Negative AdS energy and its implications for large N gauge theories

(Myers, G.H.)

<u>Claim</u>: There are smooth solutions to $R_{\mu\nu} = \Lambda g_{\mu\nu}$ $(\Lambda < 0)$ which are asymptotically AdS but have <u>negative</u> total energy.

Positive energy theorems use spinors which are "asymptotically constant"

But non-SUSY gauge theories are described by compactifying on S^1 with anti-periodic fermions. (Witten)

Positive energy theorems don't apply

In asymptotically flat case these bdy. cond. allow solns with arbitrarily negative energy!

(Witten)
(Brill, G.H.)

Is same true for $\Lambda < 0$?

"Anything that is possible in asymp. flat spacetimes should be possible in asymp. hyperbolic spacetimes"

Yau

Example of negative energy solution (D=5)

$$ds^2 = \frac{u^2}{\ell^2}\left[\left(1-\frac{u_0^4}{u^4}\right)d\tau^2 + dx^2 + dy^2 - dt^2\right] + \frac{\ell^2 du^2}{u^2\left(1-\frac{u_0^4}{u^4}\right)}$$

$u = u_0$, $\ell^4 = 4\pi g N$

globally static

τ has period

$$\beta = \frac{\pi \ell^2}{u_0}$$

This has total energy

$$E_{SUGRA} = -\frac{N^2 \pi^2 V_2}{8\beta^3} < 0$$

Yang-Mills theory on $S^1 \times \mathbb{R}^2$ has negative Casimir energy. At weak coupling:

$$E_{YM} = -\frac{N^2 \pi^2 V_2}{6\beta^3}$$

$$\Rightarrow \boxed{E_{SUGRA} = \frac{3}{4} E_{YM}}$$

Look familiar?

Recall that for the near extremal black 3-brane

$$S_{BH} = \frac{3}{4} S_{YM}$$

(Gubser, Klebanov, Peet; Strominger)

This is <u>not</u> a coincidence:

The same partition function on $S^1 \times \mathbb{R}^3$ yields both S and E depending on whether Euclidean time is S^1 or $\mathbb{R}$

Key question:

Is this the lowest energy SUGRA soln for given periodicity β?

If so, there must be a new positive energy theorem saying $E \geq E_{SUGRA}$

If not, either large N non-susy gauge theory is unstable, or conjecture fails

Evidence for new positive energy theorem:

Solution is extremum of energy (since static)

Solution is local minimum of energy

$\Rightarrow$ small fluctuations are stable

(This was not true in asymp. flat case)

Key difference between asymp. flat + asymp. AdS

<u>Asymp. flat</u>: The S^1 has constant radius asymp., so 5D metric approaches flat space like 4D soln.

<u>Asymp AdS</u>: The radius of $S^1 \to \infty$ asymp, so space effectively de-compactifies.

Generalizations

- There are negative energy SUGRA solutions corresp. to gauge theory on $S^1 \times S^2$, $S^1 \times \Sigma_g$

E.g. take 5D Schwarzschild AdS & analytically continue in t & θ (get $S^1 \times S^2$ asymp)

Conclusions

1) There are negative energy solutions which reflect Casimir energy in gauge theory

2) AdS/CFT correspondence predicts a new (& strange) positive energy theorem in general relativity with $\Lambda < 0$