A Non-Superstring (II)

Preliminary results with:

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What we have shown so far is that :

has vanishing integrand if we make a very specific choice of insertion pts for the picture-changing operators.

But : δ_{pts} $= \int_{\mathcal{F}} \partial \omega \Rightarrow$

one should worry about contributions at the <u>boundary</u> of the relevant fundamental domain.

<u>What is $\mathcal{F}$ for this computation?</u>

At genus 2, the Teichmüller space is

$$H_2 = \{ \tau_{2\times 2} : \tau^{tr} = \tau, \ \mathrm{Im}\, \tau > 0 \}$$

(τ is the period matrix of the surface).

We need to "mod out" by the subgroup of $Sp(4, \mathbb{Z})$ that preserves $(1, 1, f, g)$ twists.

So we allow matrices that act on the homology (a_1, a_2, b_1, b_2) like:

$$\begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ x & y & 1 & 0 \\ z & w & 0 & 1 \end{pmatrix}$$

Denoting the 2×2 blocks $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$, must have:

$$\left.\begin{array}{l} A^T C = C^T A, \quad B^T D = D^T B \\ A^T D - C^T B = 1 \end{array}\right\} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ x & y & 1 & 0 \\ y & w & 0 & 1 \end{pmatrix}$$

Now, $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ acts on homology $\Rightarrow \begin{pmatrix} D & C \\ B & A \end{pmatrix}$ acts on the period matrix τ.

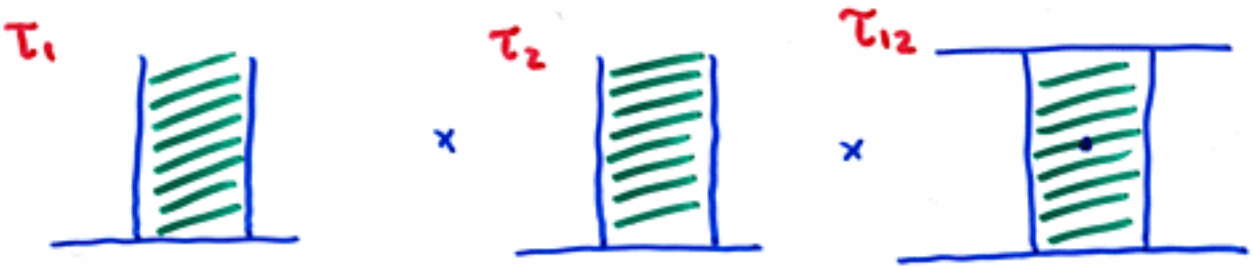
So the action on period matrices is:

$$\begin{pmatrix} \tau_1 & \tau_{12} \\ \tau_{12} & \tau_2 \end{pmatrix} \longrightarrow \begin{pmatrix} \tau_1 + x & \tau_{12} + y \\ \tau_{12} + y & \tau_2 + w \end{pmatrix}$$

In addition:

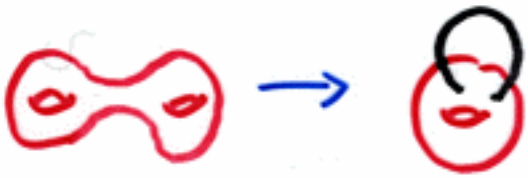
$$\mathrm{Im}\,\tau > 0 \Rightarrow \begin{cases} \cdot\ \mathrm{Im}\,\tau_{1,2} > 0 \\ \cdot\ (\mathrm{Im}\,\tau_{12})^2 < \mathrm{Im}\,\tau_1\ \mathrm{Im}\,\tau_2 \end{cases}$$

So we find for the fundamental domain:



Where are possible boundary contributions?

1) τ_1 or $\tau_2 \to i\infty$

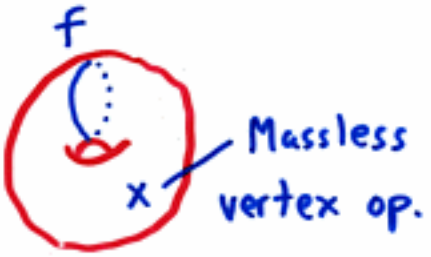
Here, 

Atick, Moore and Sen: No contribution here if there are no physical tachyons.

2) $\tau_{12} \to 0$

In this limit, the genus 2 vacuum energy is related to a product of 1-loop tadpoles.

But:



$= 0$

(the f-twist preserves 4d $\mathcal{N}=2$ SUSY)

No contribution here.

3) $\mathrm{Im}\,\tau_{1,2} \to 0$ or $(\mathrm{Im}\,\tau_{12})^2 \to \mathrm{Im}\,\tau_1\,\mathrm{Im}\,\tau_2$

The integrand contains a factor giving $e^{-\text{Action}}$ of the $g=2$ worldsheet wrapping around the $X^{3,4}$ (from g twist) and $X^{5,6}$ (from f-twist) circles.

In terms of $\operatorname{Im}\tau = \begin{pmatrix} \operatorname{Im}\tau_1 & \operatorname{Im}\tau_{12} \\ \operatorname{Im}\tau_{12} & \operatorname{Im}\tau_2 \end{pmatrix}$,

$$S \approx \frac{R^2}{\alpha'} \left\{ \frac{\operatorname{Im}\tau_1 + \operatorname{Im}\tau_2}{\operatorname{Im}\tau_1 \operatorname{Im}\tau_2 - (\operatorname{Im}\tau_{12})^2} \right\}$$

Now, positivity of $\operatorname{Im}\tau$ comes to the rescue:

- If $\operatorname{Im}\tau_1 \to 0$ at fixed $\operatorname{Im}\tau_2$, $(\operatorname{Im}\tau_{12})^2 < \operatorname{Im}\tau_1 \operatorname{Im}\tau_2$
  $\to S \to \infty$
- If $\operatorname{Im}\tau_{1,2} \to 0$, can prove denominator $\to 0$ like $(\text{numerator})^2$ so $S \to \infty$
- If $\operatorname{Im}\tau_{1,2}$ fixed and $(\operatorname{Im}\tau_{12})^2 \to \operatorname{Im}\tau_1 \operatorname{Im}\tau_2$, obvious that $S \to \infty$

Upshot: e^{-S} damps integrand, no boundary contributions here.

What about $g > 2$?

First, lets simplify the kinds of twists that we need to consider...

Twists @ higher genus

A priori at genus g, one has to consider all possible twists consistent with:

$$a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_g b_g a_g^{-1} b_g^{-1} = 1$$

- We need not worry about twists which preserve spacetime SUSY.
- We can now prove that, up to chiral shifts on bosons, one can map <u>any</u> twists which break SUSY to:

$$(\underbrace{1, \ldots, 1}_{a\text{-cycles}}, \underbrace{f, g, 1, \ldots, 1}_{b\text{-cycles}})$$

One does this using modular transformations.

First, consider the case $g=2$.

i) Suppose no "subtorus" has twists that break full SUSY. Then using $SL(2,\mathbb{Z})$ subgroups of $Sp(4,\mathbb{Z})$, can arrange to have $(1,1,*,*)$.

So the only cases to worry about are

$$(1,1,f,fg)\ ,\ (1,1,g,fg)$$

The generators of the $g=2$ modular group are:

$$D_{a_1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad D_{b_1} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$D_{a_2},\ D_{b_2},\ D_{a_1^{-1}a_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix}$$

So one easily verifies e.g.

$$(1,1,f,fg) \xrightarrow{D_{b_1}} (f,1,f,fg) \xrightarrow{D_{a_1^{-1}a_2}} (f,1,1,g)$$

$$\xrightarrow{SL(2,\mathbb{Z})} (1,1,f,g) \checkmark$$

ii) Suppose some "subtorus" does have twists that break the full SUSY. Possibilities are

$$(f, g, g, f), \quad (f, fg, fg, f).$$

$$(f,g,g,f) \xrightarrow{D_{a_1^{-1}a_2}} (f,g,f,g) \xrightarrow{SL(2,\mathbb{Z})_{1,2}} (1,1,f,g) \checkmark$$

Upshot: All $g=2$ SUSY breaking twists $\longrightarrow$ $(1,1,f,g)$ (up to chiral shifts).

What about $g>2$? Three important facts:

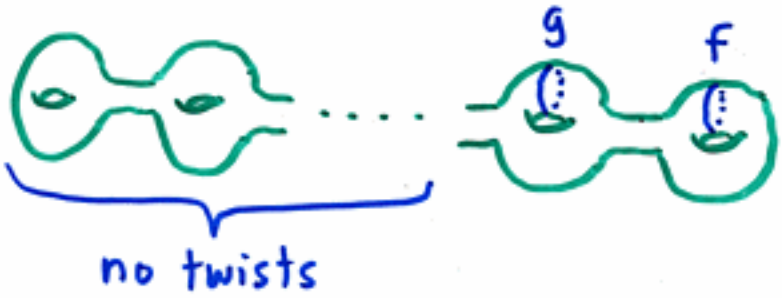
1) Among elements of $Sp(2g,\mathbb{Z})$ are matrices that allow one to permute different "subtori".

2) $\exists$ an <u>even</u> # of subtori with twists on (a_i, b_i) that break all SUSY (from $a_1 b_1 a_1^{-1} b_1^{-1} \cdots = 1$).

3) Using $Sp(4,\mathbb{Z}) \subset Sp(2g,\mathbb{Z})$, can take

$$(1,1,f,f) \longrightarrow (1,1,f,1)$$

Putting together $g=2$ facts with 1) - 3) $\Rightarrow$ only need to consider



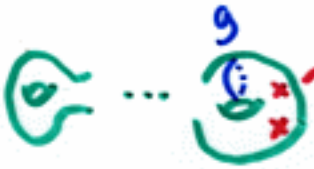
So, why might $\Lambda = 0$ at higher genus?

First, consider massless states in the f-loop:

This $\rightarrow$ trace over 2 pt functions, multiplied by

$\left\{ \begin{matrix} +1 \\ -1 \end{matrix} \right.$ for $\left. \begin{matrix} f \text{ invt.} \\ f \text{ anti-invt.} \end{matrix} \right\}$ states:

$\sum f_s$



$\sim V_0^s(z) = (\partial X^\mu + (k\cdot\psi)\psi^\mu) e^{ikX}$

$\otimes$ anti-hol.

You can convince yourself that:

- $\exists$ equal # of f invt. and f anti-invt. physical states (actually $\forall$ mass levels)
- They give equal and opposite contributions to the trace over 2 pt functions
- Unphysical states decouple (M. Peskin)

So, higher genus vanishing might be a consequence of the very symmetric nature of the spectrum.

Is there a more general story?

Maldacena; Gubser, Klebanov, Polyakov; Witten

Perhaps there is; the adS / CFT correspondences have interesting implications for non-supersymmetric models.

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E.g. the correspondences involving 4d CFTs have (in the 't Hooft limit)

$$\alpha'/R^2 \text{ expansion} \longrightarrow \text{expansion in } \frac{1}{\sqrt{g_{YM}^2 N}}$$

$$g_{st} \text{ expansion} \longrightarrow \text{expansion in } g_{YM}^2 = \frac{1}{N}$$

In cases where one has a non-supersymmetric conformal fixed line (to all orders in $1/N$ as well as $g_{YM}^2 N$), we would obtain stable non-susy string vacua that exist at arbitrary values of the coupling g_{st}.

$$\text{EOM}: \ (-g)^{1/2} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \phi \right) = -\frac{\partial V}{\partial \phi}$$

Then, stability of spacetime background $\forall \langle \phi \rangle \Rightarrow$ no g_{st} dependent vacuum energy!

Of course, in the $(AdS_5 \times S^5)/\Gamma$ cases, we are talking about the effective 10d cosmological constant. (Also, in those cases in the flat space limit the spacetime theory regains SUSY away from Γ fixed loci).

More interesting cases:

$$\{(AdS_2 \times S^2) \times \text{Internal}\}/\Gamma$$

In these cases, we could talk about the net 4d cosmological constant. The <u>internal</u> space does NOT decompactify in the $AdS_2 \times S^2 \to \mathbb{R}^4$ limit, so by breaking SUSY on the internal space one would find examples where there is no spacetime SUSY.

Comments :

1) The $AdS_2 \times S^2$ geometries of interest arise as Reissner-Nordstrom black holes. In non-susy situations it can be very difficult to find stable such solns (e.g. from wrapping branes) because

$$\mathcal{L} = \int d^4x \quad \phi F^2 + \cdots + \partial^\mu \phi \partial_\mu \phi + \cdots$$

→ scalars ϕ can be forced to take VEVs which break $SO(1,2)$ if they couple to field str.

So it is useful to start with theories with few scalars (eg asymmetric orbifolds).

2) The orbifold we've been discussing not only has

$$\Lambda_{1\text{-loop}} = \Lambda_{2\text{-loop}} \overset{?}{=} \cdots = 0$$

but also has

$$\left.\begin{aligned} \Lambda_{1\text{-loop}} &= \int d^2 m_i \; \underline{0} \\ \Lambda_{2\text{-loop}} &= \int d^6 m_i \; \underline{0} \end{aligned}\right\} \text{vanishing integrand}$$

In general, examples that might come out of AdS/CFT correspondences involving non-susy fixed lines <u>would</u> have

$$\Lambda_{1\text{-loop}} = \Lambda_{2\text{-loop}} = \cdots = 0$$

but would not necessarily have vanishing point by point on moduli space of genus 1, 2, ... surfaces. I.e., there need <u>NOT</u> be a bose-fermi degeneracy in the spectrum.