

# Non-perturbative String Theory and Supersymmetric Yang-Mills

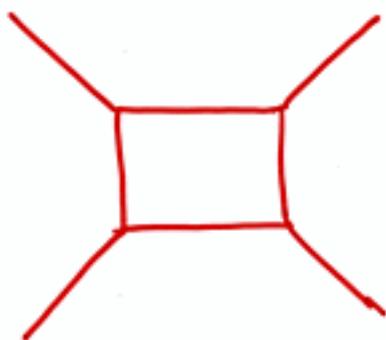
Strings '98 ITP  
Santa Barbara

- ① Nonperturbative terms in  
IIB effective action  
(with Gutperle, Vanhove, Kwon)
- ② Connection with  $N=4$   
Supersymmetric Yang-Mills theory  
(with Banks)  
D-instantons - Yang-Mills instantons  
(with Bianchi, Kovacs + Rossi)

Recall (Strings '97):

Higher derivative terms in effective action e.g.  $R^4$   
↑ Riemann

Motivated by one-loop calculation in  $d=11$  supergravity:



Compactify on  $T^2$   
volume  $V = r_B^{-4/3} e^{\phi/3}$   
↑ IIB radius      ↖ IIB dilaton

Complex structure  $\Omega = e = c^{(0)} + i e^{-\phi}$

$V \rightarrow 0$  gives IIB with  $d=10$

# IIB effective action (M.B.G., Gutperle, Vanhore)

$$\frac{1}{(\alpha')^4} \int d^{10}x \sqrt{G} e^{-2\phi} \left[ R + (\alpha')^3 \left( e^{3\phi/2} f_4(\rho, \bar{\rho}) R^4 + \text{related terms} \right) + O((\alpha')^4) \right]$$

where:

$$f_4(\rho, \bar{\rho}) = \sum_{(l_1, l_2 \neq 0, 0)} \frac{\rho_2^{3/2}}{|l_1 + l_2 \rho|^{3/2}}$$

$c^{(0)} + i e^{-\phi}$  SL(2, Z) invariant

$$\rho_2^{1/2} f_4(\rho, \bar{\rho}) \underset{e^{\phi} \rightarrow 0}{\sim} 2\zeta(3) \rho_2^2 + \frac{2\pi^2}{3}$$

tree 1-loop

$$+ \sum_{k=1}^{\infty} \mu(k) (k \rho_2)^{1/2} \left( e^{2\pi i k \rho} + e^{-2\pi i k \bar{\rho}} \right) \times (1 + O(\rho_2^{-1}))$$

$$\mu(k) = \sum_{K|M} \frac{1}{m^2}$$

Series of D-instanton terms  
Action  $S_K = 2\pi \rho_2 K$

Further evidence:

[ Berkovitz; Antoniadis, Pioline, Taylor; Pioline ]

Many other terms related by  
linearized SUSY:

"On-shell" superfield  
( $D\bar{\Phi} = 0$      $D^4\Phi = \bar{D}^4\bar{\Phi}$ )

$\theta^a$   $a=1, \dots, 16$

$(dB_{NS} + i dB_{RR})$

$$\Phi(x, \theta) = e + \theta\Lambda + \bar{\theta}\Gamma^{\mu\nu}\epsilon\theta G_{\mu\nu}$$

$$+ \theta^3 d\psi_\mu + \theta\Gamma^{\mu\nu}\epsilon\theta\bar{\theta}\Gamma^{\sigma\omega}\epsilon\theta R_{\mu\nu\sigma\omega}$$

+ ...

Weyl tensor

$$\text{Action} = \int d^{16}\theta d^{10}x F[\Phi]$$

Picks out 16- $\theta$  terms:

e.g.  $R^4$ ,  $G\bar{G}R^2$ , ...,  $\Lambda\bar{\Lambda}R^2$ ,  $\Lambda\bar{\Lambda}\psi\bar{\psi}$   
 $U(1)$  charge = 0

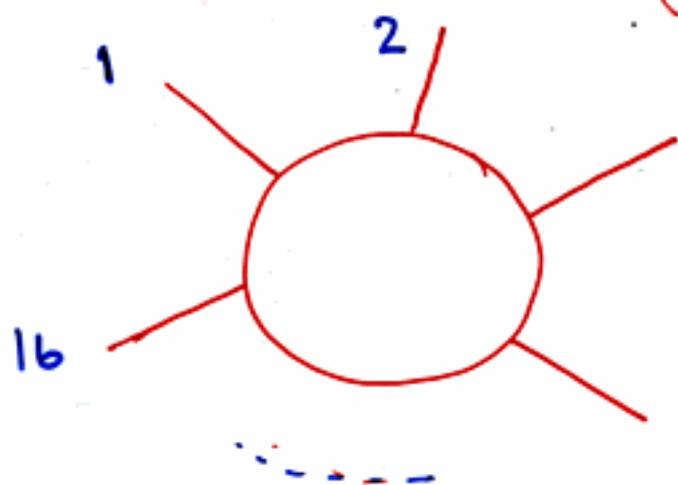
$$G^8, G^6\Lambda^4, \dots, \bar{G}^2\Lambda^4, \Lambda^{16}$$

8                  16                                  4                  24

$U(1)$  charge violation  
 $SL(2, \mathbb{R}) \rightarrow SL(2, \mathbb{C})$

$\Lambda^{16}$  (M.B.S., Gutperle + Kwon)  
term

Consider  $16$ -gravitino amp. in  $d=11$   
(on  $T^2$ )



Identify  $\Lambda^a$  in terms of  $\Psi_\mu^a$   
IIB M-theory

$$\Lambda = \frac{1}{2} (1 + \Gamma_{\bar{z}}) \mathcal{P}_{\bar{z}} \Psi_{\bar{z}}$$

$\uparrow$   
 $\Gamma_{\bar{z}} \Gamma_{\bar{z}}$

$\leftarrow z$ : complex coord on  $T^2$

$$\Gamma_{\bar{z}} = \frac{1}{2} (\Gamma^{11} + i \Gamma^9)$$

$\Lambda$  Vertex operator:  $(q^A \mathcal{P}_{\bar{z}} \Gamma_{\bar{z}} \Lambda) e^{ik \cdot X}$

$$\{q^A, q^B\} = (\Gamma \cdot p)^{AB}$$

- $$f_{16} = (e_2 D)^{12} f_4(e, \bar{e})$$

$$D F_d = \left( i \frac{\partial}{\partial e} + \frac{d}{2e_2} \right) F_d$$

Modular  
covariant  
derivative

- Charge-K D-instanton contributions satisfy (as  $e_2 \rightarrow \infty$ )

$$f_{16}^{(N)} = e_2^{12} \frac{\partial^{12}}{\partial e^{12}} f_4^{(N)}(e, \bar{e})$$

c.f. linearized superspace:

$$S = \text{Re} \int d^{10}x d^{16}\theta e_2^{-4} e^{2\pi i K \Phi} \mu(K)$$

$\uparrow$   
 $g_s^4$

$$S_{\Lambda^6} = \alpha^3 \int d^{10}x \sqrt{G} e_2^{\frac{1}{2}} f_{16} (e\bar{e}) \Lambda^6$$

where

$$f_{16} = \Gamma\left(\frac{27}{2}\right) e_2^{3/2} \sum_{l_1, l_2 \neq (0,0)} \frac{(l_1 + l_2 \bar{e})^{24}}{|l_1 + l_2 e|^{27}}$$

$$\xrightarrow{SL(2, \mathbb{Z})} \left( \frac{ce + d}{c\bar{e} + d} \right)^{12} f_{16} \quad (12, -12) \text{ Modular form}$$

discrete phase

$$e_2^{\frac{1}{2}} f_{16} \sim \Gamma\left(\frac{27}{2}\right) \zeta(3) e_2^2 \overset{\text{tree}}{+} \Gamma\left(\frac{23}{2}\right) \zeta(2) \overset{\text{1-loop}}{+} \left( \right) \sum_{\mathbf{k}} \mu(\mathbf{k}) (k e_2)^{\frac{25}{2}} e^{2\pi i \mathbf{k} e} (1 + o(e_2^{-1}))$$

$$\mu(\mathbf{k}) = \sum_{\mathbf{K} | \mathbf{m}} \frac{1}{m^2}$$

- No explicit string tree or loop calculations exist.

# General structure of interactions

$$\int d^{10}x \sqrt{G} e^{-\phi/2} F_{\mathcal{P}}^{(d)}(e, \bar{p}) \mathcal{P}(\Psi) + c.c.$$

↑  
monomial of  $p$  fields

$$e^{-\phi/2} F_{\mathcal{P}}^{(d)} = e^{-2\phi} c_{tree} + c_{1-loop} + \sum_{K=1}^{\infty} G_{K, \mathcal{P}}^{(d)}(e, \bar{p})$$

D-INSTANTON TERMS

$G_{K, \mathcal{P}}^{(d)} \sim e^{\phi \rightarrow 0}$

$$\mu^{(d)}(K) e^{-2\pi(S_K + iKc^{(0)})} S_K^{a_d + \phi}$$

$S_K = Ke^{-\phi}$

D-INSTANTON ACTION

$a_{10} = -7/2$   
 $a_6 = -3/2$   
 $a_4 = -1/2$

$$\mu^{(10)}(K) = \sum_{K|m} \frac{1}{m^2}$$

$$\left( \mu^{(6)}(K) = \frac{1}{K^2} = \mu^{(4)}(K) \right)$$

(M.B.S + Gutperle)

(evaluated directly by Moore, Nekrasov + Shatashvili)

$$\mu(K) = \sum_{\text{SUCK}} = \frac{1}{(\text{vol SU}(K))} \int d\Psi dA e^{-S_{YM}}$$

K x K matrices

zero-dimensional matrix model

Connected to Witten index for D-particles

# Connection with Yang-Mills

(Maldacena, Gubser, Klebanov, Polyakov, ...)

IIB on  $AdS_5 \times S^5$

$$(ds)^2 = \frac{L^2}{e^2} (dx^2 + de^2) + L^2 d\Omega_5^2$$

$\uparrow$   
( $x_0, \underline{x}$ )

Conformally  
flat

- Superconformal  $N=4$  Yang-Mills on boundary ( $e=0$ )

"On-shell"  
IIB amplitudes

$g_s$

$\frac{\alpha'}{L^2}$

(s-wave)

Source

$G_{\mu\nu}$

$e^{\phi}$

$C$

dilaton

$\vdots$



Correlation fns. of  
gauge in ops. in Y.M.

$g_{YM}^2$

$(g_{YM}^2 N)^{-1/2}$

$T_{\mu\nu}$

Y.M. operator

$(F_{\mu\nu})^2 + \dots$

action

$\vdots$

What is effect of  $R^4$  term?

$$O(\alpha'^3) = O(g_{\text{YM}}^2 N)^{-3/2}$$

• Note  $R^4 \equiv \int d^{16} \theta R_{\theta}^4$

where

$$R_{\theta} = \bar{\theta} \Gamma^{\mu\nu\rho} \theta \bar{\theta} \Gamma^{\sigma\omega} \theta R_{\mu\nu\sigma\omega}$$

only involves Weyl tensor

- Expand around background

$$R = R_0 + \hat{R} \quad \text{fluctuation}$$

$R_0 = 0$  in  $AdS_5 \times S^5$

- Expanding  $R^4$  around background

→ 0, 1, 2, 3-graviton terms = 0

⇒ i) Stability of background

Non-ren. at  $O(\alpha'^3)$  {

ii)  $\langle T(x_1) T(x_2) \rangle' = 0$

iii)  $\langle T(x_1) T(x_2) T(x_3) \rangle' = 0$

No interaction corrections  
to superconformal current correlators

# 4-graviton amplitude

$$\frac{e^{-2\phi}}{(\alpha')^4} \left[ \underset{\substack{\uparrow \\ \text{Einstein-Hilbert}}}{A_4^{(1)}} + \underbrace{(\alpha')^3 e^{3\phi/2}}_{N^{-3/2}} f(e, \bar{e}) \underset{\substack{\uparrow \\ R^4}}{A_4^{(2)}} \right]$$

⇒ AdS/YM correspondence

$$\langle T(x_1) T(x_2) T(x_3) T(x_4) \rangle$$

$$= A_4^{(1) \text{ YM}} + \frac{1}{N^{3/2}} f(S, \bar{S}) A_4^{(2) \text{ YM}}$$

$\uparrow$   $S = \frac{\theta}{2\pi} + i \frac{4\pi}{g_{\text{YM}}^2}$

SL(2, Z) Duality (Olive, Montonen, Witten, Sen)

Does not commute with 't Hooft limit :

$g_{\text{YM}}^2 \sim N$  fixed

$$A_4^{(1) \text{ YM}} + A_4^{(2) \text{ YM}} \left[ \frac{2\zeta(3)}{(g_{\text{YM}}^2 N)^{3/2}} + \frac{1}{N^2} \frac{2\pi^2}{3} (g_{\text{YM}}^2 N)^{1/2} \right]$$

$$+ \frac{(4\pi)^{3/2}}{N^{3/2}} \sum_{k=1}^{\infty} \mu(k) k^{1/2} \left( e^{-k \left( \frac{8\pi^2}{g_{\text{YM}}^2} + i\theta \right)} + \text{c.c.} \right)$$

Charge-k instanton

$(1 + O(g_{\text{YM}}^2/k))$

Asymptotic series of fluctuations

# Comments on instanton terms

(Bianchi, M.B.G., Kovacs, Rossi)

i) N=4 Yang-Mills instanton (SU(2))

$$F_{\mu\nu}^{-a} = \frac{\eta_{\mu\nu}^a \rho_0^2}{((x-x_0)^2 + \rho_0^2)^2}$$

← 't Hooft symbol  
scale,  $\rho_0$

$$\bullet (F^-)^2 = \frac{\rho_0^4}{((x-x_0)^2 + \rho_0^2)^4} = K_4 (z_0^M; x^\mu)$$

identical to bulk-to-boundary <sup>(massless)</sup> scalar

S-wave propagator on AdS<sub>5</sub> × S<sup>5</sup>!

Propagating from  $(x_0^\mu, y_0^i)$  to  $(x^\mu, \rho=0)$   
 $|y_0| = \rho_0$

• Measure

$$\int \frac{d^4 x_0 d\rho_0}{\rho_0^5}$$

$$g_{\text{YM}}^8 \pi d^8 \eta d^8 \xi$$

SUSY

Special SUSY

- Use broken SUSY to determine supermultiplet of zero modes:

e.g.

$$\delta \lambda_{\alpha}^A = -\frac{1}{2} F_{\mu\nu}^{-} \sigma_{\alpha}^{\mu\nu \beta} \lambda_{\beta}^A$$

$$\delta \phi^{AB} = \frac{1}{2} \lambda^{\alpha[A} \lambda_{\alpha}^{B]}$$

SU(4)

where

$$\lambda_{\alpha}^A = \frac{1}{e^{1/2}} \left[ \rho_0 \eta_{\alpha}^A + \sigma_{\alpha\dot{\alpha}}^M \bar{\lambda}^{A\dot{\alpha}} (x_0 - x) \gamma^M \right]$$

$$= \frac{1}{e^{1/2}} \left( \frac{1-\gamma^5}{2} \right) \gamma^M (z_0 - z)_M \lambda_{(5)}^A$$

↑ AdS<sub>5</sub> coordinate
 ↑ SO(5) spinor

Killing spinor on AdS<sub>5</sub>

- Consider component of superconformal current multiplet

$$\hat{\lambda}_{\alpha}^A = -\sigma_{\alpha}^{\mu\nu \beta} F_{\mu\nu}^{-} \lambda_{\beta}^A$$

$$= K_4(z_0^M; x^M) \lambda_{(5)}^A$$

↑
↑

(F<sup>-</sup>)<sup>2</sup>

- c.f. Bulk-to-boundary propagator of spin- $\frac{1}{2}$  fermion in  $AdS_5$  with dimension  $\Delta = \frac{7}{2}$   
(Henningson + Sfeetsos)

$\Lambda_\alpha^A$  IIB "dilatinos"

$f_{16}(\rho, \bar{\rho}) \Lambda^{16}$  interaction contains a  $K=1$

D-instanton contribution

$$t_{16} e^{-\frac{2\pi}{g_s} \int \frac{d^4 x_0 d\rho_0}{\rho_0^5} \text{Vol}(S^5) \prod_{p=1}^{16} [K_4(z_0; x_p) \gamma \cdot (z_0 - x) \rho_0^{-1/2}]} \Lambda_P^{A_p}$$

Integral over  $AdS_5$  (pointing to the integral)

Volume of  $S^5$  (pointing to  $\text{Vol}(S^5)$ )

Which has the same form as Yang-Mills instanton contribution.

BUT where is the  $S^5$  in Yang-Mills picture??

- IIB expression provides  $SL(2, \mathbb{Z})$ -invariant completion of correlation fn.