The particle spectrum of 4d $N=4$ gauge theory and (p,q) webs

Barak Kol

Strings 98

Mostly based on

Aharony, Hanany, BK	9710 116
BK, Rahmfeld	9801 067
Bergman	9712 211
Bergman, BK	9804 160

In this talk, rather than review the subject of (p,q) webs, I would like to discuss a specific application.

The subject is the particle spectrum of 4d $\mathcal{N}=4$ gauge theory with a broken gauge group $SU(N_c) \to U(1)^{N_c-1}$.

By 'spectrum' I mean a description of both the masses and the multiplet structure (spins) of stable particles.

For the BPS spectrum the suggested picture will be quite complete :

- BPS states exist in regions of moduli space bounded by marginal stability walls.
- For a state with given charges, I will provide a bound on the spin, and a method to determine the multiplet structure in principle.

The stable non-BPS spectrum is much less understood, but there are reasons for optimism :

- The non BPS states are continuously connected to 'BPS' states.
- Perturbative calculations can be done both at weak and at strong coupling.

The talk is organized as follows:

- Set-up and review of $N_c = 2$.
- $N_c = 3$.
- $N_c \geq 4$ – somewhat speculative.

Disclaimer: the discussed phenomena will be demonstrated in the framework of a certain web model and in some cases it remains to be checked whether they survive the field theory limit. As I believe that the marginal stability, spin bound, and stable non-BPS states were not known so far, I think these considerations are useful as motivation for further research.

Set-up - the brane realization

Consider N_c D3 branes in IIB string theory:

The 3+1 worldvolume directions are common, and are not shown.

The coupling constant is inherited from IIB:

$$\theta/2\pi + i\frac{4\pi}{g_{YM}^2} = \tau_{YM} = \tau_{IIB} = \chi/2\pi + i/\lambda$$

The moduli space is given by $\vec{\phi}_a \in \mathbb{R}^6$ $1 \le a \le N_c$, and are related to the locations of the branes through

$$\vec{\phi}_a = \vec{x}_a T_s / g_{YM}$$

We factorize the center of mass $U(1)$ by $\sum_{a=1}^{N_c} \vec{\phi}_a = 0$

We have 16 preserved susy.

To reach the field theory limit we take

$T_s \to \infty$, $\vec{\phi}$ fixed, so $\vec{x}_a \to 0$.

The charges are prescribed by (p_a, q_a) or (n_{e_a}, n_{m_a}) $1 \le a \le N_c$, subject to $\sum_{a=1}^{N_c} \begin{bmatrix} p \\ q \end{bmatrix}_a = 0$.

$G = SU(2)$

The spectrum is known to consist of the BPS (p,q) dyons. $p, q \in \mathbb{Z}$, $(p,q) = 1$.
Their mass is $m = |p + \tau q| \, T_s \, |\vec{\Delta x}|$.
They all lie in the vector multiplet of $N=4$ (2^4 states) as they are all related to the W particles through $SL(2,\mathbb{Z})$, and preserve $8\ (1/2)$ susy.
The brane realization is

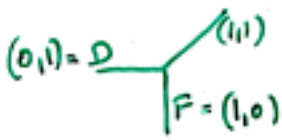
Strong evidence for this picture was supplied by Sen 9402 032 who demonstrated the existence of $(2n+1, 2)$ dyons by a semiclassical method.

Before proceeding to $SU(3)$, recall the definition of a (p,q) web:

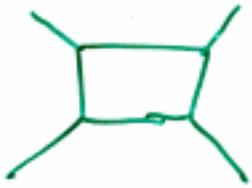
Aharony Hanany 9704170
See also:
Aharony Sonnenschein Yankielowicz 9603009
Schwarz 9508143, 9607201
Aharony Hanany BK.

Consider a collection of lines in the (x,y) plane each carrying a (p,q) label satisfying

1. slope $\quad \Delta x + i\Delta y \parallel p + \tau q$
2. junction $\quad \sum \begin{bmatrix} p \\ q \end{bmatrix} = 0$



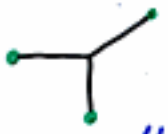
examples:



note:

- The web preserves 8 susy (Y_4)
- The conditions (+ the tension formula) imply mechanical equilibrium of junctions.
- A (p,q) web can be realized in IIB both by 5-branes and by strings.

Now we can consider a web bound on D3 branes. For example



It preserves $1/4 \times 1/2 = 4$ susy, and thus is expected to describe a 1/4 BPS state in the 4d theory.

The mass of the web models the BPS bound

$$m_{web} = m_{BPS}$$

as long as the web exists.

Note that these two quantities are constructed differently: $m_{BPS} = \max\{z_1, z_2\}$

where z_1, z_2 are the 'eigenvalues' of the central charge matrix Z, $Z^I = \sum_{a=1}^{N_s} (p + \tau q)_a \phi_a^I$

whereas

$$m_{web} = \sum (\text{length})_i \, (\text{tension})_i$$

The mass equality was shown first for the basic junction Bergman, and then generalized to an arbitrary web Bergman, BK.

For $G = SU(3)$ the D3 branes always lie in a plane, and therefore a BPS state is always possible (any charges, any moduli) except for:

<u>Marginal stability</u>

example: Bergman

critical/marginal no possible web

- So marginal stability occures whenever a D3 brane meets a junction.

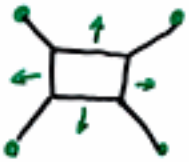
- For given charges, there is a wall of marginal stability in moduli space (codim 1 $\Rightarrow$ wall).

Multiplet structure (spin)

In order to study the web quantum mechanically one has to find the bosonic and fermionic zero modes (BZM, FZM).

The multiplet structure is given by the ground state of the quantum mechanics.

BZM:

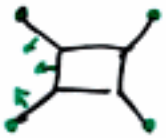


1/face

+ space translations

FZM 8/face from 8 locally preserved susy

4/external leg from "non BZM"



$n_{FZM} = 8F + 4E_{ext}$ total.

The BZM don't carry worldvolume spin, but the FZM do. We get the following bound on the spin

$$j \leq n_{FZM}/8 = F + \tfrac{1}{2} E_{ext}$$

In order to get the precise multiplet structure one would need to solve for the ground states of the quantum mechanics.

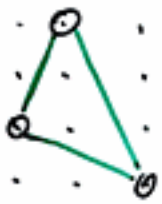
Example

look for a monopole with charges

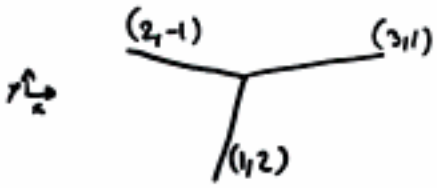
$$(2,-1) \quad (1,2) \quad (-3,-1).$$

These 2d integer vectors define a

grid diagram

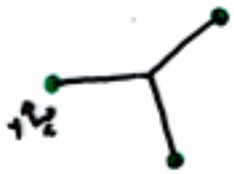


The web is the dual graph:



we can blow up hidden faces:

Finally, the web should be rotated in the plane, so that it ends on the D3 branes:



The spin $\quad j \leq F + \frac{1}{2} E_{cat} = 5/2.$

What about the existence of these states in the field theory limit?

- Appropriate classical monopole solutions were found Hashimoto Hata Sasakura Kawano Okuyama Lee Yi
- Maybe a relevant Witten index $n_B(\omega=0) - n_F(\omega=0) \neq 0$ protects the states in the field theory limit.
- Is it true that the field theory does not have states outside the marginal stability wall?

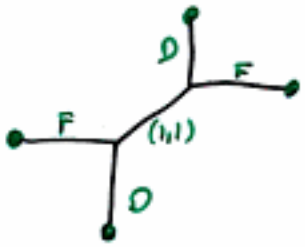
SU(3) summary

- Given charges $\exists$ BPS state within the marginal stability wall.
- In some cases the multiplet structure is known, and otherwise we have a bound on the spin.

$N_c \geq 4$

Now the configuration is generally non-planar.

For example



and move a D3 off the plane. The state cannot be BPS anymore. We may look for the lightest state carrying these charges. Such a state would be stable non-BPS.

First note that the planar, 'BPS' configuration changes to non BPS as moduli are changed. So far we saw a BPS state decaying to other BPS states. We know that short/medium multiplets cannot become long.

Puzzle?

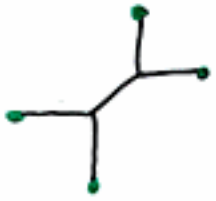
The resolution is that the planar states must lie in long multiplets as well, as can be seen from $n_{FZM} \geq 4E_{ext} \geq 16$.

One should check whether these "classically BPS" states survive the field theory limit. Again, they might be protected by a Witten index.

As the non-BPS states are continuously connected to long BPS states, it is reasonable to expect them to exist in some neighborhood of the latter.

It is not known how to compute the mass of the non BPS monopoles.

One may start by considering a non-planar web, without D3 branes and before taking the field theory limit. Recall Sen



When the distances between junctions are large (in string units) one expects a "classical" configuration of straight strings to minimize the energy. Of course $m > m_{BPS}$.

For smaller distances one may expect the quantum effective action to introduce forces between non parallel strings. Such forces would bend the strings.

Summary of $SU(N_c)$ $N_c \geq 4$

- Long BPS exist (in the web picture) on the submanifold in moduli space of planar config. The masses are known, and the spins are computable in principle.

- Stable non-BPS states are expected (generically) Their masses are unknown.
  Their spins are expected to be related to the spins of the relevant long BPS states.

Related subjects

- Translation to math: it should be possible to translate the discussion to a study of supersymmetric cycles in $K3 \times T^2$, since the same theory can be described by $II/K3 \times T^2$ in the field theory limit.

- Non BPS states in $N=2$ 4d? Bergman Fayyazuddin Mikhailov Nelson Sethi
  The junction realization of these states suggests the existence of stable non-BPS states. It is possible to test that in the $n_m = 2$ sector, following Sen's method.

Conclusion

Can we determine the spectrum of 4d $N=4$?