

# Background Geometry of Matrix Theory and Holography

(Strings '98)

- based on works with

Youngjai Kiem, Hyeonjoon Shin

S. Hyun

(KIAS, Korea)

## Maldacena's proposal

CFT of SYM  $\xleftrightarrow{\text{duality}}$  M/string theory  
on AdS space

This is observed by studying appropriate limit  
which gives

- ① conformal limit of SYM  
(bulk deg. of freedom decoupled and  
only worldvolume d.o.f remains)
- ② near horizon (or large charge) limit  
of SUGRA sol.

The limit considered by Maldacena is exactly the same limit considered in DLCQ M theory on  $T^P$  in the decompactification limit.

This is natural since the whole idea of DLCQ M theory is to extract the worldvolume degrees of freedom of p-brane decoupled from all other deg. of freedom by going discrete light cone quantization and taking T-dualities and appropriate limit.

The natural question is then,

"What does the original BFSS Matrix model ( $N \rightarrow \infty$  limit of DLCQ M theory) describe?"

DLCQ of M theory (BFSS, Susskind)  
 (Seiberg)

$$x^\pm = x \pm t$$

$t$  11<sup>th</sup> coord.

light-like compactification of M theory  
 with radius R

$$x^- \sim x^- + R \rightarrow \text{spatial coord.}$$

$$x^+ \rightarrow \text{time coord.}$$

=  $\infty$  boosts of M theory on a small spatial circle with radius  $R_s$

$$x'' \sim x'' + R_s$$

$t$

$$\binom{x^+}{x^-} \sim \binom{x^+}{x^-} + \binom{R_s}{R_s} \rightarrow \binom{x^+}{x^-} \sim \binom{x^+}{x^-} + \binom{0}{R}$$

Lorentz boosts

$$\lambda = \frac{R}{R_s}$$

$$x^+ \rightarrow \bar{\lambda} x^+$$

$$x^- \rightarrow \lambda'' x^-$$

(4)

Seiberg's prescription (Seiberg, Sen)

DLCQ of M with  $R, M_p$

=  $\tilde{M}$  with  $\tilde{R}, \tilde{M}_p$  with the following identifications

$$\tilde{R} \tilde{M}_p^2 = R M_p^2 : \text{fixed in the limit } \tilde{R} \rightarrow 0$$

$$\tilde{R}_i \tilde{M}_p = R_i M_p : \dots$$

$\tilde{R}_i$ : characteristic length of transverse 9d space.

$\tilde{M}$  theory has

$$\tilde{g}_s = (\tilde{R} \tilde{M}_p)^{\frac{3}{2}} \sim \tilde{R}^{\frac{3}{4}} \rightarrow 0$$

$$\tilde{M}_s^2 = \tilde{R} \tilde{M}_p^3 \sim \tilde{R}^{-\frac{1}{4}} \rightarrow \infty$$

$\Rightarrow$  weakly-coupled string theory

N sector of DLCQ M theory

= theory of N D0 branes living in  
a small transverse space

$$\tilde{R}_i \sim \tilde{R}^{\frac{1}{4}} \rightarrow 0$$

## SUGRA solution of N D0 branes

Hyun ~~Kim~~

Hyun, Y. Kim, H. Shin

~~Kim~~

$$ds_{10}^2 = -\frac{1}{f} dt^2 + \sqrt{f} (dx_1^2 + \dots + dx_9^2)$$

$$\bar{e}^{2\phi} = f^{-\frac{3}{2}}$$

$$A_t = 1 - \frac{1}{f}$$

$$f = 1 + \frac{r_0^7}{r^7} \quad r_0^7 = \frac{Q l_p^9}{R_s^2}, \quad Q \propto N$$

$\Rightarrow$  lift to 11 d.  $x \sim x + R_s$

$$ds_{11}^2 = -dt^2 + dx^2 + (f-1)(dt-dx)^2 + dx_1^2 + \dots + dx_9^2$$

$$= dx^+ dx^- + (f-1) dx^-{}^2 + dx_1^2 + \dots + dx_9^2$$

$\Rightarrow \infty$  boosting  $x^- \sim x^- + R$   $R : \text{finite}$   
 $R_s \rightarrow \infty$

$$ds_{11}^2 = -d\tau^2 + dx^-{}^2 + (h-1)(d\tau - dx^-)^2 + dx_1^2 + \dots + dx_9^2$$

$$h = \frac{r_0^7}{r^7} \quad r_0^7 = \frac{Q l_p^9}{R^2}$$

Now we can apply the prescriptions on DLCQ M-theory

The background geometry can be considered as the background geometry of  $\tilde{M}$  theory with the following identifications

$$\text{metric : } \frac{1}{\tilde{\alpha}'} d\tilde{s}^2 = \frac{1}{\alpha'} ds^2 \quad \alpha' = l_s^{-2}$$

$$\text{string coupling : } \frac{\tilde{g}_s}{g_s} = \left( \frac{\tilde{\alpha}'}{\alpha'} \right)^{\frac{3}{2}}$$

$\Rightarrow$

$$d\tilde{s}^2 = -\frac{1}{\tilde{f}} + \sqrt{\tilde{f}} (d\tilde{x}_1^2 + \dots + d\tilde{x}_9^2)$$

$$e^{-2\tilde{\phi}} = \tilde{g}_s^{-2} \tilde{f}^{-\frac{3}{2}}$$

$$\tilde{f} = \left( \frac{\tilde{M}_p}{M_p} \right)^2 h = -\frac{Q \tilde{l}_p^9}{R^2 \tilde{r}^7}$$

## Interpretation

- correspondence between

$$\text{DLCQ M theory} \longleftrightarrow \tilde{\text{M}} \text{ theory}$$

- Thus, DLCQ M theory is the theory of D0-branes.

$$\begin{array}{c} \wedge \\ N \end{array}$$

- We are considering the theory with  $N$  D0-branes in the  $\mathbb{R}^3$  background

$\Rightarrow$  The background geometry is not flat Minkowskian.

- $x^a \sim x^a + R \Rightarrow$  true spatial 11-th direction

- What would we get in the large  $R$  limit? (decompactification)

(2-1)

DL CQ on  $T^3$  and  $AdS_5 \times S^5$

$\Rightarrow$  After T-duality on  $T^3$ , it becomes

theory of N D3 branes on  $\tilde{T}^3$  with radii

$$\Sigma_i = \frac{1}{\tilde{R}_i \tilde{M}_5} = \frac{1}{R_i R M_p} = \text{finite}$$

$$\tilde{g}_s' = \tilde{g}_s (\Sigma_i \tilde{M}_5) \sim \tilde{R}^{-\frac{p-3}{4}} \rightarrow \text{finite for } p=3$$

$$h = \frac{Q \tilde{l}_p^9}{R^2 R_1 \cdots R_p \tilde{r}^{7-p}}$$

$$\Rightarrow \tilde{f} = \left(\frac{\tilde{M}_5}{M_p}\right)^2 h = \frac{Q \tilde{l}_p^9}{\tilde{R}^2 \tilde{R}_1 \cdots \tilde{R}_p \tilde{r}^{7-p}}$$

$$d\tilde{s}^2 = \tilde{\alpha}' \left[ \underbrace{\frac{U^2}{\sqrt{Q \tilde{g}_s}} (-d\tilde{t}^2 + d\tilde{x}_1^2 + d\tilde{x}_2^2 + d\tilde{x}_3^2)}_{AdS_5} + \underbrace{\sqrt{Q \tilde{g}_s} \left( \frac{dU^2}{U^2} + dS_5^2 \right)}_{S^5} \right]$$

where  $U \equiv \frac{\tilde{r}}{\tilde{\alpha}'} = (RM_p) r$  (with  $\tilde{x}_i \sim x_i + \Sigma_i$ )

$$\tilde{x}^i = \frac{\alpha'}{\tilde{\alpha}'} \tilde{x}_i \quad i=1,2,3$$

$$\Sigma'_i = \frac{\alpha'}{\tilde{\alpha}'} \Sigma_i \sim \tilde{R}^{-\frac{1}{2}} \rightarrow \infty : \text{non-compact}$$

(2-2)

In the original DLCQ M theory on  $T^7$   
it reads,

$$ds^2 = \frac{r^2}{\sqrt{Qg_s'} \alpha'} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{Qg_s'} \alpha' \frac{dr^2}{r^2} \\ + \sqrt{Qg_s'} \alpha' d\Omega_5^2$$

$$x_i \sim x_i + \Sigma_i , \quad \Sigma_i = \frac{1}{R_i M_s^2}$$

If  $R_i \rightarrow 0$ ,  $\Sigma_i \rightarrow \infty \Rightarrow AdS_5 \times S^5$   
↳ conformal limit of SYM

32 Killing spinors  $\Rightarrow$  M-theory background.

In general,

$$ds^2 = \frac{1}{f} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{f} (dx_4^2 + \dots + dx_9^2)$$

$$f = \sum_a \frac{Q_a g_s' \alpha'^2}{|\vec{r} - \vec{x}_a|^4}$$

Asymptotically  $AdS_5 \times S^5$

(8)

# DLCQ of M theory on $T^9$

For vanishingly small  $\rightarrow T^9$ , we take

T-duality.

	$(\tilde{M})$ rel. deg. freedom.	string coupling	$(M)$ background
$T^0$ :	$D0$ (IIA)	weak	
$T^1$ :	$D1$ (IIB)	weak	$\xrightarrow{TS \uparrow}$ KK momentum (IIA) "DVV"
$T^2$ :	$D2$ (IIA)	$\xrightarrow{\text{lift to } II^d}$	$M2$ (M) $AdS_4$
$T^3$ :	$D3$ (IIB)		$AdS_5$
$T^4$ :	$D4$ (IIA)	strong	$\xrightarrow{\text{lift to } II^d}$ $M5$ (M) $AdS_9$
$T^5$ :	$NS5$ (IIA) (+ FI)	$\longrightarrow$	$D1 + D5$ (IIB)

If we take the large  $N, R$  limit of  
DLCQ M theory, we would end up  
with the decompactification limit of  
BFSS Matrix theory.

The theory is described by

0+1 dimensional quantum mechanics.  
 $\stackrel{\wedge}{\text{Do}}$

In the  $R \rightarrow \infty$  limit, it becomes  
superconformal Q.M. which has

$$OSp(16/2, R) \supset Sp(2, R) \times SO(16)$$

symmetry. (Güneydin & Minic)

$$Sp(2, R) \sim SO(2, 1) \quad \text{0+1 dim'nal conformal symmetry.}$$

$$SO(16) \quad R\text{-symmetry}$$

10

# Background geometry of BFSS Matrix theory

$$ds_n^2 = dx^+ dx^- + \frac{N}{R^2} \frac{1}{r^n} dx^- dx^- + dx_1^2 + \dots + dx_9^2$$

Let  $P_- = \frac{N}{R}$  = fixed and take  $R \rightarrow \infty$  limit

Note :

As  $R \rightarrow \infty$ , we need to include

the contribution from all the massive

KK modes

$$\sum_{n=-\infty}^{\infty} \delta(x^- + nR) \xrightarrow[R \rightarrow \infty]{\quad} \frac{1}{R} \int_{-\infty}^{\infty} dt_0 \delta(x^- + t_0) = \frac{1}{R}$$

$$\Rightarrow \frac{1}{R} \xrightarrow[R \rightarrow \infty]{} \sum_{n=-\infty}^{\infty} \delta(x^- + nR) \xrightarrow[R \rightarrow \infty]{} \prod \delta(x^-)$$

$\Rightarrow$

$$ds_n^2 = dx^+ dx^- + \frac{P_- \delta(x^-)}{r^n} (dx^-)^2 + dx_1^2 + \dots + dx_9^2$$

Aichelburg & Sexl metric

## Proposal

BFSS Matrix theory in the large N limit

describes M theory on  $\checkmark^{11D}$  A-S spacetime.  
 $\uparrow$   
 no "D" !!

### NOTE

- ① This would be thought as "T-dual" version of  $CFT^F/AdS$  correspondence.
- ② In contrast to flat Minkowskian geometry, AS spacetime has time-like boundary at  $x^- = 0$ .  
 $(x^+ \sim \text{time-like}, x^- \sim \text{space-like coord.})$
- DO-CFT can be thought as the theory on this boundary, treating the transverse part ( $x^1 \dots x^9$ ) as the internal one.

## Supporting argument

① Since  $g_{\mu\nu}$  depends on  $r$ , the transverse part has  $SO(9)$  isometry.

② In the bulk ( $x^- \neq 0$ ), it has 32 supercharges (Killing spinors).

$\Rightarrow$  two 16's  $\Rightarrow SO(16)$  R-symmetry.

③ The geometry is invariant under

$$\cdot x^+ \rightarrow x^+ + \epsilon \quad (\text{time translation})$$

$$\begin{cases} x^+ \rightarrow \lambda x^+ \\ x^- \rightarrow \bar{\lambda} x^- \end{cases} \quad (\text{time dilatation})$$

$$\begin{cases} x^+ \rightarrow x^+ + \frac{\epsilon}{2} x^{+2} \\ x^- \rightarrow x^- - \epsilon x^+ x^- \end{cases} \quad (\text{special conformal transf.})$$

In the last case, the background geometry is not invariant, but

$$ds^2 \rightarrow ds^2 + O(x^-) \xrightarrow{x^- \rightarrow 0} 0$$

## Conclusions

Superconformal Limit of BFSS Matrix ~~theory~~

describes M theory on AdS spacetime

(not on flat Minkowskian spacetime)

It implies the holographic nature of  
the matrix theory, just like its

"T-dual cousins" : CFT/AdS

In analogy with GKP, Witten for CFT/AdS  
case

one can study the SUGRA on this  
background.

(To appear soon, hoping not like W9901\*\*\*)