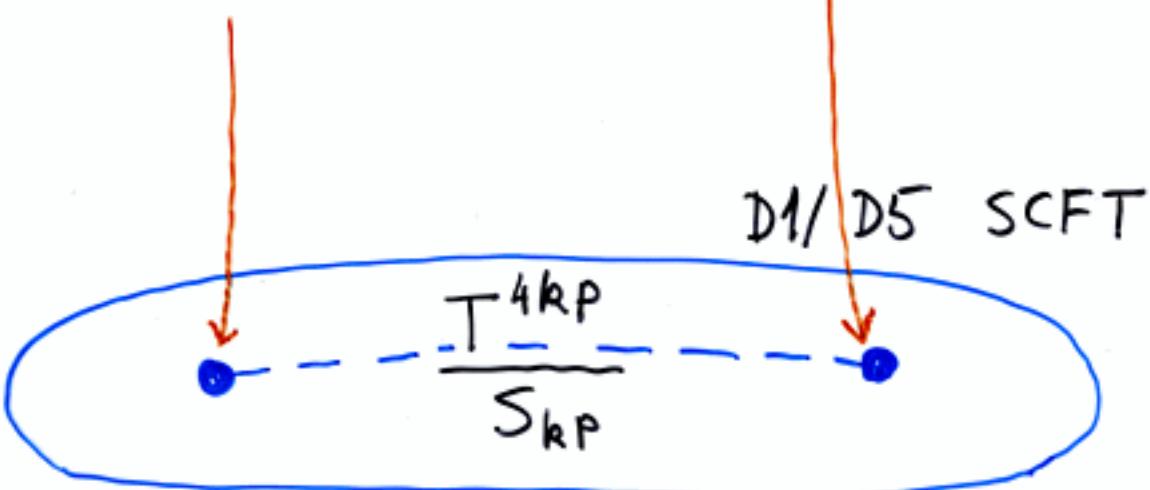
$k \times \text{NS5} + p \times \text{F1}$ $Q_5 \times \text{D5} + Q_1 \times \text{D1}$ 

throat limit

$$\frac{v k}{P} \ll 1$$

weakly coupled strings

$$\frac{P}{k v} \ll 1$$

superstring on
 $\text{AdS}_3 \times S^3 \times T^4_v$ with
RR backg., $\hat{v} = \frac{P}{k}$ 

BTZ black holes

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 (N^\phi dt + d\phi)^2$$

$$N^2 = \left(\frac{r}{\ell}\right)^2 - 8\ell_p M + \left(\frac{4\ell_p J}{r}\right)^2$$

$$N^\phi = -\frac{4\ell_p J}{r^2}$$

- $M=J=0$ bh \equiv R vacuum of spacetime SCFT
- $M\ell = -\frac{\ell}{8\ell_p} = -\frac{c}{12} = -\frac{kP}{2} \equiv$ NS vacuum

$$M\ell = L_o + \bar{L}_o \quad J = L_o - \bar{L}_o$$

Bekenstein-Hawking entropy:

$$S = \frac{A}{4\ell_p} = \pi \sqrt{\frac{\ell(\ell M + J)}{2G_3}} + \pi \sqrt{\frac{\ell(\ell M - J)}{2G_3}}$$

$$= 2\pi \sqrt{kPL_o} + 2\pi \sqrt{kP\bar{L}_o}$$

compare to spacetime CFT:

$$S_{\text{CFT}}(L_0, \bar{L}_0) = 2\pi \sqrt{\frac{c L_0}{6}} + 2\pi \sqrt{\frac{\bar{c} \bar{L}_0}{6}}$$

Remarks

- M, J bh \leftrightarrow multi-states with L_0, \bar{L}_0 total $\leftrightarrow M, J$ corresponding in string theory to

Fundamental string states

- $S_{\text{CFT}} \leftrightarrow$ unitary CFT; for superstring on $AdS_3 \times S^3 \times T^4$: spacetime theory is unitary (at least at weak coupling: $P \gg 1$)
- Mass gap of BTZ bh:

$$M \sim \frac{L_0}{\ell} \sim \frac{1}{\ell} \sim \frac{1}{\ell_p k P} \ll \frac{1}{\ell_p}$$

at large

$$k \equiv Q_S$$

$$P \equiv Q_I$$

natural mass scale of the theory