

Brane Box Models

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Based on works with

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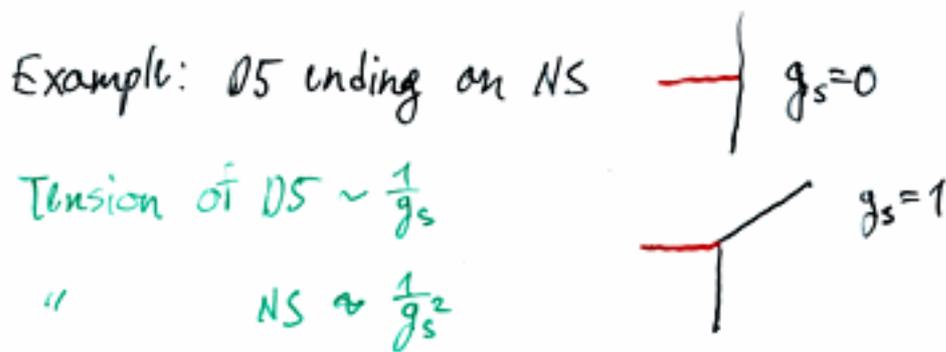
Related work

Gimon-Grolemund 9803033

Armoni-Brandhuber 9803186

Randall-Shirman-von Unge 6092

- * Construct a large class of $N=1$ 4d chiral gauge theories using branes in Type IIB
- * Study the strong coupling dynamics of these theories
- * Dynamical supersymmetry breaking - geometrical picture?
- * Typical configuration involves a D-brane ending on NS brane. This causes bending on the NS brane



The problem of brane bending is very complicated and at the moment not solved.

We describe the construction for $g_s = 0$

In this limit the branes look like straight lines and simplify the description

Finite g_s will teach us about quantum dynamics.

For small values of g_s the branes will bend slightly reflecting the one loop contribution to the theory.

Large g_s is expected to teach us about non-pert. corrections

In some cases, there will be configurations with no bending, we will find that they correspond to finite gauge theories

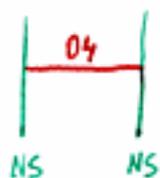
We construct a large class of finite gauge theories using this principle of no bending

These theories have a set of marginal couplings which are acted by some duality group

We count the marginal couplings and describe the duality group

Brane Construction

* Basic idea: Take 5d theory with 16 susy's on $\mathbb{R}^4 \times I$
 B.C. impose reduction to 4d in the IR with half susy
↑
internal
HW

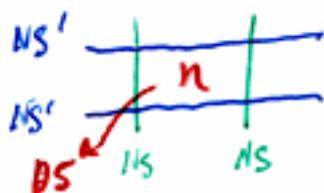


NS impose B.C.
 get $N=2$ theory on the w.r. of D4 brane

Improve to get $N=1$ theories: HZ

Take 6d theory with 16 susy's on $\mathbb{R}^4 \times I \times I$

B.C. impose reduction to 4d in the IR with $\frac{1}{4}$ susy



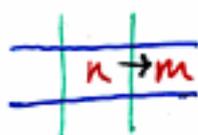
$N=1$ $SU(n)$ theory on the w.r. of D5 brane

setup: Type IIB superstring theory

NS 5-branes	012345
NS' 5-branes	0123 67
D5 branes	01234 6

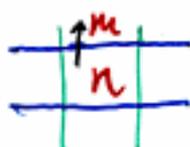
D5-brane finite in 4, 6 directions (rectangle) \rightarrow 3+1 dim. in IR
 each brane breaks $\frac{1}{2}$ susy
 $\Rightarrow \frac{1}{8}$ susy = 4 susy's $\Leftrightarrow N=1$ in 4d

Matter Fields & Superpotentials

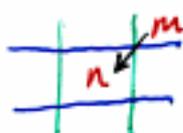


$$SU(n) \times SU(m)_{i+1, j}$$

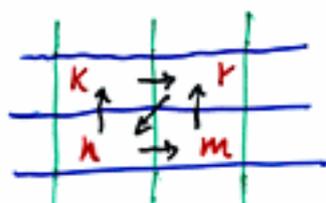
H	\square	$\bar{\square}$
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V	\square	$\bar{\square}$
---	-----------	-----------------



D	$\bar{\square}$	\square
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$$SU(n) \times SU(m) \times SU(k) \times SU(r)$$

H_1	\square	$\bar{\square}$		
H_2			\square	$\bar{\square}$
V_1	\square		$\bar{\square}$	
V_2		\square		$\bar{\square}$
D	$\bar{\square}$			\square

$$W = H_1 V_2 D - V_1 H_2 D$$

a: cubic term per triangle
upper (lower) with - (+) sign

The (3,2) model

As an example we demonstrate two models

Take 3 NS branes & 2 NS' branes

2	3	2	1

$SU(3) \times SU(2)$ with matter

$R_i, i=1,2$ (3,1)

L (1,2)

Q (3,2)

$W=0$

Compare this model to

			1
1	3	2	

with same matter content but

$W = R_1 Q L$

This model is supposed to break supersymmetry dynamically whereas model A does not.

Note that model A has classical flat directions (3) and Model B does not as expected in the field theory

This is expected to be seen by briding & finite g_s
model A is supersymmetric while model B is not.

A generic configuration

* choose 446 to be on torus with radii R_4, R_6
crucial to finiteness

k NS branes

k' NS' "

divides torus to kk' boxes

each box place arbitrary # of D5-branes N_{ij}
 $i=1, \dots, k$
 $j=1, \dots, k'$

gauge group $\prod_{ij} SU(N_{ij})$

matter fields H_{ij}, V_{ij}, D_{ij} in $(\square, \bar{\square})$ of

$SU(N_{ij}) \times SU(N_{i+1, j})$; $SU(N_{ij}) \times SU(N_{i, j+1})$; $SU(N_{ij}) \times SU(N_{i-1, j-1})$

$$W = \sum_{ij} H_{ij} V_{i+1, j} D_{i+1, j+1} - \sum_{ij} H_{i, j+1} V_{ij} D_{i+1, j+1}$$

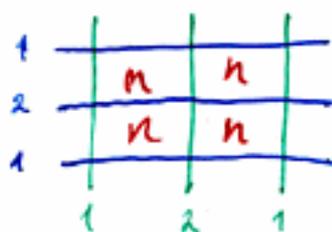
gauge couplings: k positions in X_6 X_6^i $i=1 \dots k$
 k' " " X_4 X_4^j $j=1 \dots k'$

$$\begin{array}{lll} k \text{ intervals} & a_i = X_6^i - X_6^{i-1} & \sum_i a_i = R_6 \\ k' \text{ "} & b_j = X_4^j - X_4^{j-1} & \sum_j b_j = R_4 \end{array}$$

$$\frac{1}{g_{ij}^2} = \frac{a_i b_j}{g_s l_s^2}$$

$k \cdot k'$ gauge couplings given by $k+k'-1$ param.
 $k+k'$ - 2 choice of origin + 1 Area of Torus

Example: $k=k'=2$; each box contains n D5 branes



recall identification on torus

12 matter fields (4 boxes \times 3 fields H, V, D)

8 Yukawa interactions (4 vertices \times $2 \pm$ triangles)

$U=1$ gauge theory $SU(N)^4$

$$H_{1,1} = \square, \bar{\square}, 1, 1 = \bar{H}_{2,1}$$

$$V_{1,1} = \square, 1, \bar{\square}, 1 = \bar{V}_{1,2}$$

$$D_{1,1} = \square, 1, 1, \bar{\square} = \bar{D}_{2,2}$$

$$H_{1,2} = 1, 1, \square, \bar{\square} = \bar{H}_{2,2}$$

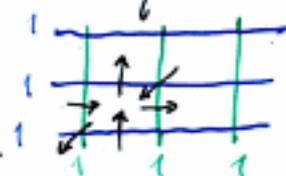
$$V_{1,2} = 1, \square, 1, \bar{\square} = \bar{V}_{2,2}$$

$$D_{1,2} = 1, \square, \bar{\square}, 1 = \bar{D}_{2,2}$$

non-chiral model

Example: $N=4$ $SU(N)$ gauge theory $k=k'=1$

* Single gauge group

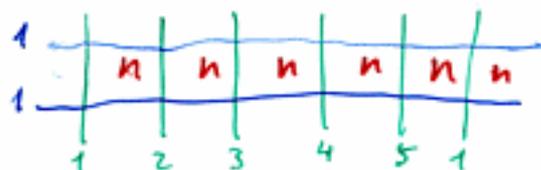


* H, V, D are charged under same group \rightarrow adjoints

$W = H[V, D]$ as for $N=4$ interaction

$$\frac{1}{g^2} = \frac{R_4 R_6}{g_s l_s^2} \quad \mathcal{O} = \int_B \frac{1}{46}$$

* Example: $N=2$ models choose $k'=1$



$SU(N)^k$ with b_i -fundamentals V_i -adj in V -plet

* Classical flat directions k -box model \rightarrow $k-1$ box model
 remove one NS brane
 UV to bifund. field

* Can proceed down to 1-box model \Leftrightarrow $N=4$ theory
 as in the field theory

Models with non-trivial Identification

* Example:

$$k=3, k'=1, p=1$$

2	0	1
1	2	0
0	1	2

$SU(N)_1 \times SU(N)_2 \times SU(N)_3$

$$\begin{array}{l} Q_{1a} \ 3 \times \bar{\square} \quad \square \\ Q_{2a} \ 3 \times \square \quad \bar{\square} \\ Q_{3a} \ 3 \times \quad \square \quad \bar{\square} \end{array}$$

A chiral Model
with 9 Matter fields $Q_{ia} \ i=1,2,3, a=H,V,D$

$$W = \epsilon^{ijk} Q_{iH} Q_{jV} Q_{kD}$$



The unit box is 3×1 but is shifted by one unit when going around the circle

One NS brane, One NS' brane
only the area of the torus is a parameter
one marginal parameter

* More general models

given k, k', p
how many marginal
parameters for the model?



H, V, D permutations

* Recall, a model defined by k, k', p
has 3 types of fields H, V, D
with

$$W = \sum HVD - \sum HVD$$

#parameters for $p=0$ was $k+k'-2+1$
 $k-1$ vertical lines
 $k'-1$ horizontal lines
diagonal?

* Asymmetry: H, V, D fields but only H, V parameters

Field theory: Flat directions along H directions
V directions
also D "

consider a permutation
 $H, V, D \rightarrow V, H, D$

Does not change the matter content
exchanges $NS \leftrightarrow NS'$
vertical lines \leftrightarrow horizontal lines

6 possible permutations

$H, V, D \rightarrow H, D, V$ is more interesting

*Example: $H, V, D \leftrightarrow H, D, V$ For $k=4, k'=2, P=0$

1	2	3	4
5	6	7	8
1	2	3	4
5	6	7	8

1	2	3	4	
6	7	8	5	
	1	2	3	4
	6	7	8	5

Count parameters

4 vertical
 2 Horizontal
 2 Diagonal
 -3 for com
 +1 Area of torus
 6

$k=4, k'=2, P=2$

4 Diagonal
 2 Horizontal
 2 vertical
 -3
 +1
 6

general formula: #diagonals = $\gcd(k, k') \equiv r$ ($P=0$)

#parameters = $(\#H-1) + (\#V-1) + (\#D-1) + 1 = k + k' + r - 2$

For $P \neq 0$:

#parameters = $k' + \gcd(k, P) + \gcd(k, k'+P) - 2$

check: $4 + 2 + 2 = 4 + 2 + 2$

Singularity to Brane Box

* Rules for obtaining Gauge theory data are mapped from one picture to the other

* Given a discrete group Γ & its action on \mathbb{C}^3 i.e. specifying the rep $\underline{3} = (A_1, A_2, A_3)$ construct a Brane Box model

* Example: $\Gamma = \mathbb{Z}_3$ with $\underline{3} = (1, 1, 1)$ associate matter fields as (H, V, D)

start with irrep 0

proceed in H

direction

0	1	2
2	0	1

next in V direction

D direction is already build in

Get the $k=3, k'=1, p=1$ model in Brane Box

* H, V, D permutation corresponds to z_1, z_2, z_3 perm. in \mathbb{C}^3

* Example: $\Gamma = \mathbb{Z}_k$ $\underline{3} = (1, p, -p-1)$

p	p+1		k-1	0		k-1	p
0	1	...	k-p-1	k-p	...	k-1	0

Get the $k'=1, k, p$ Brane Box model

* Example: $\Gamma = \mathbb{Z}_k \times \mathbb{Z}_{k'}$ $\underline{3} = (0 \oplus 0, 1 \oplus -1, 1) \leftrightarrow k, k', p=0$

Branes Cubes and Brane hypercubes

Generalize the construction to higher dimensional intervals $I_n = \underbrace{I \times I \times \dots \times I}_n$

Pick a D5-brane 012345 in Type IIB

$n=0$: 16 susy's

$d = p+1 \leftarrow$ dim of field theory

interval	$n=1$: add one NS brane	012346	8 susy's	$d=p$
Box	$n=2$: add another NS'	012357	4 susy's	$d=p-1$
Cube	$n=3$: add " NS''	012458	2 susy's	$d=p-2$
hypercube	$n=4$: NS'''	013459	1 susy	$d=p-3$

See Uranga - Garcia Emtaran for (0,2) construction

Marginal Operators

* Conditions for having dim-less couplings & associated marginal operators

* A field theory has at least one marginal coupling if for every box vertex

$$a+d=b+c \quad (*)$$

$$\begin{array}{c|c} a & b \\ \hline c & d \end{array} \quad (*)$$

* Generic theory is chiral; however condition (*) implies anom. free theories

* If we want a theory with no divergences need to require all one-loop β -func. to vanish
 \Rightarrow all $n_{ij} = n \Rightarrow$ no bending

Conclusions

- * Brane-Box Models - tools for studying dynamics of $N=1$ 4d chiral gauge theories
- * Bending - open problem, will teach us on strong coupling dynamics
- * no Bending \leftrightarrow zero β -functions - finite gauge theories
- * Count # of marginal operators
- * Brane Boxes $\xleftrightarrow{\text{T-duality}}$ Branes on singularity
- * T-dual pairs are calculated using 4d $N=1$ Field theory data
- * Brane cubes & Brane hypercubes