

Consistency Condition for
Fivebrane in M theory on
 $\mathbb{R}^5/\mathbb{Z}_2$ Orbifold

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Realization of SUSY Gauge theories on Branes + superstring dualities

→ { new point of view
new results.

{ SO/Sp Gauge Group
Real / Pseudo-real representation

requires **ORIENTIFOLD PLANES.**

... NOT well-understood

- e.g - Strong string coupling limit?
- intersection with D/NS branes?

Study of O4-planes

by giving/using M theory realization

→ Fivebranes in $\mathbb{R}^5/\mathbb{Z}_2$ orbifold

- Consistency conditions for
Fivebranes in $\mathbb{R}^5/\mathbb{Z}_2$ orbifold
- Proposal of M th. realization of O4
- Application
 - Intersection of O4 with D6/NS5
 - Montonen-Olive duality
 - $N=2, 1$ SUSY & $\mathcal{N}=1$ SUSY Gauge theory

Flux Quantization Condition ^{Witten}

C_3 potential, $G = dC_3$ field strength

$$2 \int_{S \subset X''} G/2\pi \equiv \int_S w_4(X'') \pmod{2}$$

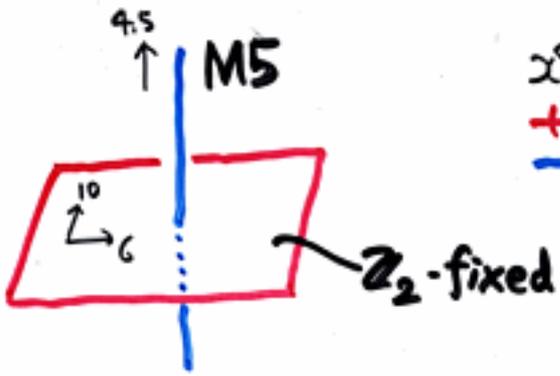
e.g. $X'' = \mathbb{R}^6 \times (\mathbb{R}^5/\mathbb{Z}_2 - o) \supset S = \mathbb{R}P^4 = S^4/\mathbb{Z}_2$

$$\int_{\mathbb{R}P^4} w_4(X'') = 1 \pmod{2}$$

$$\therefore 2 \int_{\mathbb{R}P^4} G/2\pi = \int_{S^4} G/2\pi \text{ must be odd}$$

M on $T^5/\mathbb{Z}_2 \rightsquigarrow$ each \mathbb{Z}_2 fixed plane ^{Dasgupta-Mukhi} ^{Witten} has fivebrane charge -1

$n \times M5$ on top of \mathbb{Z}_2 fixed plane:
 n must be even



$$\begin{array}{cccccccccccc} \chi^0 & \chi^1 & \chi^2 & \chi^3 & \chi^4 & \chi^5 & \chi^6 & \chi^7 & \chi^8 & \chi^9 & \chi^{10} \\ + & + & + & + & - & - & + & - & - & - & + \end{array}$$

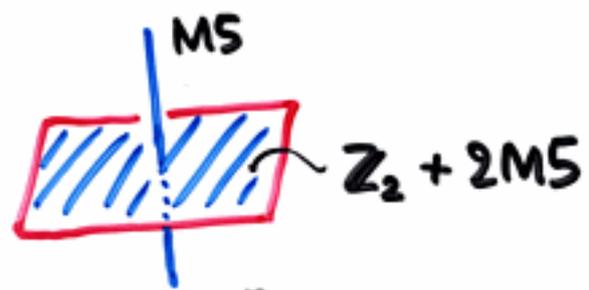
8 spinors with $\Gamma_{012345} = \Gamma_{01236,10} = 1$

Compactify $\chi^{6,10}$ on a torus T^2

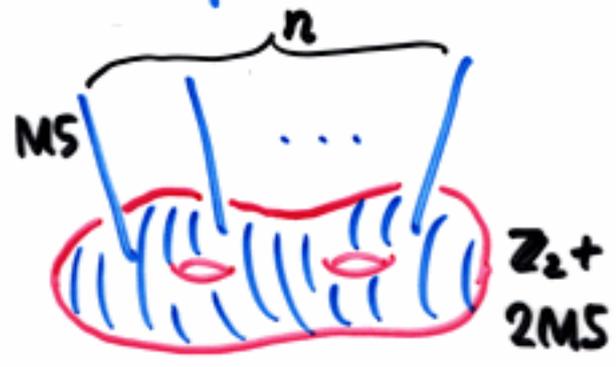
$$S = T^2 \times \mathbb{R}P^2 ; \quad \mathbb{R}P^2 = \left\{ \begin{array}{l} \chi^{4,5} = 0 \\ |\chi^{7,8,9}| = \text{const} \end{array} \right\} / \mathbb{Z}_2$$

$$\left. \begin{array}{l} 2 \int_S G/2\pi = 1 \\ \int_S W_4 = 0 \end{array} \right\} \rightarrow \text{INCONSISTENT}$$

$$\left\{ \begin{array}{l} U = \chi^4 + i\chi^5 \\ Z = \chi^6 + i\chi^{10} \end{array} \right\} \left\{ ZU^2 = \epsilon, \chi^{7,8,9} = 0 \right\} \xrightarrow{\epsilon \rightarrow 0}$$



CONSISTENT



CONSISTENT
only for even n

04 - planes Type IIA on $\mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \cong \{1, \Omega P\}$ ⁵

$$\begin{cases} \text{SO type (D4 brane charge } -1, Z(\mathbb{R}P^4) = +) \\ \text{Sp type (D4 brane charge } 1, Z(\mathbb{R}P^4) = -) \end{cases}$$

04⁻ (SO type 04):

$$M \text{ on } \mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1$$

04⁰ (SO type 04 + D4):

$$M \text{ on } \mathbb{R}^5 \times (\mathbb{R}^5 \times S^1) / \mathbb{Z}_2$$

$\xrightarrow{+\pi}$

04⁺ (Sp type 04):

$$M \text{ on } \mathbb{R}^5 \times \mathbb{R}^5/\mathbb{Z}_2 \times S^1$$

+ 2 × M5 frozen at \mathbb{Z}_2 -fixed plane

$\widetilde{04}^+$ (Sp type 04):

$$M \text{ on } \mathbb{R}^5 \times (\mathbb{R}^5 \times S^1) / \mathbb{Z}_2$$

$\xrightarrow{+\pi}$

+ 1 × M5 stuck at the \mathbb{Z}_2 -inv. cylinder

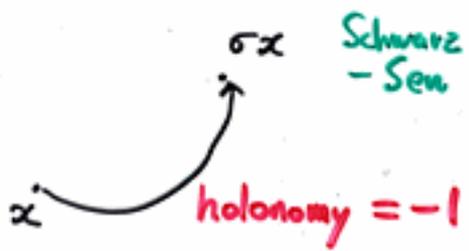
Flux Q.C. OK

also when even # of M5's are added

RR U(1) Gauge Field

$$M \text{ on } (\widehat{X}^{10} \times S^1) / \mathbb{Z}_2$$

$$= \text{Type IIA on } \widehat{X}^{10} / \mathbb{Z}_2$$



$04^-, 04^+$: trivial RR U(1)

$04^0, \tilde{04}^+$: non-trivial RR U(1)

$$\widehat{X}^{10} / \mathbb{Z}_2 \cong \mathbb{RP}^4 \quad H^2(\mathbb{RP}^4, \mathbb{Z}) = \mathbb{Z}_2 = \{0, 1\}$$

NS-NS Two Form

$$B^{NS}, \quad H^{NS} = dB^{NS}$$

$$Z(\mathbb{RP}^2) = \pm \iff H^3(\mathbb{RP}^4, \mathbb{Z}^0) = \mathbb{Z}_2 \ni [H^{NS}/2\pi] = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Witten

$$C_3 = A_3^{RR} + \frac{dx^{10}}{2\pi} \wedge B^{NS}$$

$$G = F_4^{RR} + \frac{dx^{10}}{2\pi} \wedge H^{NS} \quad \text{or} \quad [H^{NS}/2\pi] = \oint_{S^1} [G/2\pi]$$

$$H^4(X'', \mathbb{Z}^0) \xrightarrow{\int_{S^1}} H^3(X^{10}, \mathbb{Z}^0)$$

$$X^{10} \cong \mathbb{RP}^4$$

$$X'' \cong \begin{cases} \mathbb{RP}^4 \times S^1 & 04^-, 04^+ \\ (S^4 \times S^1) / \mathbb{Z}_2 & 04^0, \tilde{04}^+ \end{cases}$$

$$\begin{array}{ccc}
 H^4((S^4 \times S^1)/\mathbb{Z}_2, \mathbb{Z}^0) \cong \mathbb{Z} & \supset \text{even, odd} & \\
 \downarrow \phi_{S^1} & \downarrow \text{mod } 2 & \downarrow \quad \downarrow \\
 H^3(\mathbb{RP}^4, \mathbb{Z}^0) \cong \mathbb{Z}_2 & \supset 0, 1 &
 \end{array}$$

$O4^0 \quad [G/2\pi] = 0 \Rightarrow [H^{NS}/2\pi] = 0$
 $\tilde{O}4^+ \quad [G/2\pi] = 1 \Rightarrow [H^{NS}/2\pi] = 1$

$$\begin{array}{ccc}
 H^4(\mathbb{RP}^4 \times S^1, \mathbb{Z}^0) \cong \mathbb{Z} \oplus \mathbb{Z}_2 & & \\
 \downarrow \phi_{S^1} & \circlearrowleft \downarrow \text{id} & \\
 H^3(\mathbb{RP}^4, \mathbb{Z}^0) \cong \mathbb{Z}_2 & &
 \end{array}$$

$G/2\pi \equiv \frac{1}{2} \omega_4$ ill-defined

$$\begin{array}{ccc}
 \underbrace{\epsilon^{ijkl} R_{ij} \wedge R_{kl}}_{\cong} \in H^4(BSO(4), \mathbb{Z}) & \leftrightarrow & \tilde{\chi} \in H^4(BO(4), \mathbb{Z}^{w_1}) \\
 & & \text{BSpin}(4) \qquad \text{BPin}(4) \\
 \chi, \tilde{\chi} & \longrightarrow & w_4 \text{ mod } 2
 \end{array}$$

$$\tilde{G}/2\pi = \frac{1}{2} [2G/2\pi - \tilde{\chi}] \in H^4(\mathbb{RP}^4 \times S^1, \mathbb{Z}^0)$$

Claim

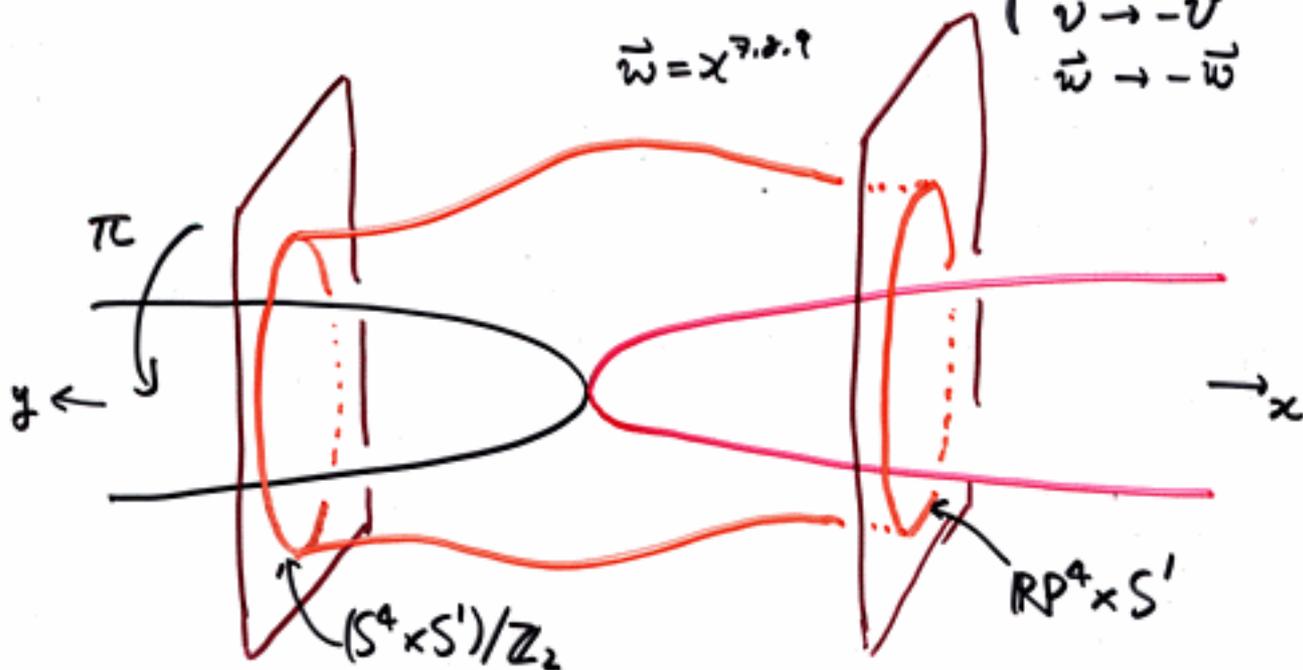
$O4^- \quad [\tilde{G}/2\pi] = (-1, 0) \Rightarrow [H^{NS}/2\pi] = 0$
 $O4^+ \quad [\tilde{G}/2\pi] = (0, 1) \Rightarrow [H^{NS}/2\pi] = 1$

Taub-NUT Space $yx = \nu$

$$\vec{\omega} = \chi^{7,8,9}$$

$$\mathbb{Z}_2 \begin{cases} y \rightarrow -y \\ x \rightarrow x \\ \nu \rightarrow -\nu \\ \vec{\omega} \rightarrow -\vec{\omega} \end{cases}$$

8



$$\int_{(S^3 \times S^1)/\mathbb{Z}_2} \tilde{G}/2\pi = \begin{Bmatrix} 0 & 04^0 \\ 1 & 04^+ \end{Bmatrix} \Rightarrow \int_{\mathbb{RP}^3 \times S^1} \tilde{G}/2\pi = \begin{Bmatrix} 0 & 04^- \\ 1 & 04^+ \end{Bmatrix}$$

04^+

torsion component of $\tilde{G}/2\pi \neq 0$

\Leftrightarrow FREEZING of 2-M5

cf. Sp-type 06 : frozen D_4

Sp-type 07 : frozen D_8

\uparrow
F theory with non-zero flux

Landsteiner Lopez

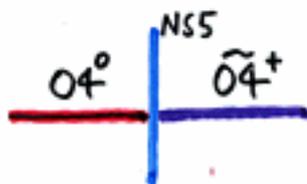
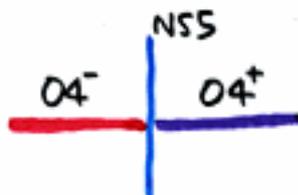
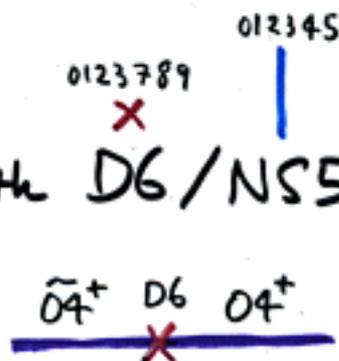
Witten

Berkshsky-Pantev-Sadov
Berglund-Klemm-Mayr-
Theisen

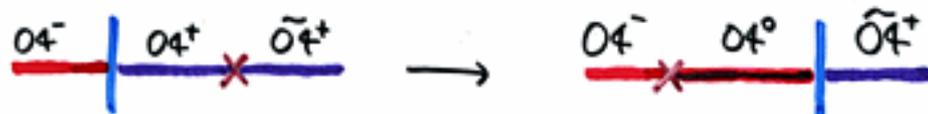
Application

① Intersection of $O4$ with $D6/NS5$

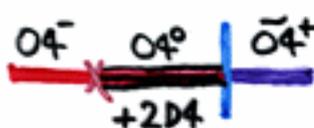
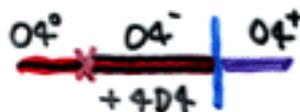
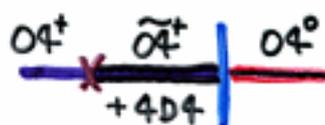
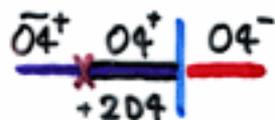
possibility



brane creation



s-rule

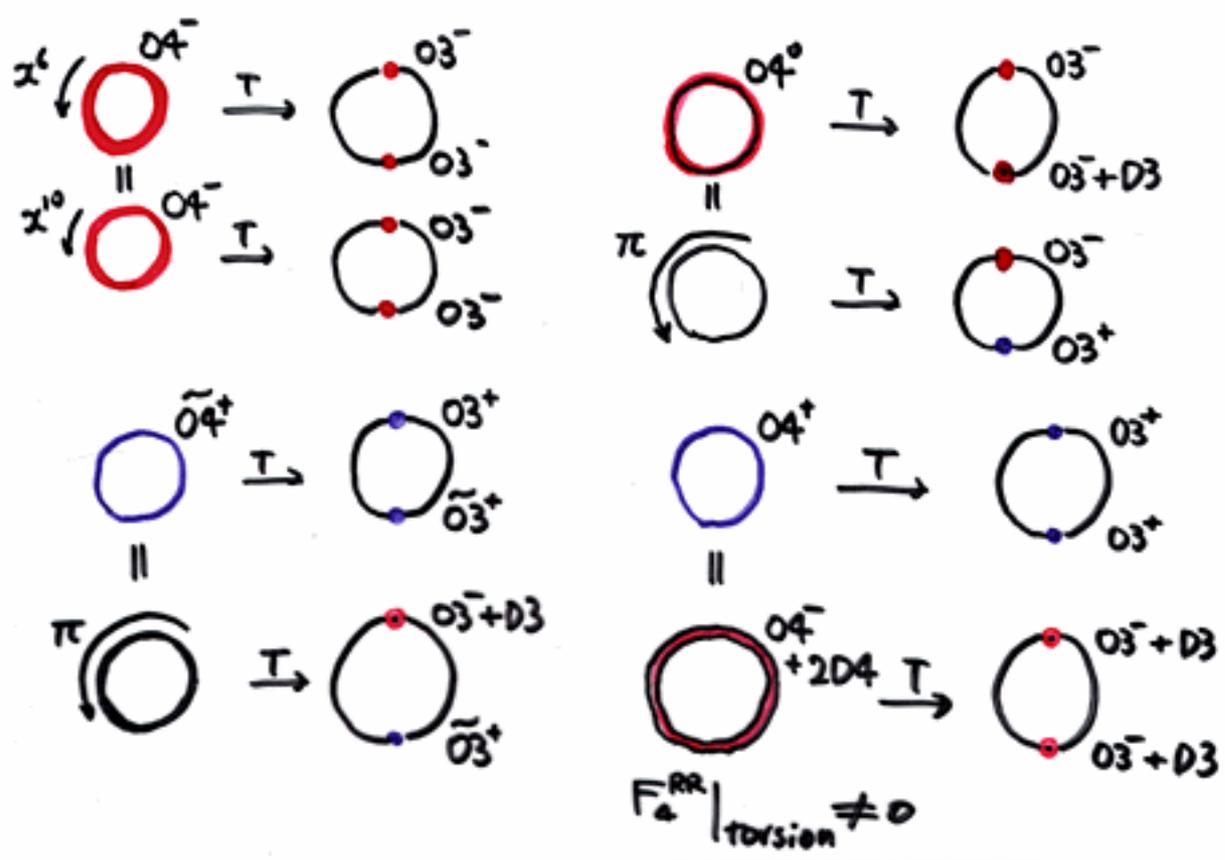
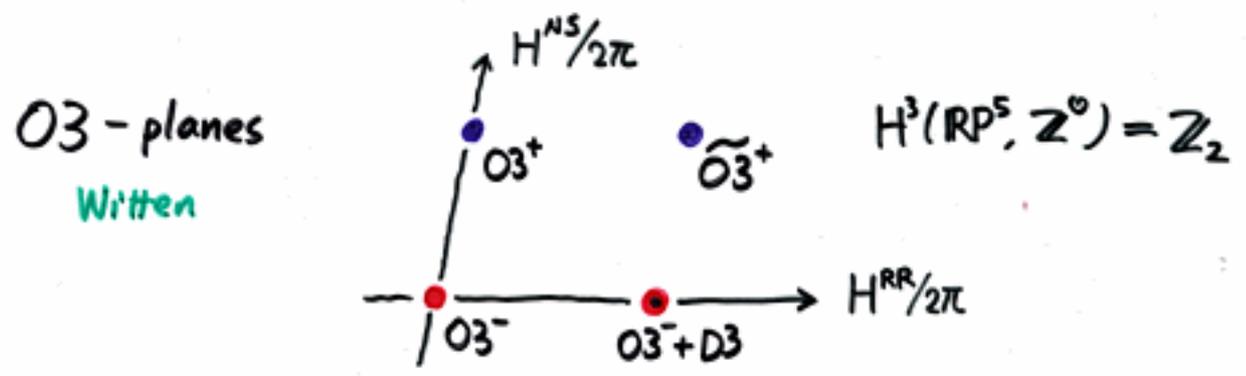


..... SUSY breaking

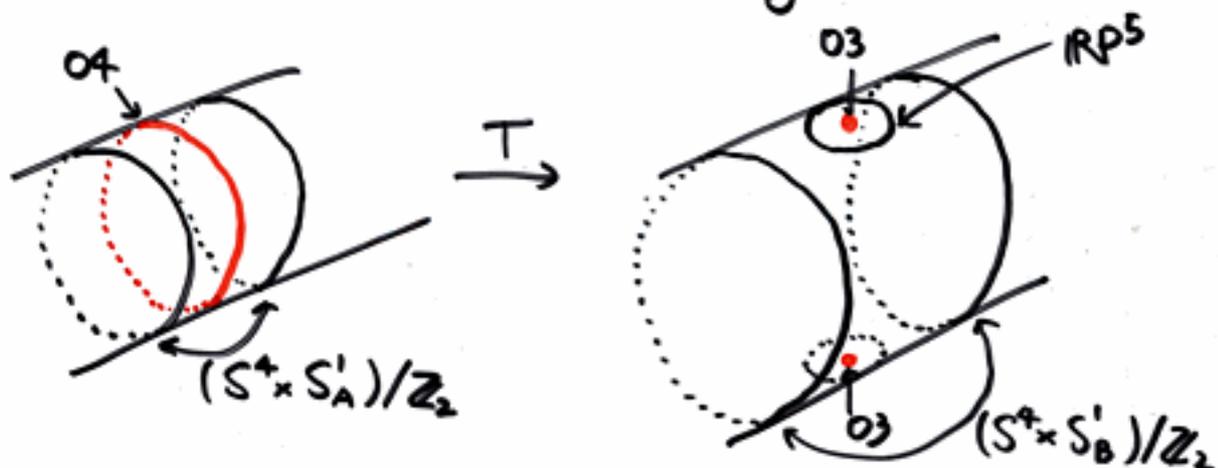
② Montonen-Olive Duality

$$M \text{ on } \mathbb{R}^4 \times S^1_{(11)} \times (\mathbb{R}^5 \times S^1_{(21)}) / \mathbb{Z}_2$$

$$\text{IIBO on } \mathbb{R}^4 \times \underline{\tilde{S}^1_{(11)}} \times \underline{\mathbb{R}^5} / \mathbb{Z}_2 \quad \text{IIBO on } \mathbb{R}^4 \times \underline{\tilde{S}^1_{(21)}} \times \underline{\mathbb{R}^5} / \mathbb{Z}_2$$



RR fluxes under T-duality



$$F_A^{RR} = \oint_{S_B^1} \text{ch}(\mathcal{P}) F_B^{RR}$$

$$F_B^{RR} = \oint_{S_A^1} \text{ch}(\mathcal{P}^\vee) F_A^{RR}$$

\mathcal{P} : Poincaré bundle
over $Z \xrightarrow{S_A^1 \times S_B^1} \mathbb{RP}^4$

cf. Fourier-Mukai transform for RR sources.

③ 4d Gauge theories with $N < 4$

- SW curve of SO/Sp $N=2$ QCD AS
LLL
- Quantum modified constraint S, IP
of $N=1$ $Sp(N_c)$ QCD $N_f = N_c + 1$
- Dynamical SUSY breaking of IYIT model

