

NEAR HORIZON SUPERSPACE

R. Kallosh, Strings 98

Recent results

R.K. and A. Van Proeyen : hep-th/9804099

R.K. and A. Rajaraman: hep-th/9805041

R.K., A. Rajaraman and J. Rahmfeld: hep-th/9805217

Part of the program

TOWARDS SUPERCONFORMAL DYNAMICS OF BRANES

as developed in

P. Claus, R. K, J. Kumar, A. Van Proeyen, P. Townsend
in hep-th/9711161 , hep-th/9801206 , hep-th/9804177
(Talk of A. Van Proeyen)

Outline

- Conformal symmetry of supergravity in adS spaces in the bulk and at the boundary
- From tree level supergravity to include HIGHER DIMENSIONAL OPERATORS
- EXACT VACUA of type IIB string theory:
 $adS_5 \times S^5$; of M-theory: $adS_4 \times S^7$ and
 $adS_7 \times S^4$



FIXED POINTS IN SUPERSPACE

- i) flat superspace based on SuperPoincaré algebra
- ii) near horizon superspace based on Super-Conformal algebra

- Application: Closed form of Green-Schwarz type string action on adS

see also A. Tseytlin talk₂

AdS/ CFT

Maldacena's conjecture as developed in

Gubser, Klebanov, Polyakov ; Witten ;
Ferrara, Frossel, Zaffaroni

can be specified more and PARTIALLY PROVED

under ONE CONDITION:

Use the BACKGROUND FIELD METHOD
for supergravity in adS

We found

- The background invariant PATH INTEGRAL of supergravity IN THE BULK has conformal symmetry due to the ISOMETRY of the adS space
- From the symmetry in the bulk one can derive a RIGID CONFORMAL SYMMETRY of the generating functional for ALL CORRELATORS AT THE BOUNDARY

A PUZZLE (solved) concerns conformal symmetry of correlators of vector, graviton, gravitino etc.

The conjecture is

$$Z[\phi_b] = e^{-I_s[\phi(\phi_b)]}$$

This can not be literally true for gauge fields as $S_{\text{gaugefixing}}$ has to be specified even for tree approximation. Face the OLD PROBLEM: QED IN FLAT SPACE has no conformal covariant propagator

$F_{\mu\nu}^2$ is inversion invariant, $(\partial^\mu A_\mu)^2$ is
not!

Masterpieces of calculations

Freedman, Mathur, Matusis, Rastelli
Liu, Tseytlin

NICE SURPRISE: in adS space there is a conformally covariant propagator for vectors and there is a partial evidence of conformal symmetry for gravitons!

What about correlators of graviton, gravitino, form fields? n-point correlators? higher dimensional operators?

$$S_{g.f.} = \int \sqrt{g^{adS} + g} \left(D^\mu [g^{adS}] A_\mu \right)^2$$

This is INVARIANT under

$$\underline{\mathcal{L}_{\hat{\xi}_k} g_{\mu\nu}^{adS} = 0}$$

$$A_\mu \rightarrow A_\mu + \mathcal{L}_{\hat{\xi}_k} A_\mu \quad g_{\mu\nu} \rightarrow g_{\mu\nu} + \mathcal{L}_{\hat{\xi}_k} g_{\mu\nu}$$

$\mathcal{L}_{\hat{\xi}_k}$: LIE DERIVATIVE WITH RESPECT TO

THE KILLING VECTOR ON ADS

$$\hat{\xi}_k(x, r) = a^m + \lambda_M^{mn} x_n + \lambda_D x^m$$

transl. Lorentz dilat

$$+ (x^2 \Lambda_K^m - 2x^m x \cdot \Lambda_K) + \frac{R^4}{r^2} \Lambda_K$$

special conf modified
special
conformal
in the BULK

x^m are coordinates on the boundary, r is the coordinate in the bulk

CONFORMAL SYMMETRY of supergravity
effective action on adS space in the BULK

$$\Gamma(\phi_{\text{backgr}}, \tilde{\Phi}) = \Gamma(\phi_{\text{backgr}}, \tilde{\Phi} + \mathcal{L}_{\hat{\xi}_k} \tilde{\Phi})$$

$$\mathcal{L}_{\hat{\xi}_k} \phi_{\text{backgr}} = 0$$

In the tree approximation

$$\Gamma(\phi, \tilde{\Phi}) = S_{\text{Cl}}(\phi + \tilde{\Phi}) + S_{g.f.}(\phi, \tilde{\Phi})$$

$$\frac{\delta \Gamma_{\text{tree}}(\phi, \tilde{\Phi})}{\delta \tilde{\Phi}} + J(x^m, r) = 0$$

In the limit when the SOURCES are located
only at the BOUNDARY of adS

$$J(x^m, r)_{r \rightarrow \infty} = J^{\text{bound}}(x^m)$$

$$\Gamma(\phi, \tilde{\Phi}[J^{\text{bound}}]) = \Gamma(\phi, \tilde{\Phi} + \mathcal{L}_{\hat{\xi}_k} \tilde{\Phi})$$

RIGID CONFORMAL SYMMETRY

$$\hat{\xi}_k(x^m) = a^m + \lambda_M^{mn} x_n + \lambda_D x^m$$

$$+ (x^2 \Lambda_K^m - 2x^m x \cdot \Lambda_K)$$

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$$\hat{\xi}_k(x, r) \underset{r \rightarrow \infty}{=} \xi_k(x)$$

STATUS of AdS/CFT CORRESPONDENCE CONJECTURE

- The path integral of supergravity on the boundary has RIGID CONFORMAL SYMMETRY IFF defined in adS background according to the rules of the background field method. This part of the conjecture is established!

Not a conjecture anymore (apart from possible anomalies all future calculations should support this proof)

Background covariant
propagators and vertices!
in the Bulk

- Supergravity correlators on $adS_5 \times S^5$ correspond to $N \rightarrow \infty$ $\mathcal{N} = 4$ SYM

- ?
- OPEN ISSUE supported so far by plausible arguments and calculations available. Even $SU(2,2|4)$ superconformal symmetry does not require unique correlators. There is a freedom in normalization between various superconformal contributions etc. Howe, West

New ideas and/or
new calculations are required!

HIGHER DIMENSIONAL OPERATORS (HDO)

IIB string theory will add to tree level supergravity HDO, e.g.

Gross, Witten;
Grisaru, Van de
Veen, Zanon

$$(\alpha')^3 e^{-\phi} f(\rho, \bar{\rho}) (R^4 + \dots)$$

Banks, Green
proved exactness
up to $(\alpha')^3$

How HDO will affect AdS/CFT CORRESPONDENCE?

The background field method is consistent only if the background solving classical equations will solve also equations with account of HDO.

The proof that the VACUA of type IIB string theory:
 $adS_5 \times S^5$ + 5-form; and of M-theory: $adS_4 \times S^7$ + 4-form and $adS_7 \times S^4$ + dual 4-form are EXACT, is available, see next

$adS_{p+2} \times S^{d-p-2}$ / SCFT
correspondence survives HDO
corrections!

Bosonic $adS_5 \times S^5$ + self-dual 5-form
EXACT VACUUM OF IIB STRING THEORY

Consider the on shell supergravity superspace

Hawke, West

TORSION, CURVATURE, FORMS

$$T^{\hat{C}}(x, \theta), \quad R^{ab}(x, \theta), \quad F_3(x, \theta), \quad G_5^+(x, \theta)$$

Solution of constraints, field equations and
Bianchi identities depends on 2 superfields

$$\Lambda_\alpha(x, \theta) = \lambda_\alpha(x) + \dots$$

$$G_{abcde}^+(x, \theta) = g_{abcde}^+(x) + \dots$$

For our background one can show that

$$\Lambda_\alpha(x, \theta) = 0$$

$$G_{abcde}^+(x, \theta) = g_{abcde}^+(x)$$

The superfield
is covariantly
constant

The proof is based on 32 UNBROKEN 
SUPERSYMMETRIES
(more general than conformal flatness) ⁹

UNIVERSAL FIXED POINTS IN SUPERGRAVITY SUPERSPACE

$$Z^M = (x^m, \theta^\alpha)$$

$$\frac{\partial}{\partial Z^M} \left(T^{\bar{C}}(x, \theta), \quad R^{ab}(x, \theta), \quad F_{p+2}(x, \theta) \right) = 0$$

All components of torsion, curvature and
form superfields take constant values with
vanishing derivatives in (x, θ)

GENERALIZATION TO SUPERSPACE of
the concept of
SUPERSYMMETRIC ATTRACTORS

Ferrara, R. K., Strominger

recent study in
G. Moore talk

For M-theory analogous situation based on
the study of 11d supergravity in superspace

Cremmer, Ferrara ; Brink, Howe

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$adS_4 \times S^7$
+ 4 form

$adS_7 \times S^4$
+ dual 4 form

STABILITY of the SUPERSPACE FIXED POINTS with respect to QUANTUM CORRECTIONS

- Superfield eqs. of motion for fermions always carry an odd # of spinorial indices
- Superfield eqs. of motion for bosons are derivable from fermionic ones as covariant derivatives
- Generic corrections to eqs. of motion are covariant in superspace \Rightarrow depend on components of $T^{\bar{C}}(x, \theta)$, $R^{ab}(x, \theta)$, $F_{p+2}(x, \theta)$ and their covariant derivatives
- Our backgrounds have no fermionic fields, only $(\frac{\partial}{\partial \theta} + \dots) (T^{\bar{C}}, R^{ab}, F_{p+2})$ may carry an odd # of fermionic indices

ALL SUPERFIELD CORRECTIONS VANISH
FOR OUR FIXED POINT VACUA!

Critical systems have
Universality classes :

Near horizon geometries for branes with
enhancement of supersymmetry

$d=11$ $adS_4 \times S^7$ $adS_7 \times S^4$
 electric M2 magnetic M5

$d=10$ $adS_5 \times S^5$
 self-dual D3

?

$d=6$ $adS_3 \times S^3$
 self-dual string

$d=5$ $adS_2 \times S^3$ $adS_3 \times S^2$
 electric black holes magnetic string

$d=4$ $adS_2 \times S^2$
 self-dual black holes

Branes interpolate between

2 maximally supersymmetric vacua.

1982 Gibbons; R.K., Peet

Gibbons, Townsend

1997 Gibbons, Chemissany, Ferrara, P.K.

Now we may also add:

Between two fixed points in superspace

(1) Minkowski vacuum
 $SO(1, d-1)$

spont.
down to
broken
 $SO(1, p+1) \times$
 $SO(d-p-2)$

$r \rightarrow \infty$
Physical states:
representations
of superPoincaré
group

(2) New exact vacua

$adS_{p+2} \times S^{d-p-2}$
Singletons!

Dirac; Flato, Fronsdal;
Bergshoeff, Duff, Pope
Sezgin; Gunaydin, van Nieuwenhuizen, Warner, Nicolai, ...

$r \downarrow 0$

Physical states on the
worldvolume: boundary of
 $adS \rightarrow$ representations of
superConformal group

Conceptually simple!

SUPERGRAVITY IN GENERIC POINT OF
THE LORENTZ SUPERSPACE

$$D = d + \omega^{ab} J_{ab} \equiv E^{\hat{C}} D_{\hat{C}}$$

Lorentz generators
convection

$$[D_{\hat{A}}, D_{\hat{B}}] = T_{\hat{A}\hat{B}}^{\hat{C}} D_{\hat{C}} + R_{\hat{A}\hat{B}}^{ab} J_{ab}$$

Torsion *Lor. curvature*

+ CONSTRAINTS

+ FORM FIELDS $A \Rightarrow F \equiv dA, dF = 0$

LORENTZ SUPERSPACE: LORENTZ
CURVATURE

$$R^{ab} \equiv d\omega^{ab} + (\omega \wedge \omega)^{ab}$$

Components of torsion, curvature and forms
depend on (x, θ)

SUPERGRAVITY VIELBEIN IN SUPERSPACE

$$E_M^{\hat{A}}(x, \theta) = \begin{vmatrix} e_m^a(x, \theta) & \psi_m^\alpha(x, \theta) \\ e_{\underline{\alpha}}^a(x, \theta) & e_{\underline{\alpha}}^\alpha(x, \theta) \end{vmatrix}$$

graviton gravitino
 $E_M^{\hat{A}}|_{\theta=0} = \begin{vmatrix} e_m^a(x) & \psi_m^\alpha(x) \\ 0 & \delta_{\underline{\alpha}}^\alpha \end{vmatrix}$

at $\theta=0$
 Wess-Zumino gauge

SUPERGRAVITY AT THE FIXED POINTS IN SUPERSPACE

Consider a superalgebra of the supergroup G

$$[B_A, B_B] = f_{AB}^C B_C$$

$$[F_\alpha, B_B] = f_{\alpha B}^\beta F_\beta$$

$$[F_\alpha, F_\beta] = f_{\alpha\beta}^C B_C$$

Remember:
**Universality
of critical
points**

(1)

(2)

It can be SuperPoincaré or SuperConformal
for Minkowski *for near horizon*

$$\mathcal{D} = d + L^A B_A + L^\alpha F_\alpha$$

$$\mathcal{D}^2 = 0$$

The coset superspace construction

$$\frac{G}{H}$$

supergroup G
Lorentz subgroup ₁₆

Homogeneous superspace.

UNIVERSALITY OF THE SUPERGRAVITY SOLUTION FOR THE VIELBEINS AND CONNECTIONS AT THE SUPERSPACE FIXED POINTS

$$L_M{}^{\hat{A}}(x, \theta) = \begin{vmatrix} e_m^a(x) & \omega^{ab}(x) & 0 \\ e_{\underline{\alpha}}^a(x, \theta) & \omega_{\underline{\alpha}}^{ab}(x, \theta) & e_{\underline{\alpha}}^\beta(x, \theta) \end{vmatrix}$$

$$e_{\underline{\alpha}}^\alpha(x, \theta) = \left(\frac{\sinh \mathcal{M}}{\mathcal{M}} \right)_\beta^\alpha e_{\underline{\alpha}}^\beta(x)$$

$$(\mathcal{M}^2)_\beta^\alpha = -\theta^\delta(x) f_{\delta A}^\alpha \theta^\gamma(x) f_{\gamma\beta}^A.$$

structure
constants of G

Maximal dimension space-time Killing spinors

$$\epsilon_k^\alpha(x) = e_{\underline{\alpha}}^\alpha(x) \epsilon_{\text{const}}^{\underline{\alpha}}$$

$$\theta^\alpha(x) \equiv e_{\underline{\alpha}}^\alpha(x) \theta^{\underline{\alpha}}$$

are known for all adS_{p+2} ¹⁷
 $\text{adS}_{p+2} \times S^{d-p-2}$ Lu, Pope, Townsend

Lu, Pope, Rahmfeld

ALTERNATIVE FORM OF THE SOLUTION

$$L^\alpha = \left(\frac{\sinh \mathcal{M}}{\mathcal{M}} \right)_\beta^\alpha (D\theta)^\beta$$

$$L^A = L_{\theta=0}^A + 2\theta^\alpha f_{\alpha\beta}^A \left(\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \right)_\gamma^\beta (D\theta)^\gamma.$$

$$(\mathcal{M}^2)_\beta^\alpha = -\theta^\gamma f_{\gamma A}^\alpha \theta^\delta f_{\delta\beta}^A$$

Wess-Zumino gauge

$$D\theta \equiv d\theta + L^A|_{\theta=0} B_A \theta$$

Killing spinor gauge

fermion-fermion part of the
superspace at $\theta=0$ is **FLAT**

$$(D\theta)^\alpha = e_{\underline{\alpha}}^\alpha(x) d\theta^{\underline{\alpha}}$$

fermion-fermion part
of the superspace at $\theta=0$ is **CURVED**

Application

GS IIB STRING ACTION on $adS_5 * S^5$

In generic supergravity background

Grisaru, Howe,
Mezincescu,
Nilsson, Townsend

On $\frac{SU(2,2|4)}{SO(1,4) \times SO(5)}$ coset Tseytlin, Metsaev

We

- Identified the coset as a fixed point of the generic IIB superspace starting from supergravity side
- Found the closed form action describing both fixed points: FLAT SUPERSPACE AND ADSXS

$$-\frac{1}{2} \int_{\partial M_3} d^2\sigma \sqrt{g} g^{ij} L_i^a L_j^a + i \int_{M_3} L^a \wedge \bar{L} \gamma^a \wedge K L$$

(for Cartan forms)

In eq. defining M the structure constants f are either for superPoincaré (flat superspace) or superConformal (near horizon superspace)

$$(\mathcal{M}^2)_{\beta}^{\alpha} = \theta^r f_{\gamma a}^{\alpha} \theta^s f_{\delta \beta}^a + \theta^r f_{\gamma a \delta}^{\alpha} \theta^s f_{\beta}^{ab}$$

SuperPoincaré

$$[Q, P] = 0$$

$$f_{\gamma a}^{\alpha} = 0$$

$$\{Q, Q\} \sim P$$

$$f_{\gamma \delta}^{ab} = 0$$

$$\mathcal{M}^2 = 0$$

SuperConformal

$$[Q, P] \sim Q$$

$$f_{\gamma a}^{\alpha} \neq 0$$

$$\{Q, Q\} \sim P + J$$

$$f_{\gamma \delta}^{ab} \neq 0$$

$$\begin{matrix} \text{non-vanishing} \\ \text{constant torsion} \end{matrix} \quad \mathcal{M}^2 \neq 0$$

Hopefully

'When you do the right things AdSxS isn't
much worse than flat space'

J. Schwarz

At the FIXED POINTS the generic Lorentz superspace of supergravities acquires an ALTERNATIVE description as a coset

$$\frac{G}{H}$$

- 1) Flat superspace: G is superPoincaré and $H = SO(1, d - 1)$
 - 2) Near Horizon superspace: G is superConformal and $H = SO(1, p + 1) * SO(d - p - 2)$
- Examples

M-THEORY

$$\frac{OSp(8|4)}{SO(1,3) \times SO(7)}$$

$$\frac{OSp(6,2|4)}{SO(1,6) \times SO(4)}$$

TYPE IIB STRING and D3 BRANE

$$\frac{SU(2,2|4)}{SO(1,4) \times SO(5)}$$

BLACK HOLES in $d = 4$

$$\frac{SU(1,1|2)}{SO(1,1) \times SO(2)}$$