

ASPECTS OF LARGE N GAUGE DYNAMICS AS SEEN BY STRING THEORY

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- PROPERTIES of Wilson Loops $\left\{ \begin{array}{l} \text{Glueballs + Cont. Limit} \\ \text{Higher Reps} \\ \text{BARYONS} \end{array} \right.$
- SHORT DISTANCE SINGUL + LOOP EQS
- LARGE N Phase Transitions
& BRINKY Exclusion Principle.

AdS \longleftrightarrow CFT

[Maldacena]

String Theory on $= N=4$ SUSY-YM
 $\text{AdS}_5 \times \text{S}_5$ in $d=4$

$$g_s = \frac{\lambda}{N}$$

$$R/\sqrt{\alpha'}$$

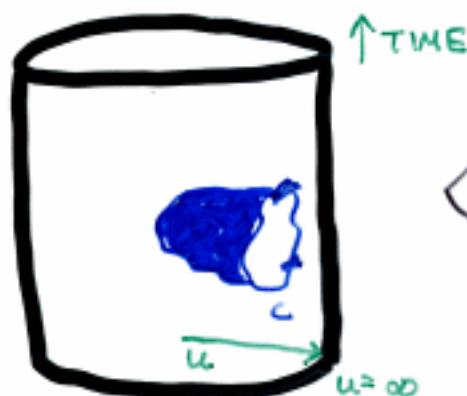
$$g_{YM}^2$$

$$(g^2 N)^{1/4} = \lambda^{1/4}$$

$Z_{IIB}[\phi_B]$
 Fields $\phi \rightarrow \phi_B$
 $u \rightarrow \infty$

$\langle \exp \int \phi_B \hat{\theta} \rangle$
 CFT
 Gubser, Klebanov, Polyakov
 Witten

$$l_s^{-2} ds^2 = R^2 \frac{du^2}{u^2} + u^2 d\bar{x}^2 + R^2 d\text{S}_5^2$$



$$\langle W_C \rangle = \int dX e^{-S_{IIB}}$$

$$X[u=\infty] = X_C$$

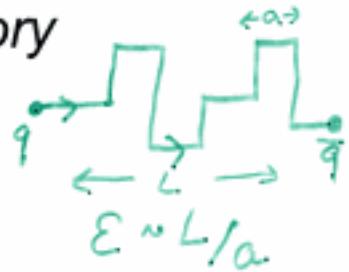
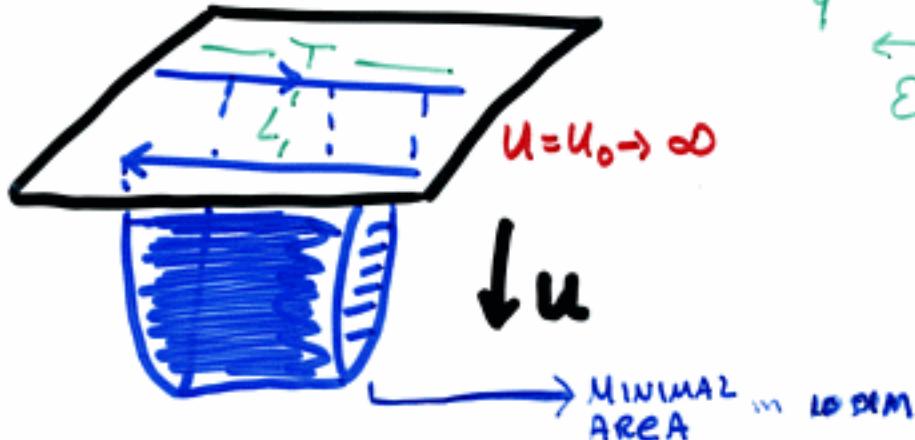
SURPRISES :

- The Large N Limit of a Non-Confining YM theory (Coulomb phase) = String Theory

Expected for a confining theory

- The Existence of a simple strong coupling expansion for continuum YM theory = α' expansion of String Theory

True for a Lattice gauge theory



$$ds^2 \propto u^2 d\bar{x}^2$$

MALDACENA

$$u=0$$

$$\langle W(L) \rangle \sim e^{-T(g_s^2)^\frac{1}{4} [L U_0 - C_L]}$$

QCD₄ AT FINITE T (T → ∞ ≡ QCD₃)

Compactify SUGRA on CIRCLE + BREAK SUSY BY
AP boundary conditions on fermions

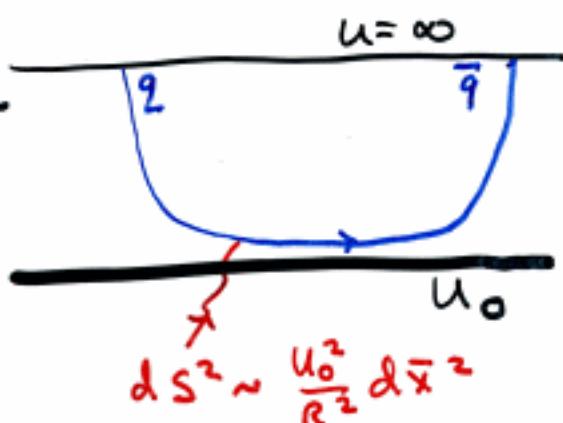
$$\frac{MF}{MS} \sim \frac{T}{g^2 T} \rightarrow \infty$$

$$l_s^{-2} ds^2 = \frac{R^2}{u^2} \frac{du^2}{1 - u_0^4/u^4} + \frac{u^2}{R^2} \left(1 - \frac{u_0^4}{u^4} \right) d\gamma^2 + d\bar{x}^2 + R^2 d\mathcal{R}_5^2$$

$$u_0 = \sqrt{g^2 N} \cdot T$$

$$\langle W_{\text{SPATIAL}}^{(c)} \rangle \sim e^{-A(c)\sigma}$$

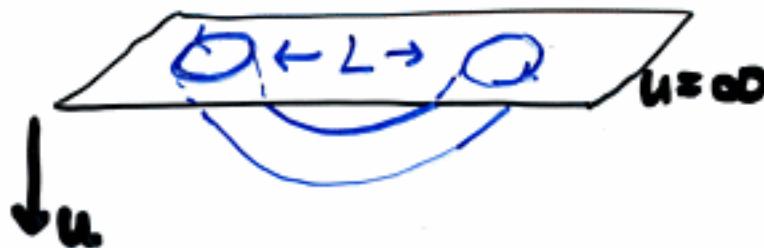
$$\sigma = \text{string tension} = \alpha \sqrt{g^2 N} \cdot T^2$$



$$g_3^2 = g_4^2 \cdot T - \quad \text{CONT. Limit need to take} \\ g_3^2 \sim \frac{\Lambda}{T}, T \rightarrow \infty$$

- Finite T -phase transition -
"Electric" (Screening) Mass
- * MASS GAP
- GLUE BALL SPECTRUM

Rey + Yee
 Minahan
 Brandhuber, Ishiki,
 Sonnenburg
 + Yankielowicz
 M-Li
 D F, Ooguri
 Cacciatori, Ooguri, Oz, ...
 :



Beyond LcR - NO AREA MINIMIZING WORLD SHEET

$$\propto R_{\text{loop}}$$



\Rightarrow (numer)

graviton, dilaton, ...
 = zero size string



spectrum governed by
 'masses' of Laplacian on SUPER BACKGND FOR Large coupling.

QCD₄

mmmm



$$M_{\text{glue}} \sim \Pi \cdot \sqrt{g_T^2 N}$$

$$\Pi f(g_T^2 N)$$

CONTINUUM

$$g^2 N \sim \gamma_B n T$$

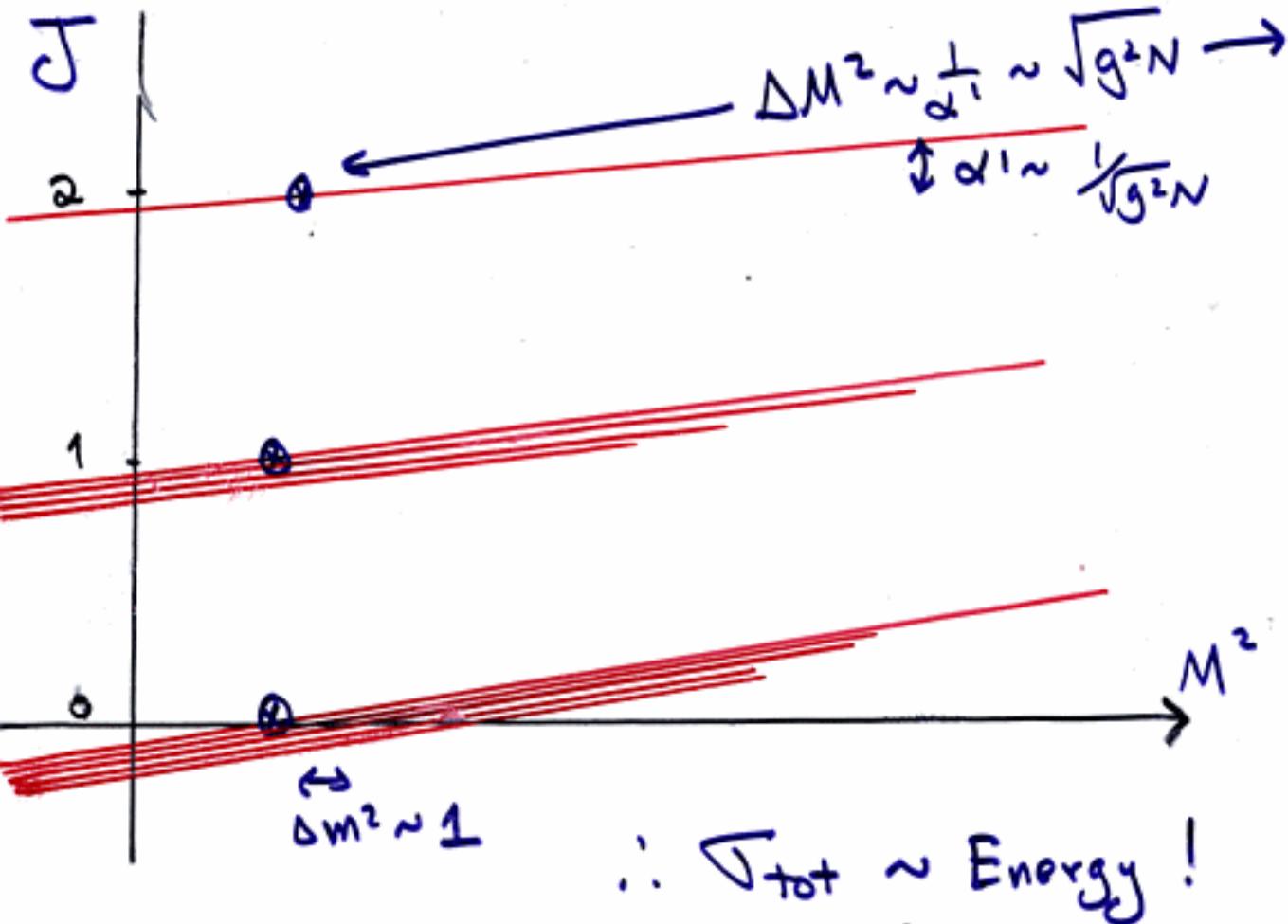
HOPE THAT: $f \sim \exp[-\frac{1}{N}]$!!

$$m_g \sim \frac{1}{a} 4 \ln g_a^2 N$$

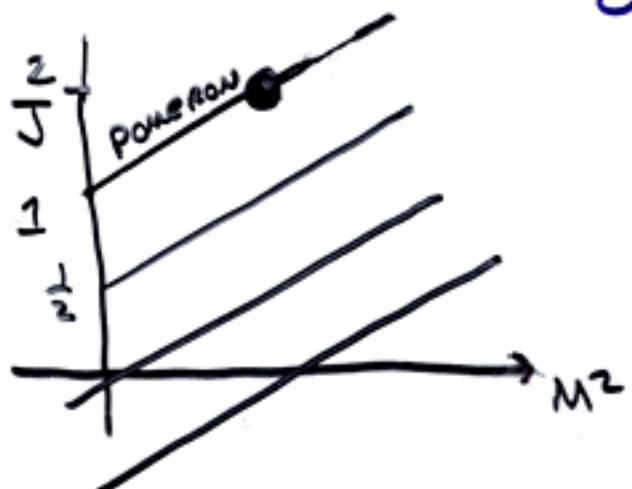
$$\frac{1}{a} f(g_a^2 N)$$

$$a \rightarrow 0$$

$$g^2 N \sim \gamma_B n$$



REAL WORLD:



$$\sigma_{\text{tot}} \sim \text{const.} (\ln E)^2$$

Higher Reps. $\chi_R(u)$

Expect that: $\langle \text{Tr}_R e^{\oint_c A(x)} \rangle \sim e^{-g^2 C_2(R) A(c)}$

To calculate use: (Frobenius)

$$\chi_R(u) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma) \prod_{i=1}^{K_\sigma} \text{tr } u^{k_i}$$

$K=4$ 

$$0 0 \dots 0$$

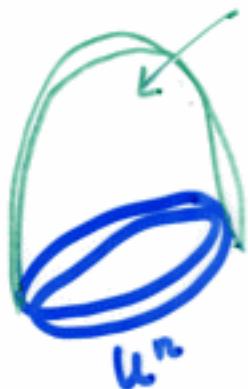
$k_1 \quad k_2 \quad \dots \quad k_\sigma$

e.g.

$\chi_s(u) = \frac{1}{2} [(1+u)^2 + \text{tr } u^2]$	$C_2(s) = 2N + 2 - \frac{4}{N}$
$\boxed{\text{H}}$	
$\chi_A(u) = \frac{1}{2} [(\text{tr } u)^2 - \text{tr } u^2]$	$C_2(A) = 2N - 2 - \frac{4}{N}$
$\boxed{\text{B}}$	

$$\langle \text{tr } u^2 \rangle = \chi_s - \chi_A \sim N^2 e^{-2g^2 \pi N A(c)} (-4g^2 A(c) + \frac{L}{N} + \dots)$$

AdS/CFT



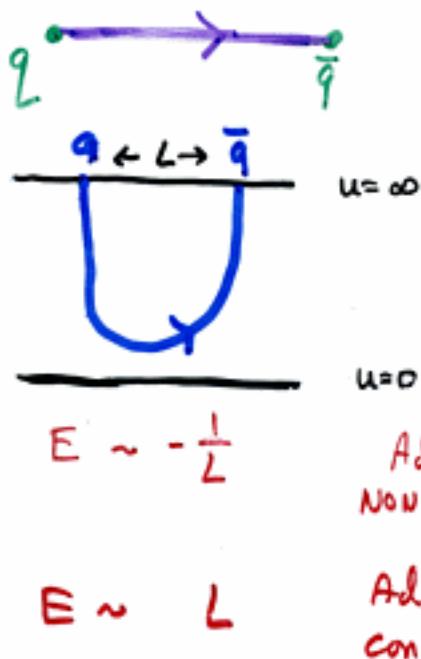
MINIMAL AREA = n. discs $(n-1) \mathbb{P}_2$
BRANCH POINTS

$$e^{-n\sqrt{g s_N} A(c)} (-)^{n-1} A^{n-1}$$

NS-NS n-odd + \rightarrow
RR n-even - entropy of
branch
points

BARYONS

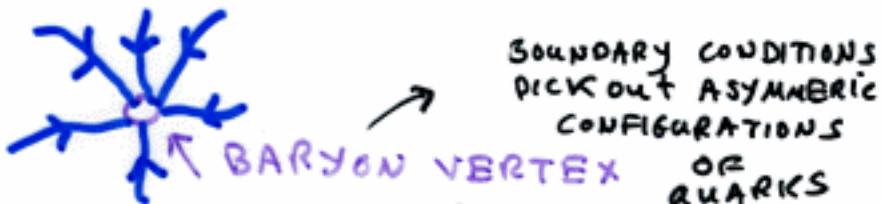
MESONS



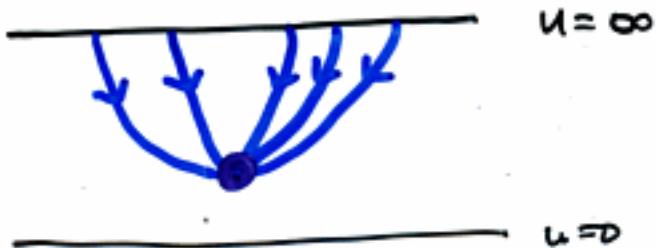
What about $(q^{a_1} \dots q^{a_N} \epsilon_{a_1 \dots a_N}) = \text{BARYON}$

NEED soliton of SUGRA THAT CAN EAT N units of fluxes due to $B_{\mu\nu}$ charge ^{DG, Ooguri WITTEN}

- Possible due to Chern-Simons term in SUGRA
- = D5 BRANE wrapped on S^5 .



∴ BARYON



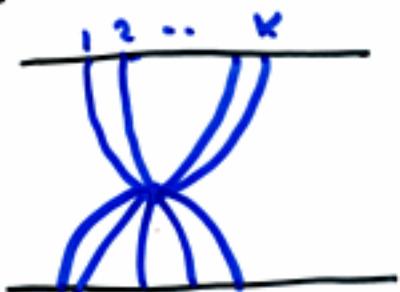
DG Ooguri
Witten

CONFINING CASE

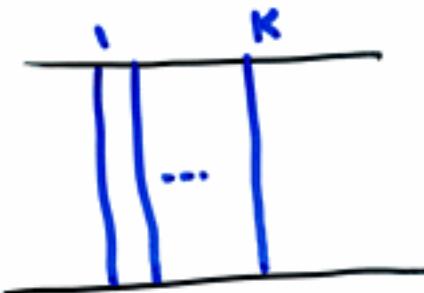


NON CONFINING

CASE



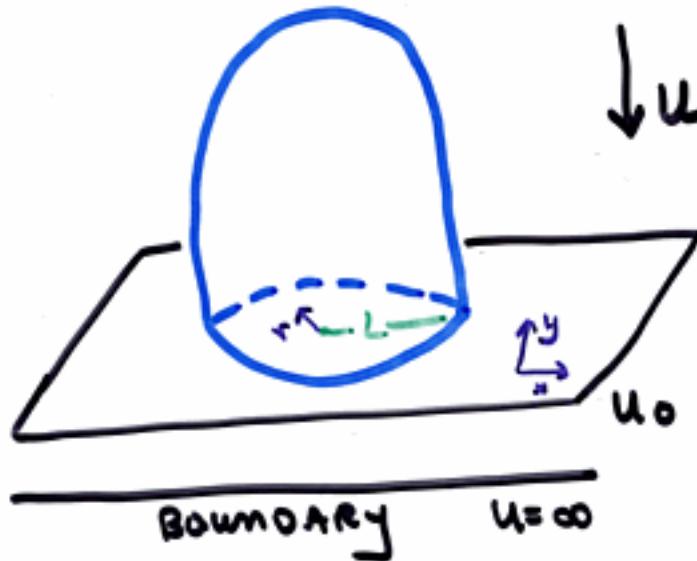
K quark bound
state



STABLE for $\frac{5N}{8} < K < N$

Brandhuber
Itzhaki
Sonenshein
Yankelowicz

CIRCLE Wilson Loop



$$u(r) = \frac{1}{\sqrt{L^2 + \frac{1}{u_0^2} - r^2}}$$

$$\langle W_{u_0}(L) \rangle = e^{-2\pi\sqrt{g^2 N} [\sqrt{L^2 u_0^2 + 1} - 1]}$$

$$\rightarrow 1 \quad L \rightarrow 0 \quad \text{FOR FIXED } u_0 = \frac{\text{Area}}{\text{CUTOFF}} = \frac{uv}{\text{CUTOFF}}$$

$$\rightarrow C^{-2\pi\sqrt{g^2 N} L \cdot u_0 + 2\pi\sqrt{g^2 N}}$$

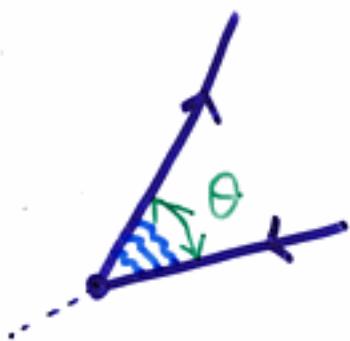
Length $\cdot (\Lambda_{uv})$

QED

$$\langle W_\epsilon(L) \rangle = e^{-e^2 \frac{2\pi L}{\epsilon} + 2\pi^2 e^2}$$



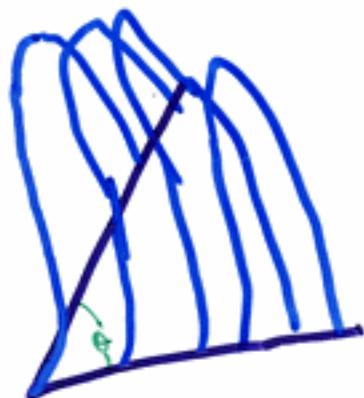
CORNER DIVERGENCES



IN QCD₄, WILSON LOOP HAVE
LOG. DIVERGENCES AT
CORNERS

$$\langle W \rangle \sim e^{+g^2 N [(\pi - \theta) \ln \theta + 1]} \times \ln \Lambda_{uv} + \dots$$

in CFT/AdS



$$\langle W_{u_0} \rangle \sim$$

$$\exp -\sqrt{g^2 N} [\text{Length} \cdot u_0 - f(\theta) \ln u_0 - \text{finite}]$$

Evaluating in terms of Elliptic functions.

$$\approx (\pi - \theta) \ln \theta + 1$$

$$\sim \frac{1}{\theta} \quad \theta \rightarrow 0.$$



ZIG-ZAG Sym.

$\text{Tr } e^{i \int ds \dot{x}^\mu(s) A_\mu(s)}$

is classically

INVARIANT $S \rightarrow F(s)$, even for $F' > 0$, i.e.

$$\langle \text{ } \text{ } \text{ } \text{ } \rangle = \langle \text{ } \text{ } \text{ } \text{ } \text{ } \text{ } \rangle$$

IN QCD, QED, ... W_{REG} obeys ZIG-ZAG SYM.

BUT NOT $W_{\text{REN.}}$

$$\overbrace{\text{ }}^{\text{ }} \equiv \left| \begin{array}{l} \\ \end{array} \right| \text{ IF } d \ll \frac{L}{\Lambda_{\text{UV}}}$$

FOR LARGE $g_{\text{YM}}^2 N$, QGS/CFT

\Rightarrow ZIG ZAG INVARIANT LOOPS FOR U_0 FIXED

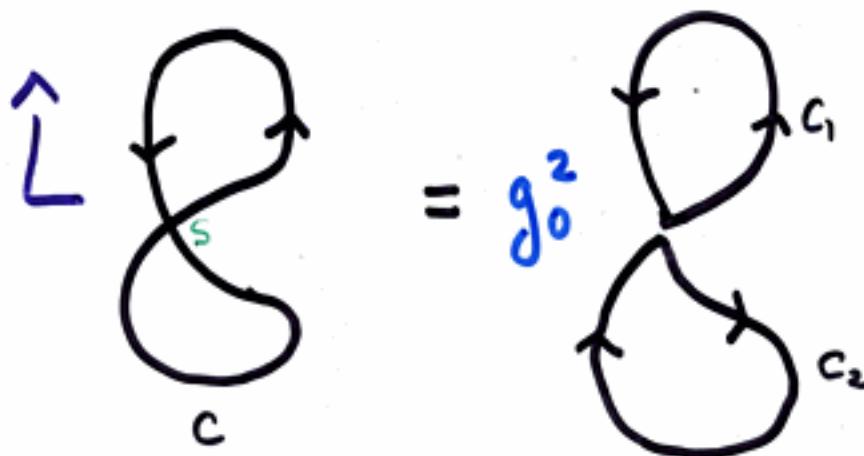
As expected.



Loop Equations

MIGDAL
MAKEENKO.

$N = \infty$



$$\hat{L} w(c) = \int dy^m \dot{x}_{\mu}(s) \delta^4(x(s)-y) w(c_1) \cdot w(c_2)$$

HOLDS FOR
REGULATED
LOOPS.

$$\hat{L} = \lim_{\epsilon \rightarrow 0} \int_{-\epsilon}^{\epsilon} dt \frac{\delta^2}{\delta y_\mu(s+t) \delta x^\mu(s-y)}$$

AdS/CFT : Leading

$$e^{-U_0(\text{Length})} + \sum_{\text{corners}, i} \ln U_0 f(\theta_i)$$

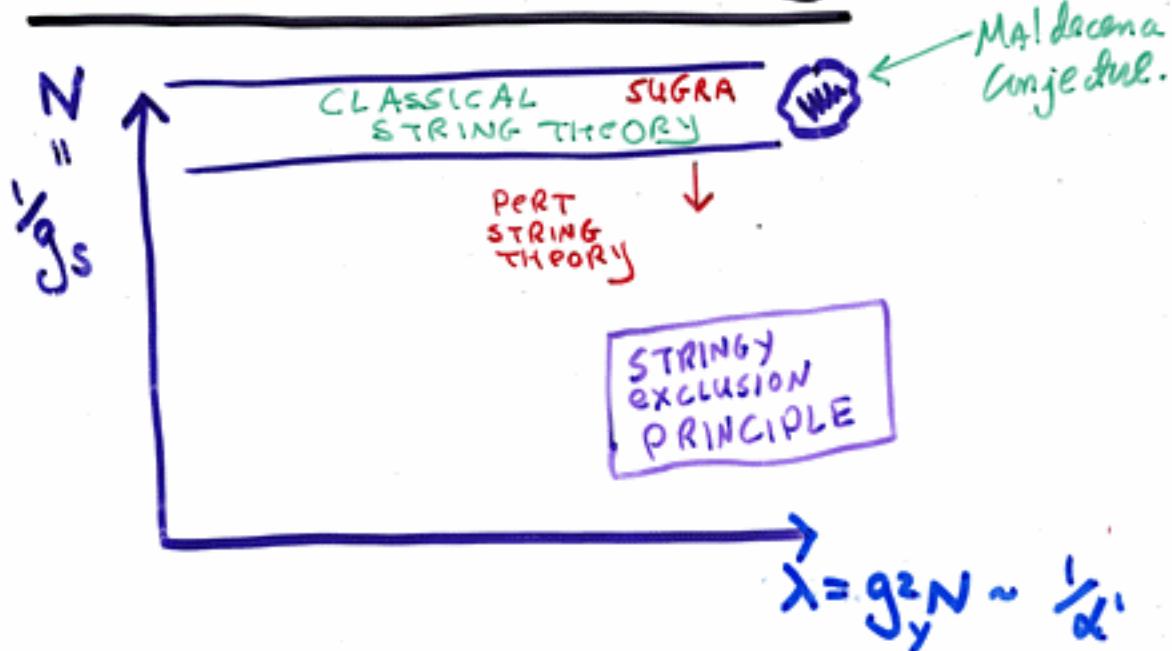
MATCH.

ONLY TRUE BECAUSE THE q-q forces cancel, so



$$\overbrace{\quad}^{\text{scalar}} + \overbrace{\quad}^{\text{gluon}} = 0.$$

Phase Transitions 5



1. $N = \infty$ Large N phase transition
at $\lambda = \lambda_{CR}$.

As in:

ONE PLAQUETTE MODEL

DG + Witten

$$\langle \text{tr} U \rangle = \frac{1}{\lambda} \Big| = 1 - \frac{\lambda}{4}$$

$$\int dU \text{tr} U e^{-\frac{1}{g_s^2} \text{tr}(U+U^\dagger)} \lambda \rangle, \lambda_{critic} = 2 \Big| \quad \lambda \leq 2$$

3-order transition:



$\lambda \ll 1$



Eigenvalue
Distribution

QCD₂

Douglas-Kazakov
DG Matytsin

Phase transition on a sphere of radius A

$$\lambda A < C$$

QCD perturbation Theory
NON CONFINING.

$$\lambda A > C$$

Strings

$$F_{N^2} = \sum Z_i (\lambda A)^i + \left(f(\lambda) e^{-\frac{1}{\lambda A}} \right)^N$$

$$\hookrightarrow$$

$$\begin{cases} < 1 \\ > 1 \end{cases}$$

$$\begin{cases} \lambda A < C \\ \lambda A > C \end{cases}$$

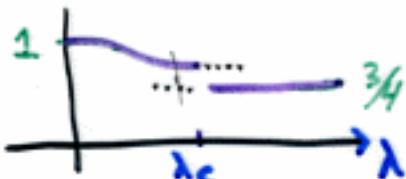
LATTICE GAUGE THEORY FOR QCD₄.

STRONG COUPLING LATTICE
STRING PICTURE BREAKS DOWN
IN CONTINUUM ($g^2 N \sim \frac{1}{\alpha m a}$) limit.

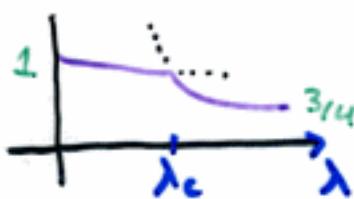
- Might still get string for small λ , but not the one seen by continuing the large $\lambda = R^2 \alpha'$ expansion to small λ or large α'

- Observables might be non-smooth in λ

S



or



STRINGY EXCLUSION PRINCIPLE

$$\# \text{ of D.O.F} < N \approx \frac{1}{g_s}$$

MEANS STRING NOT FUNDAMENTAL

EXCEPT in Asymptotic Expansion in $\frac{1}{N} = g_s$

(like QCD) \hookrightarrow Eg FOR $N=1$ NO
DOF

BUT BOUNDARY FIELD THEORY even more
BACKGROUND DEPENDENT.

→ WHAT ARE FUND. DOF?

WHAT IS THE BKRD
INDEPENDENT FORMULATION
OF THE THEORY?

THANK You ALL
FOR COMING TO
ITP / STRINGS '98

—

See you all at

Strings '99 Potsdam

ITP PROGRAM 8/99-12/99

"Dynamics of SUSY
Gauge Theories"

D. Kutasov
M. Shifman
L. Randall.