The **D-Star and its Decays**

- a phenomenological parton model of
  - the bound states of heavy D-particles

- Banks, Fischler, Veneziano & Susskind (BFKS) hep-th/9708051, 9710005
- G. Horowitz, EJM 9710217
- M. Li, GJM 9801070
- M. Li, EJM, V. Sahakian ?
- H. Liu, A. Tseytlin 9712063

1. **YM Phase transitions & their SUGRA counterparts**
2. **Mean field theory & Black hole thermodynamics**
3. **Matrix string limit & the correspondence principle**
• Matrix theory (BFSS) asserts that low-energy dynamics of many (say N) Dp-branes on $\mathbb{T}^p$ reproduces 11D SUGRA on $\mathbb{T}^p$ in the infinite momentum frame (IMF). The appropriate kinematic regime is (BFSS, Sen, Seiberg, Naldaena,...)

\[
\begin{align*}
M\text{-theory} & \\
G_s \sim (\frac{R}{l_{pl}})^{3/2} & \rightarrow 0 \\
R_1, \ldots, R_p & \rightarrow 0 \\
E & \rightarrow 0 \\
N & \rightarrow \infty
\end{align*}
\]

\[
\begin{align*}
\text{SYM} & \\
\tilde{R}_i = \frac{l_{pl}}{R_i} & \text{ fixed} \\
G_{\mu\nu} = \frac{R_i^2 \tilde{R}_1 \ldots \tilde{R}_p}{l_{pl}^6} & \text{ fixed}
\end{align*}
\]

• The 11th dimension is the hidden circle of $M \rightarrow$ IIA reduction, size $R = G_s^{2/3} l_{pl}$. The system carries longitudinal momentum $P = N/R$ in this direction.

• The generic object in the theory is a black hole. Want to describe it as a metastable bound state of the Dp-branes: a D-star. The above limit freezes stringy excitations, leaving YM on the branes.

• Expect that generic (thermal) state of SYM describes a BH in $D=11-p$ dimensions in the IMF...
Problem - typical BH has horizon size

\[ r_0 \sim \left( G_0 M \right)^{\frac{1}{D-3}} \gg R \quad (G_0 = \frac{R_1 \cdots R_p}{4!^D}, \quad D = 11-p) \]

and the entropy

\[ S_{\text{BH}} \sim G_0^{-1} r_0^{D-2} \text{ can be } \gg \quad S_{\text{YM}} (g, N, E, R_i) \]

thus typically the BH

a) does not fit in the longitudinal box of size R
b) has more available states than YM at a given energy

BFKS proposed a solution to these two problems:

a) for sufficient boost, "Lorentz contraction" will "squeeze" the BH into size R or less; one needs

\[ R > \left( \frac{M}{F} \right) r_0 = \frac{S_{\text{BH}}}{F} = R \cdot \frac{S_{\text{BH}}}{N} \]

\[ e^{\gamma - \alpha} \quad \alpha = \text{rapidity} \]

is \( N > S_{\text{BH}} \) "fits" the BH into the box

b) thus, automatically have enough d.o.f.'s to describe the BH
YM picture:

\[ N \ll S_{\text{BH}} \quad (P \ll S/R) \]

reason for transition: thermal wavelength grows larger than effective box size \(\Rightarrow\) system freezes into moduli space dynamics

GR picture:

\[ N \ll S_{\text{BH}} \quad (P \ll S/R) \]

reason for transition: for given mass black object, string has more entropy in fixed box size \(R\) for hole \(\Rightarrow\) system transitions to black hole dynamics
Aside on AdS/CFT correspondence:

A similar transition occurs for $AdS_5$ Schwarzschild

\[ r_0 \gg R_{AdS} = R_{S^5} \]

\[ T_{YM} \sim T_H \gg R_{AdS} \]

$YM$ interpretation: there exist long-lived, metastable states of strongly coupled 3+1d $N=4$ SYM at large $N$, with

\[ E \sim N^2 \frac{r_0^2}{R_{AdS}} \]

\[ S \sim N^2 \left( \frac{r_0}{R_{AdS}} \right)^8 \]

lifetime $\sim N^2 \frac{r_0^4}{R_{AdS}^8}$

\[ \ell_5 < r_0 < R_{AdS} \]

(get macroscopic strings for smaller energy)

these states spontaneously break the $R$-symmetry (localize on $S^5$) and are only present for strong coupling $(g_s N)^{\frac{4}{3}} = \frac{R_{AdS}}{\ell_5} \gg 1$
QM of SYM zero modes

Fix a superselection sector N. The ground state (at $E_{\text{IMF}} = 0$) is the graviton of momentum $P = \frac{N}{R} = \text{threshold bound state of N D-particles}$. One can roughly think of the e-values of the SYM matrices as position coords for the D-particles.

This state has a highly coherent wavefunction. Expect that if we perturb it by adding energy, some coherence is lost.

- **Hypothesis #1**: As system disorders w/increasing energy, it maintains coherence over clusters of D-particles of size $k$ ($k = N$ for threshold bound state $k = 1$ at hole/string transition $N = S_{\text{BH}}$)

- **Hypothesis #2**: Mean field theory of clusters approximates the dynamics of BH's with $S_{\text{BH}} \leq \text{PR} = N$

- **Hypothesis #3**: Can use known (and expected) effective interactions of matrix theory at their limit of validity.
• Let \( \frac{N}{k} = \# \text{clusters} \equiv S \) (we'll see that \( S = S_{BH} \))

• Suppose cluster c.o.m. has wavefn. of spread \( \ell \sim r_0 \)
  
  \[ E_{\text{kin}} \sim (\# \text{clusters}) \cdot \frac{\ell^2}{P} \sim S \cdot \frac{r_0^{-2}}{N_{SR}} \sim \left( \frac{S}{r_0} \right)^2 / (\frac{N}{R}) \]

• Simplest cluster interaction is via 1-loop matrix integral (i.e. static interaction of zero longitudinal momentum exchange)
  
  \[ V_{\text{int}}^{\text{1-loop}} \sim \left( \frac{N_1}{R} \frac{\ell_1^2}{R} \right) \left( \frac{N_2}{R} \frac{\ell_2^2}{R} \right) \frac{G_0}{R r_0^{-2}} \]

  Since clusters are comingled, need to include longitudinal momentum exchange (parton exchange between clusters)

---

• **Hypothesis #3a**: Interactions are approximately Lorentz covariant at sufficiently large \( N \).

  \[
  \int_0^{2\pi R} \frac{dx_{10}}{2\pi R} \sum_{n \in \mathbb{Z}} \frac{\exp[-ip \chi_{10}]}{[(x_{10}^2 + 2\pi R n)^2 + r^2]^{d/2}} = \frac{(2\pi R)^{d-1}}{\sqrt{\pi} \Gamma(d/2)} \frac{1}{R^{d-1}} K_{d-1}(pr)
  \]

  \[
  \sim \begin{cases} 
  \frac{1}{R r_0^{d-1}} & \text{pr} \ll 1 \\
  \exp[-pr] & \text{pr} \gg 1 
  \end{cases} \quad (\text{want } d-1 = D-4)
  \]

  **In the boosted frame, longitudinal momentum exchange interactions are same as** \( V_{\text{int}}^{\text{1-loop}} \) **up to scale of long. resolution** \( S \chi \sim \frac{RS}{N} \)
YM picture:

\[ E_{\text{pot}} \sim G_0 \sum_{R \geq P=0}^N \sum_{a,b=1}^S \frac{(\vec{P}_a^2/\rho_d)_a (\vec{P}_b^2/\rho_d)_b}{R r_{ab}^{D-4}} \]

\[ \sim E_{\text{kin}} \cdot G_0 S \frac{r_0^{D-2}}{r_0} \]

Virial theorem: \( E_{\text{pot}} \sim E_{\text{kin}} \Rightarrow \boxed{S \sim G_0^{-1} r_0^{D-2}} \)

Moreover, \( \frac{M^2}{P} = E_{\text{IMF}} \sim E_{\text{kin}} + E_{\text{pot}} \sim \left( \frac{S}{r_0} \right)^2 / P \Rightarrow M = \delta r_0 \)

**Punchline**: IMF description of a black hole is a "star" composed of D-particles. Simple arguments reproduce the expected mass-entropy and entropy-area scalings.
The interactions are a cartoon. In this regime, all loop interactions \( \left( \frac{N_c e^{i/2}}{r^{d-1}} \right)^{2d+1} \) are of the same order. The loop expansion breaks down, since nonabelian d.o.f.'s are light & excited (the state has a significant membrane component). Nevertheless, expect mean field theory to predict the scale at which this takes place.

(cf. also refined analysis Liu-Tsajlin 9712063.)

In order to claim \( S = \# \text{clusters} = S_{\text{tree}} \), need to treat clusters as distinguishable objects. This is implied by the previous remark - clusters interact with a diffuse background of nonabelian matrix modes which break any permutation symmetry of clusters (each cluster interacts with a different set of off-diagonal matrix elements)
Charged Black Holes:

\[ M \sim G_0^{-1} r_0^{d-3} \, \text{ch}^2 \chi \]
\[ Q \sim G_0^{-1} r_0^{d-3} \, \text{sh} \chi \, \text{ch} \chi \]
\[ S \sim G_0^{-1} r_0^{d-2} \, \text{ch} \chi \]
\[ \text{Horizon area} \]

- One finds that Black String / Black Hole transition takes place at \[ P = S \, \text{ch} \chi / R \]
  (M. Li, GJM, H. Amoda)

- In YM, can introduce charge by giving D-particles net momentum along internal torus \( T^r \). Virial estimate again reproduces scaling behavior \( \varnothing \) given hypotheses 1-3a.

- In all cases, \[ |\delta p|_{\text{non-compact}} \sim \frac{1}{r_0 \, \text{ch} \chi} \sim T_H \]
  is virial temperature of clusters is the Hawking temperature. Characteristic temp of emitted quanta must be this scale - Hawking radiation is the "solar wind" of the D-star.
Probe dynamics

\[ S_{\text{probe}} = P_{\text{probe}} \int d\tau \frac{1}{g_{--}} \left[ (g_{tt} + g_{ij} u^i) - (g_{tt} + g_{ij} u^i)^2 - (g_{tt} + g_{ij} u^i + g_{ij} u^i) \right]^k \]

Expanding for large \( r \), small \( v \), with \( g_{\mu\nu} \sim y_{\mu\nu} + a_{\mu\nu} (r_0)^{d-3} \)

\[ S_{\text{probe}} = P_{\text{probe}} \int d\tau \left[ \frac{1}{2} v^2 + \frac{1}{4} (r_0)^{d-3} (a_{++} + a_{ij} u^i u^j + a_{+i} u^i \\
- (a_{++} + a_{-i} u^i) v^2 + \frac{1}{2} a_{--} u^2) + \ldots \right] \]

- Treat the source as a collection of clusters
- Allow both BH and probe to have charge \( (\tilde{u} \to u \text{ in } TP) \)
- Use leading matrix theory interaction to estimate terms in \( S_{\text{probe}} \)

\( \Rightarrow \) qualitatively reproduce all terms in \( S_{\text{probe}} \) to order \( v^4 \)
Further support for the model comes from the matrix string limit. Shrinking an $S^{\ast 2} \mathcal{C} T^{p}$ (e.g. $R_p \ll R$) one finds

\[ p+1 \text{ SYM on } \tilde{T}^p \rightarrow 1+1 \text{ SYM on } \tilde{S}^1 \quad \left( \tilde{R}_p = \frac{\tilde{R}^3}{R R_p} \gg \tilde{R}_i \right) \]

Dominant configurations are "Slinkies\textsuperscript{®}" a "string bit" description of the IMF superstring

At the "correspondence point" \( r_0 \sim l_{\text{str}} = \left( \frac{\ell_p}{R_p} \right)^{\frac{3}{2}} \), a transition occurs from BH's \((r_0 > l_{\text{str}})\) to strings \((r_0 < l_{\text{str}})\) as the entropically favored objects (Horowitz - Polchinski). At \( r_0 \sim l_{\text{str}} \) the entropies match

\[ S_{\text{BH}} \sim G_{0} r_0^{D-2} \sim S_{\text{str}} \sim \sqrt{N_{\text{str}}} \]

Now consider a BH which is simultaneously

a) at the correspondence point \( r_0 \sim l_{\text{str}} \)

b) boosted to the black hole/black string transition \( N = S \) in a longitudinal box of size \( R \)
The IMF matrix string has temperature

\[ T_{\text{str}} = \sqrt{N_{\text{osc}}}/(\frac{N^2}{R} l_{\text{str}}^2) \]

The correspondence principle asserts that this is the (boosted) Hawking temperature (since a macroscopic string decays by emitting quanta at the Hagedorn temperature \( T \sim \frac{1}{l_{\text{str}}^2} \sim \frac{1}{R_0} \sim T_H \))

\[ T_H = e^{-\alpha} \cdot \frac{1}{R_0} \sim \frac{R}{R_0^2} \]

Comparing,

\[ \frac{\sqrt{N_{\text{osc}}}}{N} \sim \frac{l_{\text{str}}^2}{R_0^2} \sim 1 \]

- Thus, each of the \( N \) "string bits" or D-particles carries one "bit of information". This is clear from \( \ast \ast \) - the thermal wavelength is \( \lambda_T \sim \tilde{R}_p = \) one string bit; a macroscopic string is a random walk of \( S \) steps of size \( \lambda_T \).

- The "Boltzmann" character of the D-particle gas is manifest in the fact that the D-particles are labelled by their position on the matrix string - this is an example of the "nonabelian background", i.e. the set of twisted B.C.'s that intertwines the string bits as one circles \( \tilde{S}^1 \).
Further boosting to \( N \gg S_{BH} \) (but remaining at the correspondence point), the string temperature boosts down by a factor \( N/S_{BH} \). The number of string bits contained in a thermal wavelength (coherence length) is \( k = N/S_{BH} \); these are the "clusters". Their virial temperature is indeed the (boosted) Hawking temperature.

Charged BH's at the correspondence point match perfectly with the cluster ansatz; one finds that at the junction of the correspondence point and the hole/string transition, the matrix string has \( N/S_{BH} \approx e^{\chi} \) string bits in a thermal wavelength - exactly as predicted by the ansatz.
Black holes are "stars" composed of D-particles, membranes, etc. They are "supported" against collapse by the uncertainty principle and by the fact that gravity ceases to be the complete IMF description of interactions within the BH.

- Characteristic size of BH is governed by available energy/momentum resolution — $r_0$ in transverse direction, $r_0$ "Lorentz contracted" to $e^{-x}r_0 = \frac{R}{N}$ longitudinally.

- Transverse density of cluster centers never exceeds the "holographic bound" of one per transverse Planck volume.

- **Locality/nonlocality** — individual D-particles have spread $|\delta x| \sim N^{1/4}$, whereas black hole wavefunction has $|\delta x| \sim r_0$ independent of $N$. Any locality properties are a result of coherence of cluster wavefunction.

(same phenomenon occurs in \textit{perturbative string theory}), \textit{AdS/CFT}