A Few Bumps on The M5

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Based on

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Outline

1. The M5 anomaly problem
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2. Bumps and Curves

3. Anomaly Cancellation

4. Applications
   A. Conformal anomaly of (2,0) theory
   B. Chern-Simons terms in gauged SUGRA
1. M5 Problem

M5 w/ w-vol. We breaks $SO(10,1) \rightarrow SO(5,1) \times SO(5)$

$TM \big|_W = TW_6 \oplus N$

Diffeos preserving $W_6$ act as

1. Diffeos of $W_6$

2. $SO(5)$ gauge transf. on connection on $N$

If M-theory is consistent these should be good symmetries, without anomalies
Anomalies in theory on $W_6$ are determined from an 8-form $I_8$ by descent:

$$I_8 = dI_f^{(0)}$$

$$\delta I_f^{(0)} = dI_6^{(11)}$$

Anomaly = $\int_{W_6} I_6^{(11)}$

For a charge 1 MS we have

$$I_8 = I_8^{\text{2,m.}} + I_8^{\text{inflow}} = \frac{p_2^{(IV)} \neq 0}{24}$$

From tensor mult of (2,0) theory

$$\delta \int_{M_{11}} C_3 \wedge I_8^{\text{in}} = \delta \int_{M_{11}} G_4 I_f^{(10) \text{ in}}$$

$$= \int_{M_{11}} G_4 dI_6^{(11) \text{ inf}} - \int_{M_{11}} dC_4 I_6^{(11) \text{ inf}}$$

$$= \int_{M_{11}} dS_{I_6^{(11) \text{ inf}}} - \int_{W_6} I_6^{(1) \text{ inf}}$$
2. Bumps + Curves

\[ \Delta y = \delta_{\xi} \]

\[ \frac{dy}{\pi G_4 / 2} \]

is too singular to allow a careful defn of D=11 SUGRA in MS backround

Given a surface \( W_6 \rightarrow M_{11} \) there is a standard framework to smooth out the source and capture its topological structure - Bott + Tu

* Remove a tubular nbhd of \( W_6 \)

\[ \text{radius } \varepsilon \]

\[ \left\{ \text{disk bundle } D_\varepsilon (W_6) \right\} \]

* Define bulk integrals

\[ \int_{M_{11}} \rightarrow \lim_{\varepsilon \rightarrow 0} \int_{M_{11} - D_\varepsilon} \]

will use \( \mathcal{J}(M_{11} - D_\varepsilon) = S_6(W_6) \) (sphere bundle)
*Smooth out source*

\[ d\mathcal{A}_y = d\mathcal{A}(r) \wedge \frac{e_y}{2} \]

radial dir. \hspace{1cm} conn. on normal bundle

\[ \text{bump form} \]

\[ E_{y/2} = \text{global angular form} \]

\[ = \frac{1}{64\pi^2} E_{a_1 \ldots a_5} \left[ \left( \partial \vec{y} \right)^{a_1} \ldots \left( \partial \vec{y} \right)^{a_5} - 2 F^{a_1 a_2} \partial \vec{y}^{a_3} \partial \vec{y}^{a_4} \partial \vec{y}^{a_5} + F^{a_1 a_2} F^{a_3 a_4} \partial \vec{y}^{a_5} \right] \]

\[ \Theta^{\alpha \beta} \Theta^{\gamma} \Theta^{\delta} \]

curvature of isotropic angular coord.

\[ E_{y/2} \text{ generalizes vol. form on } S^y \text{ in presence of sol(s) gauge fields so that } d\mathcal{A}_y = 0, \ e_y \text{ gauge cov., integral over } S^y \text{ fibres is one.} \]
3. Anomaly Cancellation

In absence of MS

\[ S_{CS} = -\frac{1}{6} \int \mathcal{L}_3 \wedge \mathcal{B}_4 \wedge \mathcal{B}_4 \quad \text{with} \quad \mathcal{B}_4 = d\mathcal{C}_3 \]

w/ MS

\[ dB_4 = dg \wedge \frac{e_2}{2} \Rightarrow \mathcal{B}_4 = \mathcal{D}_3 - dg \wedge e_3^{(0)}_{1/2} \]

like \( H_3 = dB_2 - W_3 \) in GS mechanism

\[ \Rightarrow \delta \mathcal{L}_3 = -dg \wedge e_2^{(1)}_{1/2} \]

Modify C-S to

\[ S_{CS}^I = \lim_{E \to 0} \int \frac{1}{6} (\mathcal{L}_3 - 6) \wedge d(\mathcal{L}_3 - 6) \wedge d(\mathcal{L}_3 - 6) \]

Now compute

\[ \delta S_{CS} = -\frac{1}{6} \int_{\mathcal{S}_e} \frac{e_2^{(1)}}{2} \wedge \frac{e_4}{2} \wedge \frac{e_4}{2} = -\int \left[ \frac{P_2(N)}{24} \right]^{(1)}_{W_6} \]

by Bott + Cattaneo \( dg - g_4/9 + 10001 \)

\[ \therefore \text{MS Anomaly cancels} \]
4. Applications

A. Conformal anomalies

We have shown cancellation for \( Q_5 = 1 \), for arb. \( Q_5 \) if we assume the anomaly cancels then

\[
I_9^{2.m.}(Q_5) = Q_5 I_9^{2.m.}(1) + \left( \frac{Q_3^3 - Q_5}{24} \right) P_2(N)
\]

by the multiplet of anomalies this gives an exact prediction for the conformal anomaly of (2,0) theory

\( Q^3_3 \): compatible w/ BH analysis

\( Q_5 \): \((1/Q^3_3)\) correction

Precise check by wrapping MS on 4-cycle \( P \) of CY. as in MSW

find \( C^\text{anom}_R \equiv C^\text{MSW}_R \) both \( Q_5, Q_5 \) pieces
B. C-S terms in gauged SUGRA

\[ S_{5 \times AdS_5} : \int_{AdS_5} \omega_5^{(0)} (A) \]

\[ d\omega_5^{(0)} = Tr F^3 \]

\[ S_{4 \times AdS_4} : \int_{AdS_4} \left( -\frac{1}{2} \omega_3^{(0)} \wedge \omega_4 + \omega_4^{(0)} \right) \]

\[ d\omega_3^{(0)} = \omega_4 = Tr F n F \]

\[ d\omega_4^{(0)} = Tr F^4 \]

These have been determined in the literature by Noether method in compactified theory. How do they arise from KK RED.?

Clue: \[ \int_{AdS_4} \left( -\frac{1}{2} \omega_3^{(0)} \wedge \omega_4 + \omega_4^{(0)} \right) + \int_{AdS_4} \left[ P_2 (F) \right]^{(0)} \]
Problem is to generalize Freund-Rubin to include fluctuations in metric ...

\[ G_4 = N E_4 \]

Answer: \[ G_4 = \Phi N E_4 (F) + \text{fluc in } C \]
\[ C_3 = N E_3^{(0)} \]

Then \[ SC_3 \wedge G_4 \wedge G_4 = N^3 \int_{\text{AdS}} [P_2 (N)]^{(0)} \]

by Bott-Cattaneo

Note: \[ SC_3 \wedge I^\text{inf}_8 \] gives additional C-S terms linear in \( Q_5 \) needed to match AdS-CFT anomalies

We believe a similar computation using IIB e.o.m. leads to C-S term in AdS5
\[ (G_3^2 - 1) \int \omega_5^{(0)} \]
\[ \frac{1}{Q_3^3} = \frac{g_s^2}{g_5^2 Q_5^3} \]

Classical must exist 1-loop correction to match anomaly of SU(\( Q_3 \)) CFT on bndy