

# FROM BLACK 3-BRANES TO LARGE N GAUGE THEORIES

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The talk is based mainly on

1. Gubser, IK, Peet, "Entropy and temperature of black 3-branes", Phys. Rev. D54 (1996) 3915, hep-th/9602135
2. IK, "World volume approach to absorption by noncritical branes", Nucl. Phys. B496 (1997) 231, hep-th/9702076
3. Gubser, IK, Tseytlin, "String theory and classical absorption by 3-branes", Nucl. Phys. B499 (1997) 217, hep-th/9703040
4. Gubser, IK, "Absorption by branes and Schwinger terms in the world volume theory", Phys. Lett. B493 (1997) 41, hep-th/9702005
5. Gubser, IK, Polyakov, "Gauge theory correlators from non-critical string theory", hep-th/9802109.
6. Gubser, IK, Tseytlin, "Coupling constant dependence in the thermodynamics of  $N=4$  supersymmetric Yang-Mills theory", hep-th/9805156.

Type II SUGRA admits R-R charged p-brane solutions (Horowitz and Strominger; Duff and Lu):

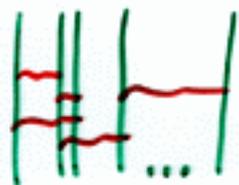
$$ds^2 = H^{-\frac{1}{2}} (-dt^2 + dx_1^2 + \dots + dx_p^2) + H^{\frac{1}{2}} (dr^2 + r^2 d\Omega_{8-p}^2)$$

$$e^{\Phi} = H^{\frac{3-p}{4}}; \quad H(r) = 1 + \frac{R^{7-p}}{r^{7-p}},$$

where the metric is given in the string frame.

These solutions are extremal: they preserve 16 of the original 32 supersymmetries.

Due to the magic of D-branes, we know a different (dual) description of the p-brane in terms of  $N$  parallel  $D_p$ -branes.



Here we find a  $U(N)$  gauge theory with 16 supercharges (Witten)

The 9-p scalar expectation values,

$\langle X^i \rangle$  label the relative transverse positions of the  $D_p$ -branes. To describe the single center object we take  $\langle X^i \rangle = 0$ : all  $N$   $D_p$ -branes are coincident.

For an appropriate choice of the parameters  $(N, g_{str}, \alpha')$  the  $N$  coincident D-branes carry the same charge under the R-R  $(p+1)$ -form gauge potential (Polchinski) and the same susy. Hence, they should be regarded as different descriptions of the same object in type II string theory.

All SUGRA p-branes are singular, due to divergence of  $\Phi$  at  $r=0$ , except for  $p=3$ . Thus, we focus on the black 3-brane as a dual description of the  $D=4$   $SU(N)$  gauge theory with 16 supercharges.

We equate the ADM tension to  $N$  times that of a D3-brane to obtain (Gubser, IK, Peet)

$$\frac{2\pi^3 R^4}{k^2} = \frac{\sqrt{\pi}}{k} N \Rightarrow R^4 = \frac{kN}{2\pi^{5/2}} \sim g_{str} N (\alpha')^2.$$

There are 2 sources of  $\alpha'$  corrections on studying string physics on the background of a black 3-brane.

Corrections due to the finite radius of the throat are  $\sim \frac{\alpha'}{R^2} \sim \frac{1}{\sqrt{g_{str} N}}$ ;

For processes with typical energy scale  $\omega$  there are also corrections in powers of  $\omega^2 \alpha'$ .

To suppress both types of string corrections to supergravity, we need to take the double-scaling limit (IK):

$$N g_{str} \rightarrow \infty ; \quad \omega^2 \alpha' \rightarrow 0.$$

In this limit SUGRA gives exact information about the  $D=4$   $SU(N)$  SYM theory!

String loop corrections are suppressed if  $g_{str} \rightarrow 0$ .

Thus, we also need to take  $N \rightarrow \infty$ .

Since  $2\pi g_{str} = g_{YM}^2$ , the black 3-brane gives us **PREDICTIONS** concerning the strong 't Hooft coupling ( $g_{YM}^2 N$ ) behavior of the large  $N$  SYM theory.

If  $g_{YM}^2 N$  is kept fixed but very large, then the stringy  $\alpha'$  corrections may be used to develop an expansion in  $(g_{YM}^2 N)^{-\frac{1}{2}}$ .

The string loop corrections are suppressed by positive powers of  $\frac{k^2}{R^p} \sim \frac{1}{N^2}$ . The large  $N$  limit is mapped onto classical string physics.

One application of these principles: by studying absorption of low-energy particles incident on the black 3-brane, we can deduce exact 2-point correlators of the strongly coupled large- $N$  SYM theory.

Consider, for instance, absorption of a dilaton ( $\Phi$ ) whose propagation is governed by  $\square \Phi = 0$ .

For s-waves (no dependence on  $S^5$ ), setting  $\rho = \omega r$ ,

$$\left[ \rho^{-5} \frac{d}{d\rho} \rho^5 \frac{d}{d\rho} + 1 + \frac{(\omega R)^4}{\rho^4} \right] \Phi(\rho) = 0.$$

For  $\rho \ll 1$  (in the throat region) the solution is

$$\Phi_{\text{throat}} = i (\omega R)^4 \rho^{-2} H_2 \left( \frac{(\omega R)^2}{\rho} \right).$$

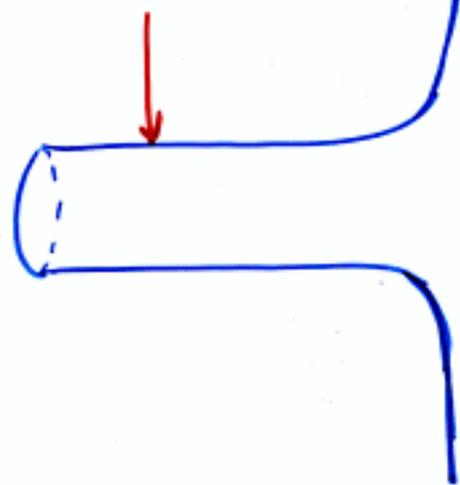
For  $\rho \gg (\omega R)^2$  (in the asymptotic region)

$$\Phi_{\text{asympt}} = \frac{32}{\pi} \rho^{-2} J_2(\rho).$$

The solutions are easily matched for  $(\omega R) \ll 1$ .

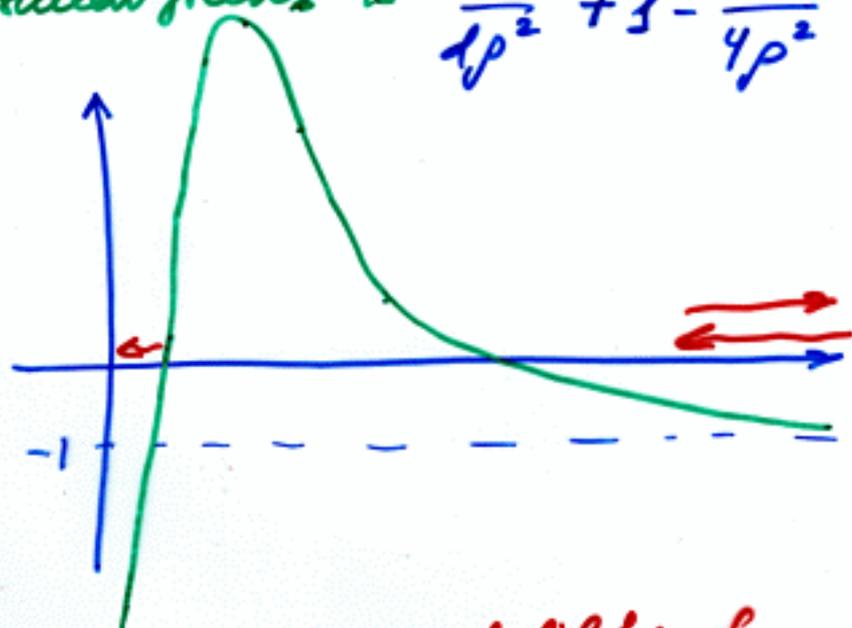
The throat of radius  
 $R$  ( $AdS_5 \times S^5$ )

Asymptotic flat space



Schematic representation  
of the 3-brane geometry.

The effective radial equation for an s-wave  
scalar field is  $\frac{d^2}{d\rho^2} + 1 - \frac{15}{4\rho^2} + \frac{(wR)^4}{\rho^4} = 0$ .



The tunneling probability from large  $\rho$  (asymptotic region) to small  $\rho$  (the throat)  $\sim (wR)^8$ .  
For small  $(wR)$  the two are almost decoupled.

The low-energy limit of the absorption probability is  $P = \frac{\pi^2}{16^2} (\omega R)^8$ . More sophisticated methods, which give an exact function of  $\omega R$  were recently used by Gubser and Hashimoto.

From  $P$  we find the low-energy absorption cross-section,  $\sigma = \frac{\pi^4}{8} \omega^3 R^8$ , which can be used for comparison with the SYM theory.

Taking the low-energy limit of the Born-Infeld action, which describes  $N$  D3-branes in external SUGRA fields, we find the bosonic part of the action,

$$S = T_{(3)} \int d^4 \sigma \text{Tr} \left[ -\frac{1}{2} \partial_\alpha X^i \partial^\alpha X^i - \frac{1}{4} e^{-\phi} F_{\alpha\beta}^2 - C F_{\alpha\beta} \hat{F}^{\alpha\beta} + \dots \right]$$

where  $C$  is the R-R scalar.

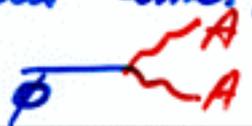
The bulk action is

$$S_{\text{bulk}} = \frac{1}{2\kappa^2} \int d^{10} x \sqrt{-g} \left( R - \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} e^{2\phi} (\partial_\mu C)^2 + \dots \right)$$

This establishes a correspondence between the fields of SUGRA and gauge invariant operators of SYM theory:

$$\phi \leftrightarrow \text{Tr} F^2; \quad C \leftrightarrow \text{Tr} F \hat{F}; \dots$$

To lowest order in  $g_{\text{eff}}$ , the dilaton absorption is

due to   $\phi$ . The matrix element comes from the diagram:  $M = \phi$  

$$\sigma = \frac{1}{2} N^2 \frac{1}{2\omega} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \delta(E_1 + E_2 - \omega) \delta^3(\vec{p}_1 + \vec{p}_2) \sum |\bar{M}|^2$$

symmetry factor

$U(N)$  factor

$$\sum |\bar{M}|^2 = |\bar{M}|^2_{\text{gauge bosons}} = \kappa^2 \omega^4$$

$$\text{Thus, } \sigma = \frac{\kappa^2 N^2 \omega^3}{32\pi} = \frac{\pi^4}{8} \omega^3 R^8$$

in exact agreement with the semiclassical SUGRA!

For the dilaton, pairs of scalars and fermions do not contribute to absorption.

Gravitons polarized along the 3-brane,  $h_{\alpha\beta}$ , are also minimally coupled scalars from the  $D=7$  point of view.

Their world volume interaction is

$$\delta S_{(3)} = T_{(3)} \int d^4 \sigma \frac{1}{2} h_{\alpha\beta} T^{\alpha\beta}$$

$$T_{\alpha\beta} = \text{Tr} \left[ F_{\alpha\gamma} F_{\beta}{}^{\gamma} - \frac{1}{4} \eta_{\alpha\beta} F^2 - \frac{i}{2} \bar{\psi}^I (\gamma_{\alpha} \partial_{\beta}) \psi_I + \partial_{\alpha} X^i \partial_{\beta} X^i - \frac{1}{2} \eta_{\alpha\beta} (\partial_{\gamma} X^i)^2 \right], \text{ the world volume stress-energy tensor.}$$

For final particle momentum along  $\hat{n}$ ,

$$|\bar{M}|^2_{\text{scalars}} = \kappa^2 \omega^4 \times 3 n_x^2 n_y^2; \quad |\bar{M}|^2_{\text{fermions}} = \kappa^2 \omega^4 (n_x^2 + n_y^2 - 4 n_x^2 n_y^2)$$

$$|\bar{M}|^2_{\text{gauge bosons}} = \kappa^2 \omega^4 (1 - n_x^2 - n_y^2 + n_x^2 n_y^2)$$

are the sums over final polarizations for incident  $h_{xy}$ .

$\sum |\bar{M}|^2 = \kappa^2 \omega^4 \Rightarrow$  the graviton absorption cross-section is the same as for dilaton, in agreement with SUGRA!

So far, our calculations are equivalent to finding the discontinuity (imaginary part) on the 1-loop diagrams

$$\text{tr} F^2 \text{ (loop) } \text{tr} F^2 \quad ; \quad T_{\mu\nu} \text{ (loop) } T_{\mu\nu} ; \text{ etc.}$$

Comparison with gravity allows to deduce the  $g_{\text{YM}}^2 N \rightarrow \infty$  limit of the exact 2-point functions:

$$\langle \text{tr} F^2(x) \text{tr} F^2(0) \rangle = k \frac{N^2}{|x|^8} ;$$

$$\langle T_{\alpha\beta}(x) T_{\gamma\delta}(0) \rangle = \frac{N^2/4}{48\pi^4} \chi_{\alpha\beta\gamma\delta} \left( \frac{1}{|x|^4} \right) \tilde{k} ;$$

We find that  $\tilde{k} = 1$  both for  $g_{\text{YM}}^2 N = 0$  and for  $g_{\text{YM}}^2 N \rightarrow \infty$ ;  $k(g_{\text{YM}}^2 N = 0) = k(g_{\text{YM}}^2 N = \infty)$ .

This is consistent with the non-renormalization of the central charge: it is related to the Adler-Bardoun theorem for the R-current anomaly by  $N=4$  SUSY.

We have calculated the exact 2-point functions by studying the tunneling from the asymptotic region of the black 3-brane into the throat region.

The throat (the  $r \rightarrow 0$  asymptotic of the 3-brane metric) is  $AdS_5 \times S^5$  (Gibbons, Townsend). In the low-energy limit the throat is almost decoupled from the asymptotic region (Maldacena).

Excitations of the throat are in 1-to-1 correspondence with excitations of the SYM by vertex operators.

A related procedure for obtaining their correlation functions is to replace the effects of the asymptotic region by boundary conditions on  $AdS_5$  (Gubser, IK, Polyakov; Witten).

The SYM theory is thus equivalent to type IIB string on  $AdS_5 \times S^5$  (Maldacena). The 5-model (confining string) formulation and the presence of the 5-th "Liouville" dimension were conjectured earlier by Polyakov.

The presence of an R-R background 5-form field strength poses a new challenge.

Nevertheless, the  $\alpha'$  expansion around the SUGRA limit can be studied via the type IIB string effective action.

These principles may be applied to the

## THERMODYNAMICS OF D=4 SYM THEORY WITH N=4 SUSY

The near-extremal black 3-brane of sufficiently low Hawking temperature  $T$  describes the SYM theory heated up to this temperature (Gubser, IK, Peet)

The IIB SUGRA solution has a constant dilaton and

$$ds^2 = H^{-\frac{1}{2}} \left[ -f dt^2 + \sum_{i=1}^3 (dx^i)^2 \right] + H^{\frac{1}{2}} \left[ f^{-1} dr^2 + r^2 d\Omega_5^2 \right];$$

$$H(r) = 1 + \frac{R^4}{r^4}; \quad f(r) = 1 - \frac{r_0^4}{r^4}.$$

The horizon is located at  $r = r_0$ .

To reach the conformal limit, we require  $T \ll \frac{1}{R}$ .

This implies  $r_0 \ll R$ . Indeed, then

$$T = \frac{r_0}{\pi R^2} \ll \frac{1}{R}.$$

The gravitational entropy is the Bekenstein-Hawking entropy,  $S_{BH} = \frac{A_h}{4G} = \frac{2\pi A_h}{K^2}$ .

The 8-dimensional "area" of the horizon is

$$A_h = \pi^3 R^5 \left(\frac{r_0}{R}\right)^3 V = \pi^6 R^8 T^3 V;$$

$$\text{Since } R^8 = \frac{N^2}{4\pi^5} K^2, \text{ we have } S_{BH} = \frac{\pi^2}{2} N^2 T^3 V.$$

This should be identified with the thermal entropy of the SYM theory in the limit of large  $g_{YM}^2 N$ .

The scaling of the entropy with  $T$  is consistent with the conformal invariance of the SYM theory.

The scaling with  $N$  signifies  $N^2$  unconfined degrees of freedom, as expected!

We may compare with the entropy for  $g_{\text{YM}}^2 N \rightarrow 0$  where there are  $8N^2$  free massless bosons and fermions,

$$S(g_{\text{YM}}^2 N=0) = \frac{2\pi^2}{3} N^2 T^3 V.$$

Thus,  $S_{\text{BH}} = \frac{3}{4} S(g_{\text{YM}}^2 N=0)$ .

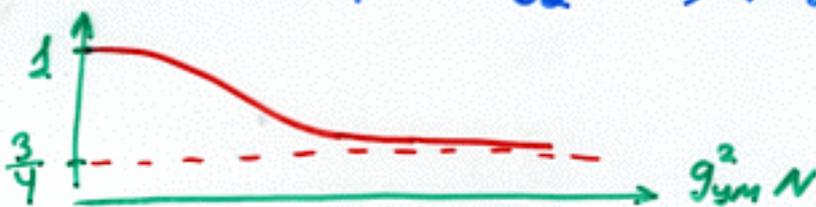
Indeed, on general grounds we expect (Gubser, IK, Tseytlin)

$$S(g_{\text{YM}}^2 N) = f(g_{\text{YM}}^2 N) \frac{2\pi^2}{3} N^2 T^3 V.$$

The function  $f$  extrapolates from  $f(0) = 1$  to  $f(\infty) = \frac{3}{4}$ .

Consistency of this prediction may be tested by calculating the strong coupling expansion of  $f(g_{\text{YM}}^2 N)$  using the stringy  $\alpha'$  corrections to SUGRA. One finds (GKT)

$$f(g_{\text{YM}}^2 N) = \frac{3}{4} + \frac{45}{32} \zeta(3) (2g_{\text{YM}}^2 N)^{-\frac{3}{2}} + \dots$$



Since the horizon is at  $r_0 \ll R$ , we may use the throat approximation to the 3-brane metric, valid for  $r \ll R$ , (Maldacena; Horowitz and Ross)

$$ds^2 = \frac{r^2}{R^2} \left[ - \left( 1 - \frac{r_0^4}{r^4} \right) dt^2 + d\vec{x}^2 \right] + \frac{R^2}{r^2} \left( 1 - \frac{r_0^4}{r^4} \right)^{-1} dr^2 + R^2 d\Omega_5^2$$

This is  $S^5 \times$  (a limit of Schwarzschild in  $AdS_5$ ).

The effective  $AdS_5$  action is

$$I_5 = - \frac{1}{16\pi G_5} \int d^5x \sqrt{g_5} \left( R_5 + \frac{12}{R^2} - \frac{1}{2} (\partial\phi)_+^2 + \gamma e^{-\frac{3}{2}\phi} W \right)$$

where  $\gamma = \frac{1}{8} \int (3) (\alpha')^3$ , and  $W$  is built out of Weyl tensor only (Banks and Green)

$$W = C^{hmnk} C_{pmnq} C_h^{rsp} C_{rsk}^q + \frac{1}{2} C^{hkmn} C_{pqmn} C_h^{rsp} C_{rsk}^q$$

Hawking and Page identify the gravitational free energy as  $F = T I_5$ .

The leading (2-derivative) term in the action requires a subtraction (written) after which we find

$$F_0 = - \frac{\pi^2}{8} N^2 V T^4, \text{ consistent with our calculation of SBH: } S_{BH} = - \frac{\partial F_0}{\partial T}$$

To find  $\delta F$  we evaluate  $W$  on the unperturbed solution to find  $W = \frac{180}{R^8} \frac{r_0^{16}}{r^{16}}$ .

$$\delta F = -\frac{1}{16\pi G_5} V \frac{(2')^3}{R^{12}} \frac{45}{2} \int(3) \int_{r_0}^{\infty} dr r^3 \left(\frac{r_0}{r}\right)^{16} =$$

$$= -\frac{\pi^2}{8} N^2 V T^4 \frac{15}{8} \int(3) (2g_{\text{YM}}^2 N)^{-3/2}$$

$$\text{Thus } F = F_0 + \delta F = -\frac{\pi^2}{8} N^2 V T^4 \left[1 + \frac{15}{8} \int(3) (2g^2 N)^{-3/2}\right]$$

By writing  $F = -f(g_{\text{YM}}^2 N) \frac{\pi^2}{6} N^2 V T^4$ , we thus identify the  $f(g_{\text{YM}}^2 N)$ .

This result is verified by carrying out the subtraction with the perturbed solution, which can be found on GKT, hep-th/9805156.

The same  $C^4$  term is generated at 1-loop and gives a  $1/N^2$  correction to the free energy.

The leading term at strong coupling appears to be

$$\delta F' \sim \sqrt{g_{\text{YM}}^2 N} V T^4$$

Similar techniques can be used to study the thermodynamics of other large  $N$  CFT's. The (Weyl)<sup>4</sup> term is generated at 1-loop in M theory and contributes at subleading orders to the free energy on M-branes.

For the CFT on  $N$  coincident M5-branes (the (0,2) theory) we find

$$F = -V_5 T^6 (a_0 N^3 + a_1 N + \dots)$$

$$a_0 = 2^6 3^{-7} \pi^3; \quad a_1 = 730 \left(\frac{2\pi}{3}\right)^8 \left(\frac{\pi}{2}\right)^{1/3}$$

For the CFT on  $N$  coincident M2-branes,

$$F = -V_2 T^3 (b_0 N^{3/2} + b_1 N^{1/2} + \dots)$$

$$b_0 = 2^{3/2} 3^{-4} \pi^2; \quad b_1 = 64 \left(\frac{2\pi}{3}\right)^5 2^{1/2} \pi^{7/3}$$

For the CFT on  $N_1$  D1-branes intersecting  $N_5$  D5-branes there is no correction due to the (Weyl)<sup>4</sup> term because the near-extremal geometry is locally  $AdS_3 \times S^3$  which is conformally flat. It is likely that all  $d'$  corrections vanish which would explain why the calculation at small  $g_{\text{str}} N$  agrees exactly with the BH entropy found for large  $g_{\text{str}} N$  (Callan and Maldacena, Horowitz and Strominger).

## CONCLUSIONS

1. Black 3-branes provide a dual description of  $N$  coincident Dirichlet 3-branes.
2. For large  $N$ , large  $g_{\text{st}} N$  and low energies this duality gives a host of PREDICTIONS about large  $N$  SYM theory at strong 't Hooft coupling.
3. The thermal free energy is proportional to a function of the 't Hooft coupling which interpolates from 1 at small  $g_{\text{YM}}^2 N$  to  $\frac{3}{4}$  at large  $g_{\text{YM}}^2 N$ . Corrections to  $\frac{3}{4}$  at strong coupling can be found from the stringy higher derivative corrections in the type IIB effective action.