A Few Bumps On The M5

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RM, GM, JH to appear

Thanks to R. Bott, E. Witten

Outline

1. The M5 anomaly problem
   EW 9610234

2. Bumps and Curves

3. Anomaly Cancellation

4. Applications

   A. Conformal anomaly of (2,0) theory

   B. Chern-Simons terms in gauged SUGRA

1. M5 Problem

M5 w/ w-vol. W_6 breaks

$SO(10,1) \to SO(5,1) \times SO(5)$

$$TM_{11}\Big|_{W_6} \simeq TW_6 \oplus N$$

normal bundle

Diffeos preserving W_6 act as

1. Diffeos of W_6

2. $SO(5)$ gauge transf. on connection on N

If M-theory is consistent these should be good symmetries, without anomalies

Anomalies in theory on W_6 are determined from an 8-form I_8 by descent:

$$I_8 = d I_7^{(0)}$$

$$\delta I_7^{(0)} = d I_6^{(1)}$$

$$\text{Anomaly} = \int_{W_6} I_6^{(1)}$$

For a charge 1 M5 we have

$$I_8 = I_8^{\text{t.m.}} + I_8^{\text{inflow}} = \frac{p_2(N)}{24} \neq 0$$

From tensor mult of (2,0) theory

$$\delta \int_{M_{11}} C_3 \wedge I_8^{\text{in}} = \delta \int_{M_{11}} G_4 I_7^{(0)\,\text{in}}$$

$$= \int_{M_{11}} G_4 \, d I_6^{(1)\,\text{inf}} = \int_{M_{11}} dG_4 \, I_6^{(1)\,\text{inf}}$$

$$= \int_{M_{11}} \text{"}\delta_5\text{"} \, I_6^{(1)\,\text{inf}} = \int_{W_6} I_6^{(1)\,\text{inf}}$$

2. Bumps + Curves

$d\mathcal{G}_4 = \delta_5$ is too singular to allow a careful defn of $D=11$ SUGRA in M5 bkgnd

$\uparrow$

$G_4/2\pi$

Given a surface $W_6 \hookrightarrow M_{11}$ there is a standard framework to smooth out the source and capture its topological structure - Bott + Tu

★ Remove a tubular nbhd of W_6

radius ϵ

} disk bundle $D_\epsilon(W_6)$

★ Define bulk integrals

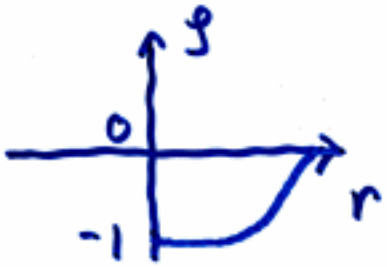
$$\int_{M_{11}} \mathcal{L} \to \lim_{\epsilon \to 0} \int_{M_{11} - D_\epsilon} \mathcal{L}$$

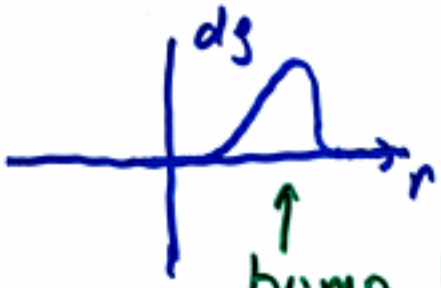
will use $\partial(M_{11} - D_\epsilon) = S_\epsilon(W_6)$ (sphere bundle)

★ Smooth out source

$$d\mathcal{G}_4 = d\mathfrak{z}(r) \wedge \frac{e_4(\theta)}{2}$$

radial dir. — conn. on normal bundle





bump form

$e_4/2$ = global angular form

$$= \frac{1}{64\pi^2}\,\epsilon_{a_1\cdots a_5}\Big[(D\hat{y})^{a_1}\cdots(D\hat{y})^{a_4}\hat{y}^{a_5} - 2F^{a_1a_2}D\hat{y}^{a_3}D\hat{y}^{a_4}\hat{y}^{a_5} + F^{a_1a_2}F^{a_3a_4}\hat{y}^{a_5}\Big]$$

$d\hat{y}^a = \theta^{ab}\hat{y}^b$ — curvature of θ — isotropic ang. coor.

$e_4/2$ generalizes vol. form on S^4 in presence of $SO(5)$ gauge fields so that $de_4 = 0$, e_4 gauge cov., integral over S_4 fibres is one.

3. Anomaly Cancellation

In absence of M5

$$S_{CS} = -\frac{1}{6}\int_{M_{11}} \mathcal{C}_3 \wedge \mathcal{G}_4 \wedge \mathcal{G}_4 \qquad G_4 = dC_3$$

w/ M5 $\quad d\mathcal{G}_4 = d\rho \wedge \frac{e_4}{2} \Rightarrow \underbrace{\mathcal{G}_4 = d\mathcal{C}_3 - d\rho \wedge e_3^{(0)}/2}_{\text{like } H_3 = dB_2 - \omega_3 \text{ in GS mechanism}}$

$$\Rightarrow \boxed{\delta\mathcal{C}_3 = -d\rho \wedge e_2^{(1)}/2}$$

Modify C-S to $\qquad \sigma \equiv \rho\, e_3^{(0)}/2$

$$S'_{CS} = \lim_{\epsilon \to 0} \int_{M_{11}-D_\epsilon} -\frac{1}{6}(\mathcal{C}_3 - \sigma) \wedge d(\mathcal{C}_3 - \sigma) \wedge d(\mathcal{C}_3 - \sigma)$$

Now compute

$$\delta S'_{CS} = -\frac{1}{6}\int_{S_\epsilon} \frac{e_2^{(1)}}{2} \wedge \frac{e_4}{2} \wedge \frac{e_4}{2} = -\int_{W_6} \left[\frac{p_2(N)}{24}\right]^{(1)}$$

by Bott + Cattaneo $\quad$ dg-ga/9710001

$\therefore$ M5 Anomaly cancels

4. Applications

A. Conformal anomalies

We have shown cancellation for $Q_5 = 1$, for arb. Q_5 if we <u>assume</u> the anomaly cancels then

from S^1_{CS}

$$I_8^{z.m.}(Q_5) = Q_5 I_8^{z.m.}(1) + \frac{(Q_5^3 - Q_5)}{24} p_2(N)$$

by the multiplet of anomalies this gives an <u>exact</u> prediction for the conformal anomaly of (2,0) theory

Q_5^3 : compatible w/ BH analysis — Gubser, Klebanov, Tseytlin

and

AdS/CFT calc. — Henningson, Skenderis

Q_5 : $(1/Q_5^2)$ correction

Precise check by wrapping M5 on 4-cycle P of CY. as in MSW

find $$C_R^{anom} \equiv C_R^{MSW}$$ both Q_5^3, Q_5 pieces

B. C-S terms in gauged SUGRA

$S_5 \times AdS_5$: $\displaystyle\int_{AdS_5} \omega_5^{(0)}(A)$

$$d\omega_5^{(0)} = \mathrm{Tr}\, F^3$$

$S_4 \times AdS_7$: $\displaystyle\int_{AdS_7} -\frac{1}{2}\omega_3^{(0)} \wedge \omega_4 + \omega_7^{(0)}$

Pernici Pilch van N

$$d\omega_3^{(0)} = \omega_4 = \mathrm{Tr}\, F \wedge F$$
$$d\omega_7^{(0)} = \mathrm{Tr}\, F^4$$

These have been determined in the literature by Noether method in compactified theory.

How do they arise from KK red. ?

Clue : $\displaystyle\int_{AdS_7} -\frac{1}{2}\omega_3^{(0)} \wedge \omega_4 + \omega_7^{(0)} \propto \int_{AdS_7} [P_2(F)]^{(0)}$

Problem is to generalize Freund-Rubin

$$G_4 = N \epsilon_4$$

to include fluctuations in metric, ...

Answer: $G_4 = N e_4(F) + \text{fluc in } C$

$$C_3 = N e_3^{(0)}$$

Then $\int_{M_{11}} C_3 \wedge G_4 \wedge G_4 = N^3 \int_{AdS_7} [P_2(N)]^{(0)}$

by Bott-Cattaneo

Note: $\int C_3 \wedge I_8^{inf}$ gives additional C-S terms <u>linear</u> in Q_5 needed to match AdS-CFT anomalies

We believe a similar computation using IIB e.o.m. leads to C-S term in AdS_5

$$(Q_3^2 - 1) \int \omega_5^{(0)} \qquad \frac{1}{Q_3^2} = \frac{g_s^2}{g_s^2 Q_3^2}$$

classical

must exist 1-loop correction to match anomaly of $SU(Q_3)$ CFT on bndy