TIMELIKE T-DUALITY, BRANES AND DE SITTER SPACE

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STRINGS 98
$R^m \times T^{n,1}$

Solution of String + M-Theories

Lorentzian Torus (Flat) in Space, 1 Time

Closed Timelike Curve

Consider Boundaries of Moduli Space

Strings on Timelike $S^1$ [Moore]

T-Duality $R \rightarrow \frac{1}{R}$

Symmetry of Bosonic + Heterotic Strings

Strange Features;

Swaps $p^0$ with Time Winding No $(p^0, n) \rightarrow (0, p^0)$ Tachyonic

If No Winding Nos. Originally

Dim Reduction on $T^{n,1} \rightarrow$ Euclidean Theory in $R^m$
IIA, IIB T-DUAL on SPACELIKE $S'$

- CHECK: REDUCTION of IIA, IIB SUGRA's GIVE SAME 8+1 SUGRA
- BUT REDUCTION on TIMELIKE $S'$ of IIA, IIB GIVE DIFFERENT SUGRAS

[CREMMER, LAURIVENKO, LU, POPE, STELLE, TRAN]

$\Rightarrow$ IIA, IIB NOT T-DUAL on TIMELIKE $S'$

- DEFINE 9+1 STRING THEORIES

IIA$^*$) TIMELIKE T-DUAL of IIB

IIB$^*$) $R \to 0$ LIMIT of IIA, IIB

IIA on TIMELIKE $S'$, RADIUS $R$
$\text{IIB}^* = \lim_{R \to 0} \text{IIA on timelikes}
= \lim_{R_1, R_2 \to 0} \text{M on Lorentzian torus}^*$

**Scalar Coset** $\text{SL}(2; \mathbb{R})/\text{SO}(1, 1)$

**RR Scalar has Wrong Sign Kinetic Term**

$\delta^{ij} \to \eta^{ij} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$H_i^j = \partial_i B_j^0$

$L \sim \eta^{ij} H_i^j \mu \nu + i \eta^{ij} \bar{\psi}_i \gamma_\mu \gamma_5 \nu \psi_j$

**Twisted Supersymmetry**:

$\{\xi^i, \xi^j\} = \frac{1}{8} \eta^{ij}$, $i, j = 1, 2$

$Q^1, Q^2$: Same Chirality in IIB*

Opposite Chirality in IIA*
$\int d^{10}x \sqrt{-g} \left( e^{-2\Phi} \left( R + 4(\nabla\Phi)^2 - H^2 \right) \right)$

Wrong

Sign

$\sum_n |dC_n|^2 + \ldots$

$C_n \rightarrow iC_n$ in Bosonic Action

$\int \text{act on}$

$\begin{align*}
\text{IIA, B* String} & \quad \text{with} \\
\text{IIA, B String} & \quad \text{with} \\
\text{iF}_L & \quad \text{Left-Handed Fermion No.}
\end{align*}$
Dp - BRANES OF TYPE II

Ep - BRANES OF TYPE II*

Neumann \( x', \ldots, x^p \)

Dirichlet \( y^{p+1}, \ldots, y^q, t \)

\( p \)-dimensional Euclidean plane at fixed \( y, t \)

Do - Brane: Particle World-Line

E1 - Brane: Tachyon World-Line

Supergravity Solution:

\[ ds^2 = H^{\frac{1}{2}} dx^2 + H^{\frac{1}{2}} (dy^2 - dt^2) \]

\( H(y, t) \) Harmonic Function
**World-Volume Theory**

**N Ep-Branes:** \( U(N) \) Super Y.M. in \( p \) Euclidean Dims, 16 Susy's

From \( 9+1 \) Super Y.M. on \( T^{9-B1} \)

\( SO(9-p,1) \) R-Symmetry

Scalar from \( A_0 \) has kinetic term of 'Wrong' Sign, 'Ghost.' Truncated theory has ghost, but if Kaluza-Klein Towers are kept, higher dimensional gauge invariance can be used to eliminate \( A_0 \).

\( A_0 \): Collective Coord for Brane-Time
N E L 4-BRANES

U(N) Super Y.M. in 4 Euclidean Dimensions
SO(5,1) R-Symmetry
SO(4) Lorentz Symmetry
SO(5,1) Conformal Symmetry

Must be Superconformal Group containing SO(5,1) x SO(5,1):

→ SU*(4/4) 32 Ferm. Gens

5-Dim. Super de Sitter Group?

This Euclidean Super Yang-Mills Theory Can Be Twisted → Topological YM

[Acharya, Figueroa-O'Farrill, O'Loughlin, Spence]
[Blau, Thompson]
IIB* has solution

5-Dim de Sitter $dS^5 \frac{SO(5,1)}{SO(4,1)}$

$\times$ 5-Dim Hyperbolic Space $H^5 \frac{SO(5,1)}{SO(5)}$

Isometry: $SO(5,1) \times SO(5,1)$

Maximally Supersymmetric: 32 Killing Spacors

Super de Sitter Invariance $SU^*(4/4)$

E4-Brane interpolates between $dS^5 \times H^5$ and flat space

$\Rightarrow$ Duality between large-N Euclidean Super Y.M.

And IIB* String in $dS^5 \times H^5$?
• Other 'New' Theories?
• Strong-Coupling Limits of IIA*B?
• M-Theory on Timelike $S^1$ 10+1

$\rightarrow$ IIAE Euclidean String Theory 10+0

2+1 Brane $\rightarrow$ 2+0 Brane, Euclid: Worldsheet

What is T-Dual of IIAE? \( (S') \) spacelike

A Euclidean IIB ?? or: IIB*

IIAE

IIA

M

M on $T^{5,1}$

$\rightarrow$ II*B* has different signature

& already has a T-dual
\[ L \sim -e^{-2\phi}(dB_2)^2 + (dC_2)^2 + \ldots \]

**Fund. Strings, 1+1 World Sheet \sim B_2**

**E2 Branes 2+0 W.S. Couples to C_2**

**Strong Coupling:** \( \phi \to -\phi, \) \( B_2 \leftrightarrow C_2 \)

\[ \rightarrow \text{II}B' \text{ String} \]

\[ L \sim e^{-2\phi}(dB_2)^2 - (dC_2)^2 + \ldots \]

**Fund. 2+0 Brane, Couples to B_2’**

**D-String 1+1 W.S., Couples to C_2’**

**Perturbative II B’: ‘Euclidean’ Strings**

**T-Dual of IIAE \rightarrow II B’**

**Euclidean World-Sheet, Instead of Target Space**
T-Duality: $X(0, r) \rightarrow \tilde{X}(0, \tilde{r})$

$\partial_a X = \varepsilon_a \partial^\alpha \tilde{X}^\alpha$

$(\partial_a X)(\delta^\alpha \tilde{X}) = \eta (\partial_a \tilde{X})(\delta^\alpha \tilde{X})$

Lorentzian W.S.: $\eta = 1$, $X, \tilde{X}$ same sign
Euclidean W.S.: $\eta = -1$, $X, \tilde{X}$ different sign

For Eucl. W.S., T-Duality changes signature

IIAE on Spacelike $S'$, radius $R$
= IIB' on Timelike $S'$, radius $\frac{1}{R}$

IIB' strong coupling dual of IIB$^{**}$

IIA in 9+1, IIAE in 10+0

Spacelike T-Duality: IIB$'_9$+1
\rightarrow Signature 8+2 ??
**Strong Coupling Limit of IIA**

- IIA* Supergravity can't be obtained from 11-D Supergravity.
- Can be obtained from $9+2$

\[ L = R + (dC_3)^2 + C_3 \wedge dC_3 \wedge dC_3 \]

'Wrong' sign.

This is only 11-D field theory giving IIA* on reduction.

- Infinite tower of $E_1$-branes or Tachyons become massless as $g_s \to \infty$, KK tower for decompactifying extra time.
$M^*$ THEORY IN 9+2 DIMS

$M$-THEORY ON $T^{2,1}$ $(T^3, (++\,-))$

IN LIMIT IN WHICH $T^{2,1}$ SHRINKS TO ZERO SIZE

$\rightarrow M^*$ IN 9+2

ON TIMELIKE $S'$: $IIA^*$ IN 9+1

ON SPACELIKE $S'$: $IIA_{8+2}$ IN 8+2

$g_{\mu\nu}$, $C_{\mu\nu\rho}$, $\Psi^\mu$ PSEUDO-MAJORANA

9+2 SUPERGRAVITY THEORY
M* \_{q+2} \text{ ON SHRINKING } T^{3,0} \\
\rightarrow M' \text{ THEORY IN } 6+5 \text{ DIMS}

D=11 \text{ M-THEORIES}
10+1, 9+2, 6+5 (1+10, 2+9, 5+6)

D=10 \text{ IIA THEORIES}
10+0, 9+1, 9+1^*, 8+2, 6+4, 5+5, 5+5^*

D=10 \text{ IIB THEORIES}
9+1, 9+1^*, (9+1^*), 7+3, 5+5, 5+5^*(5+5^*)

D=12 \text{ F-THEORIES}
11+1, 10+2, 9+3, 7+5, 6+6

\rightarrow \text{ ZERO-SCALE OR LOW 'ENERGY' LIMITS GIVES CORRESPONDING SUPERGRAVITY THEORIES.}
CONCLUSIONS

• Duality can change space-time signature, signs of kinetic terms...
  \[ IIB_{9+1}^{\ast} \]
  \[ M_{10+1} \]
  \[ ? \]
  \[ M_{9+2}^{\ast} \]
  \[ M_{6+5}^{-} \]
  all 'corners' of moduli space

• Different real forms of 'complex' \( t \times \)

• De Sitter spaces and branes of various signatures [CMH+]

• D-branes, E-branes etc

• New supergravities

• Theories have solutions \( \mathbb{R}^{p+q} \)

• Timeline compactifications used to link them