

M-Theory , Field Theory and Holography

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Since 1995: Ongoing search for non-perturbative formulation (or understanding) of **quantum M-theory**.

M-theory seems unlike anything else (e.g., not a string theory), on the face of it.

Some things we know:

- * it has an eleven-dimensional super-Poincaré vacuum, described at low energies by 11d supergravity (unique)
- * it is microscopically parity-invariant (if one believes in consistency of heterotic $E_8 \times E_8$)
- * the quantum theory should be holographic (if one believes that statistical interpretation of S_{BH} should exist)
t Hooft, Susskind
(holography at $\Lambda=0$)

"holographic field theory":

a phenomenological approach (cf. Landau's Fermi liquid theory), trying to identify symmetry principles (in some "minimal" context) that would naturally give low-en. (11d) supergravity + holography.

Surprisingly (or maybe not? - cf. QCD!), possible within
field theory.

It turns out that this phenomenological theory of holography can be formulated in a setting that generalizes the theory of 2+1 quantum gravity as Chern-Simons gauge theory. Witten,
Achucarro & Townsend,

Starting from 11d gauge theory with AdS gauge group, we will argue that this theory has a regime described at low energies by 11d supergravity of Cremmer, Scherk, Julia.

In the process (which generalizes 2+1 gravity), we encounter several surprises:

Mach's principle

Chern-Simons gauge theory in 11d, when expanded near any of its obvious solutions, does not exhibit conventional propagating effective degrees of freedom.

To find a regime that does have a conventional effective field theory description, we have to introduce a large number of partons (described in the Chern-Simons formulation by first quantized world-lines).

A large number of partons is needed to sustain a macroscopically large space-time (= Mach's principle).

Due to this relation between partons and space-time geometry, the theory is holographic:

- * N scales like the area, partons effectively 10-dimensional;
- * low-en. supergravity breaks down when Bekenstein bound saturated.

AdS gauge theory in 11 dimensions

Field theory on 11-dimensional manifold \mathcal{M} ($= \mathbb{R} \times T^{10}$, mostly)
with coordinates x^M , $M = 0, \dots, 10$.

Gauge field: a one-form A_M^a in the adjoint of gauge group \mathfrak{g}

Gauge symmetry: Yang-Mills

$$\delta A_M^a = D_M \varepsilon^a$$

No dynamical or background metric introduced;
diffeomorphism invariance required.

Extra symmetry imposed: Parity.

The most general (polynomial) Lagrangian with these symmetries:
a sum of Chern-Simons terms

$$\mathcal{L} = \sum_i \frac{1}{g_i^2} \int \omega_{11}^{(i)}(A)$$

A is of dimension one; all couplings g_i dimensionless.

$$d\omega_{11}^{(i)} = \text{Tr}_{(i)}(F_1 \dots \wedge F); \quad \omega_{11}^{(i)} \sim \text{Tr}_{(i)}(A \wedge dA \wedge \dots \wedge dA + \dots)$$

Precise definition: $\int_{\mathcal{M}} \omega_{11} = \int_B d\omega_{11}, \quad \partial B = \mathcal{M}$
 $1/g^2$ quantized

"Tr" in \mathcal{L} further restricted by parity invariance:

$P_0: x^i \rightarrow -x^i$ not a symmetry ($\int \omega_{11} \rightarrow -\int \omega_{11}$)

$$P = P_0 \cdot I,$$

and we have to choose a non-trivial I action on gauge group.

Choice of the gauge group

Supersymmetric extension of anti-de Sitter in eleven-dimensions.

Anti-de Sitter: P_A, J_{AB} .

$$\langle P_A J_{A_1 B_1} \dots J_{A_5 B_5} \rangle = \varepsilon_{A A_1 B_1 \dots A_5 B_5} .$$

Minimal supersymmetric extension with 32 supercharges:

requires introduction of extra bosonic charge $K_{A_1 \dots A_5}$

van Holten & Van Proeyen

\downarrow
 $OSp(1|32)$.

However, this is not parity invariant!

Minimal extension compatible with parity:

Q_α, Q'_α (64 supercharges; cf. D'Auria & Fré; Townsend; Bors)

$K_{A_1 \dots A_r}, r = 5, 6, 9, 10.$

$OSp(1|32) \times OSp(1|32)$ (super Lorentz in 10+2 dimensions)

The theory is formally defined by path integral:

$$A_M = V_M^A P_A + \frac{1}{2} \omega_M^{AB} J_{AB} + \sum_r B_M^{A_1 \dots A_r} K_{A_1 \dots A_r} \\ + \psi_M^\alpha Q_\alpha + \eta_M^\alpha Q_\alpha^! .$$

$$\int dA e^{i\int dA}$$

the most general Lagrangian compatible
with symmetries

Equations of motion:

$$F_1 F_1 F_1 F_1 F = 0 .$$

Theory is topological in the sense that no metric is introduced, still "local" degrees of freedom; however, no propagator and perturbation theory for particle-like excitations near free-field pt.

Strategy:

Instead of quantizing non-perturbatively this microscopic theory, we look for a regime with macroscopically large space-time, described by conventional effective low-energy field theory with particle-like excitations.

first, consider flat eleven-dimensional spacetime in the theory.

Large universes

We want to identify V_M^A in $A_M = V_M^A P_A + \dots$ as viel-bein.
 To get the dimensions right, introduce a mass scale M , write

$$A_M = M e_M^A P_A + \dots$$

the vielbein

Flat space: $\bar{e}_M^A = \delta_M^A$.

Puzzles with scenario:

- flat space does not solve equations of motion;
- de Sitter space does, but has no low-energy effective description in terms of conventional field theory
(we are not interested in such regimes)
- g is a free dimensionless parameter in theory;
no such parameters in M-theory!

Resolved when we introduce matter. Macroscopic, large universe will require large # of "partons": Mach's principle.

Interpretation of M :

$$\int_{U_{10}} V \wedge \dots \wedge V \sim 1$$

$$\int_{U_{10}} e \wedge \dots \wedge e \sim \frac{1}{M^{10}} ; \quad L = M^{-1} \begin{matrix} \text{characteristic size} \\ \text{of the universe} \\ \text{(or the surrounding box)} \end{matrix}$$

Mach's principle: Introducing partons

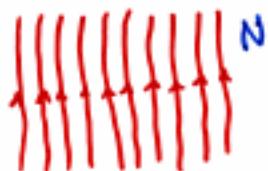
EoM in terms of the Riemannian curvature $R = d\omega + \omega \wedge \omega$:

$$(R + V_A V)^5 = 0$$

Admits AdS as a solution, but not Minkowski space.

A_M can naturally couple to particles (through Wilson lines).

Imagine N particles in a certain representation \mathcal{R} of \mathcal{G} :



Their presence will show up effectively in EoM through a source (current):

$$(R + V_A V)^5 = g^2 J$$

Microscopically, J would be a (Lie \mathcal{G} -valued) ten-form given by a sum of a large number N of $\delta(C)$ functions localized on N contours C_1, \dots, C_N .

Mean field ansatz:

At long wavelengths (to be self-consistently determined), sum of δ -functions replaced by a homogeneous field; assume there is a choice of \mathcal{R} such that

$$J|_{\text{mean field}} = c N M^{10} P^A \wedge e^{A_1} \dots \wedge e^{A_{10}} \epsilon_{A A_1 \dots A_{10}}$$

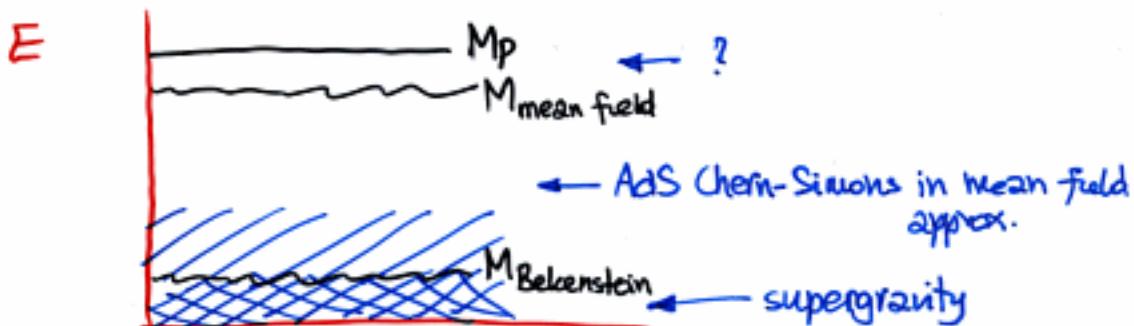
Then:

- * Flat Minkowski space-time is a solution of mean-field EoM;
- * The Chern-Simons coupling (in front of the irreducible term in \mathcal{L}) is quantized and related to N :

$$\frac{1}{g^2} \sim N.$$

One can ask what is the effective low-energy theory near Minkowski space.

We will see that (for N large):



Indications that partons are (super) singlets:

Friedan, Fazio,...
Günaydin,...

- * partons turn out to be effectively 10-dimensional;
- * individual partons carry energy $E \sim M$ just like supersinglets ($E \sim J$, instead of $E \approx J$)

Supersingleton field theory?

LOW-ENERGY EFFECTIVE FIELD THEORY IN 11 DIMENSIONS

Fluctuations near flat 11d space-time:

$$e_M^A - \bar{e}_M^A \ll 1$$

Lagrangian:

$$\mathcal{L} = -\frac{1}{g^2} \int \sum_{s=0}^5 \frac{M^{2s+1}}{2s+1} \binom{5}{s} \varepsilon_{A_1 \dots A_{11}} e^{A_1 \wedge \dots \wedge e^{A_{2s+1}} \wedge R^{A_{2s+2} \wedge A_{2s+3}} \wedge \dots \wedge R}$$

$$\sim -\frac{1}{g^2} \int (M^1 e \wedge \dots \wedge e + M^9 e \wedge \dots \wedge e \wedge R + \dots + M_{11} R \wedge \dots \wedge R)$$

At low-energies : Keep the Einstein-Hilbert term.

That identifies the low-energy Planck mass:

$$M_P \sim \frac{M}{g^{2/9}} \sim N^{1/9} M$$

Effective Lagrangian:

$$\mathcal{L} = -M_P^9 \int (e \wedge \dots \wedge e \wedge R + \# \frac{1}{g^{4/9} M_P^2} e \wedge \dots \wedge e \wedge R \wedge R + \# g^{4/9} M_P^2 e \wedge \dots \wedge e + \mathcal{O}\left(\frac{1}{g^{8/9} M_P^4}\right))$$

Limit of validity:

$$R_{AB} \ll g^{1/9} M_P^2 \equiv \frac{M_P^2}{N^{2/9}}$$

This will be interpreted in terms of the holographic principle!

Holography

Two arguments:

1. The number of partons scales like the area:

$$M = g^{2/9} M_P = \frac{M_P}{N^{1/9}}$$

characteristic size of system:

$$L \equiv M^{-1} = \frac{N^{1/9}}{M_P}$$

volume:

$$V \sim L^{10} = \frac{N^{10/9}}{M_P^{10}}$$

area of surrounding hypersurface:

$$\mathcal{A} \sim L^9 = \frac{N}{M_P^9}$$

$$N = \left(\frac{M_P}{M}\right)^9 \sim M_P^9$$

Energy/momentum at low energies carried only by the partons.

The number of degrees of freedom scales like the area; the theory is holographic.

Dramatic reduction of microscopic degrees of freedom in comparison to any conventional local field theory, (super)gravity, or even lattice theory.

2. The low-energy supergravity approximation breaks down precisely when the Bekenstein bound on energy/entropy in a given volume of space is saturated.

To see that, let's calculate the energy of configurations that saturate the condition of validity of low-en. supergravity,

$$R_{AB} \approx \frac{M_p^2}{N^{2/9}}$$

Energy-momentum density:

$$T \sim M_p^9 e_1 \dots e_n R$$

Energy: $E \sim M_p^9 \int_{M_{10}} e_1 \dots e_n R$

Energy of configuration saturating the bound:

$$E_{\max} \sim M_p^9 \frac{M_p^2}{N^{2/9}} \underbrace{\int_{M_{10}} e_1 \dots e_n}_{\text{Volume, } \sim \frac{1}{M^{10}}} R$$

$$E_{\max} \sim \frac{M_p^{11}}{N^{2/9}} \frac{1}{M^{10}} = N^{8/9} M_p$$

$$E_{\max} \sim M \left(\frac{M_p}{M} \right)^9$$

This is precisely the mass of Schwarzschild black hole with Schwarzschild radius $R_s = L \equiv M^{-1}$.

Low-energy diffeomorphisms

The microscopic theory is invariant under gauge transformations, in particular under "gauge translations" - the gauge transformations that correspond to P_A :

$$\varepsilon = \varepsilon^A P_A + \varepsilon^{AB} J_{AB} + \dots$$

$$\delta_\varepsilon A_M = D_M \varepsilon.$$

Gauge translations do not act like diffeomorphisms.

On the other hand, the low-en. effective theory (valid in the regime where curvature is sufficiently small) is NOT invariant under gauge translations; rather, gauge translations are effectively replaced by diffeomorphisms at low energies.

In more detail:

Rescale charges,

$$P_A = M^{-1} \tilde{P}_A, \quad Q_\alpha = M^{-\gamma_2} \tilde{Q}_\alpha$$

so that P_A has the appropriate dimension to be compared to diffeomorphisms.

The microscopic symmetry algebra becomes (schematically)

$$\{\tilde{Q}, \tilde{Q}\} = \Gamma^A \tilde{P}_A + \frac{M_P}{N^{1/g}} \Gamma^{AB} J_{AB} + \dots$$

$$[\tilde{P}_A, \tilde{P}_B] = \frac{M_P^2}{N^{2/g}} J_{AB} + \dots$$

Gauge transformation:

$$\varepsilon = \varepsilon^A P_A + \dots = \tilde{\varepsilon}^A \tilde{P}_A + \dots$$

↗ dimension zero ↗ dimension one

Gauge translations on the effective Lagrangian:

$$\delta L_{\text{eff}} = M_P^q \int e \wedge \dots \wedge e \wedge R$$

$$\sim M_P^q \int e \wedge \dots \wedge D \tilde{\varepsilon} \wedge R$$

$$\sim -M_P^q \int \tilde{\varepsilon} e \wedge \dots \wedge e \wedge T \wedge R \neq 0$$

(we are using the first-order formalism)

Instead, gauge translations effectively replaced by diffeomorphisms:

$$\delta_{\text{diff}} A = \delta_{\tilde{\varepsilon}} A + O(R).$$

Still, the full microscopic theory is gauge invariant under gauge translations. How is it possible?

$$\delta_{\tilde{\varepsilon}} R^{AB} \sim \frac{M_P^2}{N^{2g}} \tilde{\varepsilon}^{[A} T^{B]} \quad \text{microscopically.}$$

$$\delta_{\tilde{\varepsilon}} \left(M_P^q \int \frac{N^{2g}}{M_P^2} e \wedge \dots \wedge e \wedge R \wedge R \right) \sim$$

$$\sim M_P^q \int \frac{N^{2g}}{M_P^2} e \wedge \dots \wedge e \wedge \delta_{\tilde{\varepsilon}} R \wedge R$$

$$\sim M_P^q \int \tilde{\varepsilon} e \wedge \dots \wedge T \wedge R$$

In the effective theory, this term is absent, theory is invariant under diffeomorphisms.

Low-energy supersymmetry

Rescaled charges:

$$K_{A_1 \dots A_r} = M^{-1} \tilde{K}_{A_1 \dots A_r} ; \quad Q'{}^\alpha = M^{-3/2} \tilde{Q}'{}^\alpha .$$

The low-energy symmetry group:

Contraction of $OSp(1|32) \times OSp(1|32)$ (formally $M \rightarrow 0$).

This algebra has 64 supercharges, 32 of them ($Q'{}^\alpha$) are central.

This algebra = the "hidden supergroup" of 11d supergravity, discovered by D'Auria, Fré; van Nieuwenhuizen.

Origin of the abelian three-form C_{MNP} of 11d supergravity:
At low energies, C appears as a composite field made out of three gauge fields:

$$C \sim \frac{1}{M^3} \text{"tr"} (A \wedge A \wedge A)$$

This has to be odd under parity, and therefore each term on the RHS will contain either $B_M^{A_1 \dots A_r}$ or η_M^α .

Microscopic candidate for C :

$$M^3 C \sim \omega_3(A) \quad \text{the Chern-Simons three-form
odd under internal parity}$$

Term in the Lagrangian:

$$\int \omega_3 \wedge d\omega_3 \wedge d\omega_3 \xrightarrow{\text{low-en.}} M_P^9 \int C \wedge G \wedge G$$