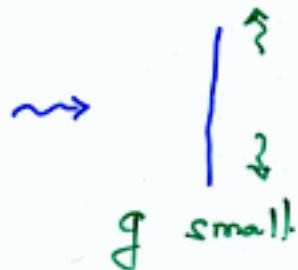
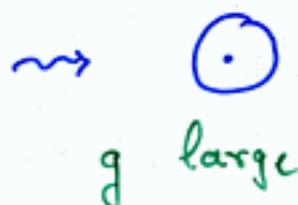


# ABSORPTION BY D3 BRANES

SAMIR MATHUR

## Black hole absorption



## 3 charge hole in 5-D

$$\sigma = A_H$$

Greybody factors

Callan - Maldacena '96  
Jas + S.D.M. '96  
Maldacena + Strominger '96

If do not get (approximate) agreement

→ Expect that effective degrees of freedom change as  $g$  small is changed to  $g$  large

## Plan of Talk:

1. Absorption of fields by D3-branes  
[ S.D.M. + A. Matusis '98 ]



2. Absorption of  $B_{\mu\nu}$  by 'test brane'

[ Das + S.D.M. + Trivedi '98 ]

(Use earlier results of Rajaraman '98,  
Das + Trivedi '98 )

- 3 Spin dependence of absorption in 3 charge black hole

→ Change in absorbed spins as  $g \rightarrow 0 \rightarrow g$  large  
[ S.D.M. '97 ]

→ Microscopic description of weak coupling phase : torons [ S.D.M '98 ]

# Absorption by D3-branes

[S.D.M. + A. Matusis '98]

Leading order in energy  $\omega$

$$ds^2 = H^{-\frac{1}{2}}[-dt^2 + dx^a dx^a] + H^{\frac{1}{2}}[dr^2 + r^2 d\Omega_5^2]$$

$$H = 1 + \frac{r^4}{R^4}, \quad F_{12r03} = H^{-2} R^4 r^{-5}, \quad F = *F$$

Linearised field equations

$$\begin{aligned} R_{\mu\nu} &= -\frac{1}{6} F_{\mu\rho\sigma\tau\kappa} \tilde{F}_{\nu}^{\rho\sigma\tau\kappa} \\ F_{\mu\nu\rho\sigma\tau} &= \frac{1}{5!} \epsilon_{\mu\nu\rho\sigma\tau\mu'\nu'\rho'\sigma'\tau'} F^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}} \\ D^{\hat{\mu}} \partial_{[\hat{\mu}} A_{\nu\rho]} &= -\frac{2i}{3} \dot{F}_{\nu\rho\sigma\tau\kappa} D^{\hat{\sigma}} A^{\hat{\tau}\hat{\kappa}} \\ D^{\hat{\mu}} \partial_{\hat{\mu}} \phi &= 0 \end{aligned}$$

$$A_{\mu\nu} = B_{\mu\nu}^{NSNS} + i B_{\mu\nu}^{RR}$$

Fields decoupled from 10-D graviton:

AdS region:  $AdS_5 \times S^5$

$\alpha, \beta \in AdS_5, \mu, \nu \in S^5$

Dilaton

$$\phi = \sum_{I_1} B^{I_1}(x) Y^{I_1}(y)$$

Scalar from 2-form

$$A_{\alpha\beta} = \sum_{I_{10}} [a_\mu^{I_{10}}(x) Y_{[\alpha\beta]}^{I_{10}}(y) + a^{I_5} D_{[\alpha} Y_{\beta]}^{I_5}(y)]$$

Vector from 2-form

$$A_{\mu\alpha} = \sum_{I_5} [a_\mu^{I_5}(x) Y_\alpha^{I_5}(y) + a_\mu^{I_1} D_\alpha Y^{I_1}(y)]$$

2-form from 4-form

$$a_{\mu\nu\alpha\beta} = \sum_{I_{10}} b_{\mu\nu}^{I_{10}}(x) Y_{[\alpha\beta]}^{I_{10}}(y)$$

2-form from 2-form

$$A_{\mu\nu} = \sum_{I_1} a_{\mu\nu}^{I_1}(x) Y^{I_1}(y)$$

# METHOD OF SOLUTION

OUTER  
REGION

INTERMEDIATE  
REGION

INNER  
REGION

$$\frac{r}{R} \gg 1$$

$$\omega r \gg 1$$

$$\frac{r}{R} \gg 1$$

$$\omega r \ll 1$$

$$\frac{r}{R} \ll 1$$

$$U = \frac{\omega R^2}{r} \ll 1$$

$$r \rightarrow 0$$

$$U \gg 1$$

Flat space

Interpolating  
region

$AdS_5 \times S^5$

$$\phi_k \sim \frac{e^{\pm ikr}}{r^{5/2}}$$

$$r^k, r^{-k-4}$$

$$r^k, r^{-k-4}$$

$$v^{3/2} e^{i\omega r}$$

$$a_k^+ \sim \frac{e^{\pm ikr}}{r'^{1/2}}$$

$$r^{k+2}, r^{-k-2}$$

$$r^{k+2}, r^{-k-6}$$

$$v^2 e^{i\omega r}$$

Match solutions across regions

| Field              | Inner powers        | Outer powers        | $\Delta$ | $\mathcal{P}$    |
|--------------------|---------------------|---------------------|----------|------------------|
| $B$                | $r^k, r^{-k-4}$     | $r^k, r^{-k-4}$     | $k+4$    | $\omega^{4k+8}$  |
| $a^+$              | $r^{k+2}, r^{-k-6}$ | $r^{k+2}, r^{-k-2}$ | $k+6$    | $\omega^{4k+12}$ |
| $a^-$              | $r^{k-2}, r^{-k-2}$ | $r^{k+2}, r^{-k-2}$ | $k+2$    | $\omega^{4k+4}$  |
| $a_1$              | $r^{k+1}, r^{-k-3}$ | $r^{k+1}, r^{-k-3}$ | $k+4$    | $\omega^{4k+8}$  |
| $a_0$              | $r^{k+1}, r^{-k-3}$ | $r^{k+2}, r^{-k-2}$ | $k+4$    | $\omega^{4k+8}$  |
| $b_{\mu\nu}$       | $r^{k+2}, r^{-k-2}$ | $r^{k+2}, r^{-k-2}$ | $k+4$    | $\omega^{4k+8}$  |
| $a_{12} - ia_{03}$ | $r^k, r^{-k-4}$     | $r^k, r^{-k-4}$     | $k+2$    |                  |
| $a_{12} + ia_{03}$ | $r^{k+4}, r^{-k}$   | $r^k, r^{-k-4}$     | $k+6$    |                  |

Table 3: Power law behavior for the fields in the inner and outer regions, and the dependence of absorption probability on  $\omega$ .

| Field  | $k$        | $\odot$    | $\mathcal{P}$   |
|--|------------|------------|---|
| $B = B^{I_1} Y^{I_1}$  | $k \geq 0$ | $k = 0$    | $\mathcal{P} = \frac{4\pi^2}{[\Gamma(k+2)\Gamma(k+3)]^2} \left(\frac{\omega R}{2}\right)^{4k+8}$    |
| $A_{\alpha\beta} = a^{I_{10,+}} Y_{[\alpha\beta]}^{I_{10,+}}$                    | $k \geq 1$ |            | $\mathcal{P}_- = \frac{4\pi^2}{[\Gamma(k+3)\Gamma(k+4)]^2} \left(\frac{\omega R}{2}\right)^{4k+12}$ |
| $A_{\alpha\beta} = a^{I_{10,-}} Y_{[\alpha\beta]}^{I_{10,-}}$                    | $k \geq 1$ | $k = 1$    | $\mathcal{P}_+ = \frac{4\pi^2}{[\Gamma(k+1)\Gamma(k+2)]^2} \left(\frac{\omega R}{2}\right)^{4k+4}$  |
| $A_{\mu\alpha} = a_\mu^{I_8} Y_\alpha^{I_8}$                                     | $k \geq 1$ |            | $\mathcal{P} = \frac{4\pi^2}{[\Gamma(k+2)\Gamma(k+3)]^2} \left(\frac{\omega R}{2}\right)^{4k+8}$    |
| $a_{\mu\nu\alpha\beta} = b_{\mu\nu}^{I_{10,\pm}} Y_{[\alpha\beta]}^{I_{10,\pm}}$ | $k \geq 1$ |            | $\mathcal{P} = \frac{4\pi^2}{[\Gamma(k+2)\Gamma(k+3)]^2} \left(\frac{\omega R}{2}\right)^{4k+8}$    |
| $A_{\mu\nu} = a_{\mu\nu}^{I_1} Y^{I_1}$  | $k \geq 1$ | $k = 1$    |   |
|  |            | $k \geq 0$ |   |

Table 2: Fields and corresponding absorption probabilities.

| $a_{12}$       | $\hat{C}_1$   | $\hat{C}_2$   | $C_1$                     | $C_2$                   |
|----------------|---|---|---------------------------|-------------------------|
| $J_{(k+4)}(v)$ | 0   | $-\frac{\pi R^2}{\Gamma(k+2)\Gamma(k+5)}\left(\frac{wR}{2}\right)^{2k+6}$ | 0                         | $-\frac{w}{k}\hat{C}_2$ |
| $N_{(k)}(v)$   | $-\frac{R^2\Gamma(k)\Gamma(k+3)}{\pi}\left(\frac{wR}{2}\right)^{-2k+2}$   | 0   | $\frac{w}{k+4}\hat{C}_1$  | 0                       |
| $N_{(k+4)}(v)$ | $-\frac{R^2\Gamma(k+4)\Gamma(k+3)}{\pi}\left(\frac{wR}{2}\right)^{-2k+6}$ | 0   | $-\frac{w}{k+4}\hat{C}_1$ | 0                       |
| $J_{(k)}(v)$   | 0   | $-\frac{\pi R^2}{\Gamma(k+1)\Gamma(k+3)}\left(\frac{wR}{2}\right)^{2k+2}$ | 0                         | $\frac{w}{k}\hat{C}_2$  |

Table 1: Scattering matrix for the two form from the antisymmetric tensor

# AN ABSORPTION PROCESS



Gravity Calculation:

- Evolve  $B_{12}(r, t)$  into AdS region
- Absorb into brane

$$S_{\text{int}} = \int d^4x \left[ -\det(G_{ij} + F_{ij}) \right]^{\frac{1}{2}} + \text{Wess-Zumino term}$$

Yang-Mills interpretation (?)



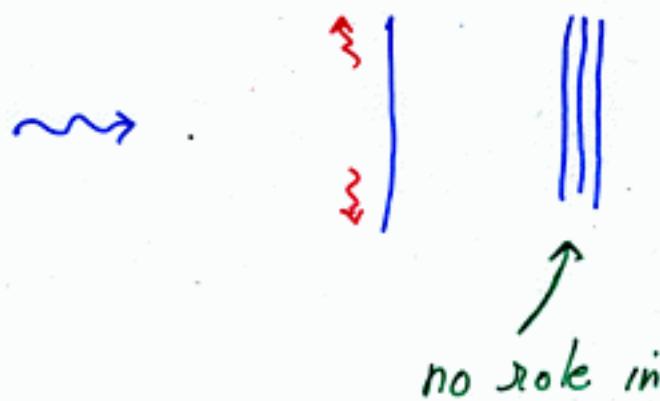
$U(1)$

$U(N)$



Absorb quantum into  $U(1)$  part

If  $gN \rightarrow 0$ .



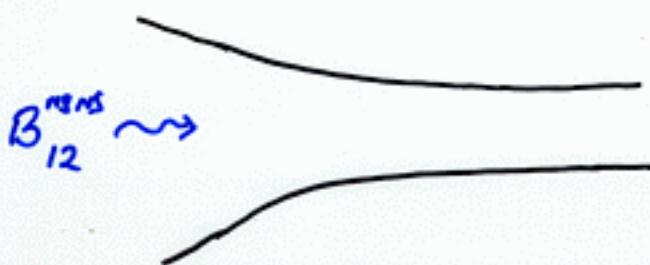
$$B_{12}(r,t) \sim e^{ikr - i\omega t} + e^{-ikr - i\omega t}$$

process

$$S_{int} = \int d^4\xi \bar{\lambda}(\xi) \Gamma_i \partial_j \lambda(\xi) B^{ij}(x(\xi))$$

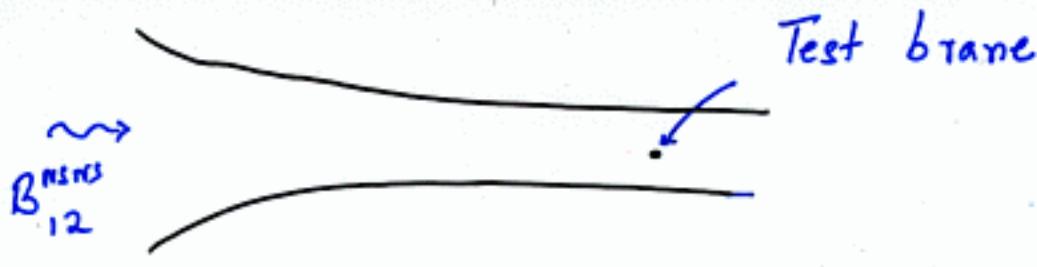
$$\sigma \sim \omega^3$$

Compare to



$$\sigma_{abs} \sim \omega^7$$

[ Rajaraman '98 ]



Related Calculation: Das + Tiwari '98

Supergravity fields  $\longleftrightarrow$  Yang-Mills operators

Find couplings using "effective brane"  
at boundary of AdS

$$S = \text{Dirac-Born-Infeld} + \text{Wess-Zumino}$$

Important effect:

$B_{12}^{new}$  at  $r \rightarrow \infty$  changes to  
a combination of  $B_{12}^{new}$  and  $B_{03}^{RR}$   
in AdS region:  $B_{03}^{RR} = B_{12}^{new}$

Change happens in intermediate  
region

$$H_{\mu\nu\sigma;\lambda}^{(RR)} = \frac{2}{3} F_{\nu\sigma\lambda\tau} H^{(NNS) \lambda\tau}$$

S-wave:  $B_{12}^{NNS} \sim \frac{e^{-ikr-i\omega t}}{r^{5/2}} \quad (r \rightarrow \infty)$

$$H_{012}^{(RR)} = -4 F_{r3012} B^{12}$$

Gauge:  $B_{r3}^{RR} = 0$

AdS region:  $F_{0123r} = \frac{r^3}{R^4}$

$$B_{12}^{NNS} = c_1 r^4 + c_2 r^{-4}$$

$\uparrow$   
dominates

$$B_{03}^{RR} = B_{12}^{NNS}$$

[ Rajaraman, '98  
Das - Trivedi '98]

→ Test brane feels both  $B_{12}^{NNS}$  and  $B_{03}^{RR}$

$B_{12}^{\text{MNS}}$  couples to  $\theta_4$

$B_{03}^{RR}$  couples to  $\theta_9$

$B_{12}^{\text{MNS}} = B_{03}^{RR}$  : couples to  $\theta_6$

Coupling:  $B_{ij}^{\text{MNS}}(x) \theta_{6,ij}(\xi)$

$$\theta_{6,ij} = \frac{R^4}{r^4} \left[ \frac{1}{8} F_{ij} F_{ke} F_{ek} - \frac{1}{4} F_{ij} \Phi_{,k}^m \Phi_{,k}^m \right]$$

$$+ \frac{1}{2} F_{jk} F_{ke} F_{ei} - F_{jk} \Phi_{,k}^m \Phi_{,i}^m \right]$$

[Das-Tirredi '98]

Free part of brane action:

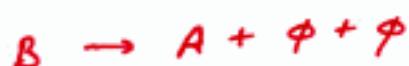
$$S_0 = -T_3 \int d^4 \xi \left[ \frac{1}{2} \Phi_{,k}^m \Phi_{,k}^m + \frac{1}{4} F_{ij} F^{ij} \right]$$

(Effect of metric drops out)

# CALCULATION OF ABSORPTION

[Das + S.D.M. + Tripathi '98]

Coupling :  $B_{ij} \Theta_{ij}$



$$\langle \Theta_{ij}^{(0)} \Theta_{ke}^{(x)} \rangle = \frac{27}{4\pi^6} \left(\frac{1}{x^2}\right)^6 \left[ J_{ik}(x) \bar{J}_{je}(x) - J_{ie}(x) \bar{J}_{jk}(x) \right]$$

$$J_{ik}(x) = \delta_{ik} - \frac{2x_i x_k}{x^2}$$

$$\langle F_{ij}(0) F_{ke}(x) \rangle = \pi^{\frac{1}{2}} (x^2)^2 \left[ J_{ik}(x) J_{je}(x) - J_{ie}(x) \bar{J}_{jk}(x) \right]$$

$$\langle \partial_i \phi^m(x) \partial_j \phi^m(x) \rangle = \frac{6}{2\pi^2 (x^2)^2} \left[ \delta_{ij} - \frac{4x_i x_j}{x^2} \right]$$

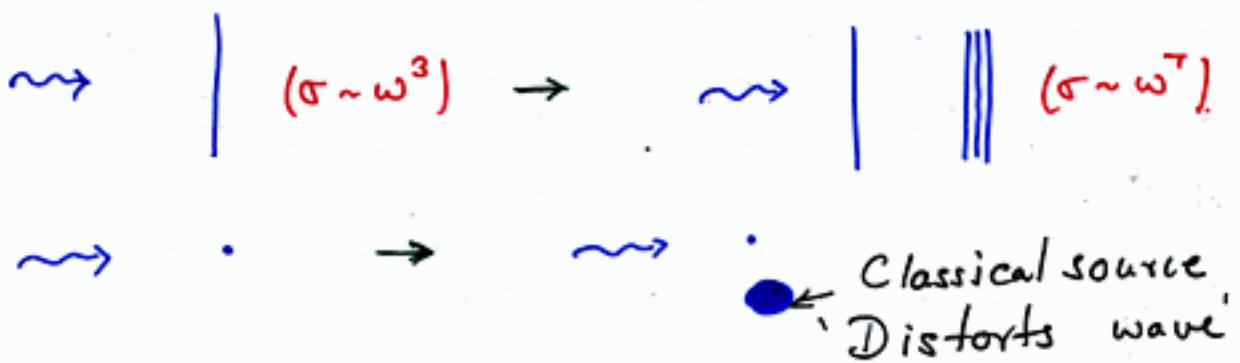
$$\sigma = \frac{2\pi^2}{2i\omega} \text{ Disc } (-s)$$

$$s = -P^2$$

Evaluate at  $P_0 = \omega$ ,  $\tilde{P} = 0$

$$\boxed{\sigma = \pi^4 2^{-10} 5^{-1} \omega^7 g^3 \alpha'^6}$$

## Picture



'Big separation'  $v = \frac{\omega R^2}{r} \ll 1$   $\phi \sim \frac{1}{r^4}$

'Small separation'  $v \gg 1$   $\phi \sim \frac{1}{r^4} e^{i\frac{\omega R^2}{r}}$

Vertex for absorption.  $\sim \frac{R^4}{r^4}$

$v \ll 1$  :  $\sigma$  independent of  $r$

$v \gg 1$  :  $\sigma$  grows as  $r \rightarrow 0$

Would be interesting to reproduce from  
a strong coupled field theory description



$$g_N \rightarrow 0, \sigma \sim \omega^3$$



$$g_N \rightarrow \infty, \sigma \sim \omega^7$$

Similar to situation for 5-D black holes

$$M^{10} \rightarrow M^5 \times T^5$$

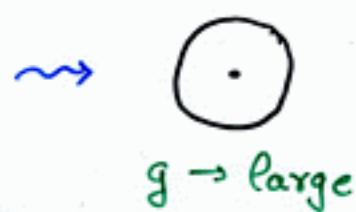


$h_{ij}$ : Scalar

$h_{i\mu} = A_\mu$ : Vector

$h_{\mu\nu} = g_{\mu\nu}$ : Graviton

$$\sigma \propto A_H$$

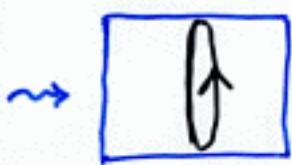


$h_{ij}$ : ~~(Scalar)~~

$$\sigma = A_H$$

$$\sigma \sim A_H (A_H \omega^3)$$

$$\sigma \sim A_H (A_H \omega^3)^2$$



2 charges

$g < g_c$ : "momentum-antimomentum" excitations

$$\sigma \propto A_H : h_{ij}, i,j=1-4$$

$h_{i\mu}$

$h_{\mu\nu}$

$$g > g_c : \sigma = A_H : h_{ij}, i,j=1-4$$

'5-5' excitations

$g_c$  = correspondence point

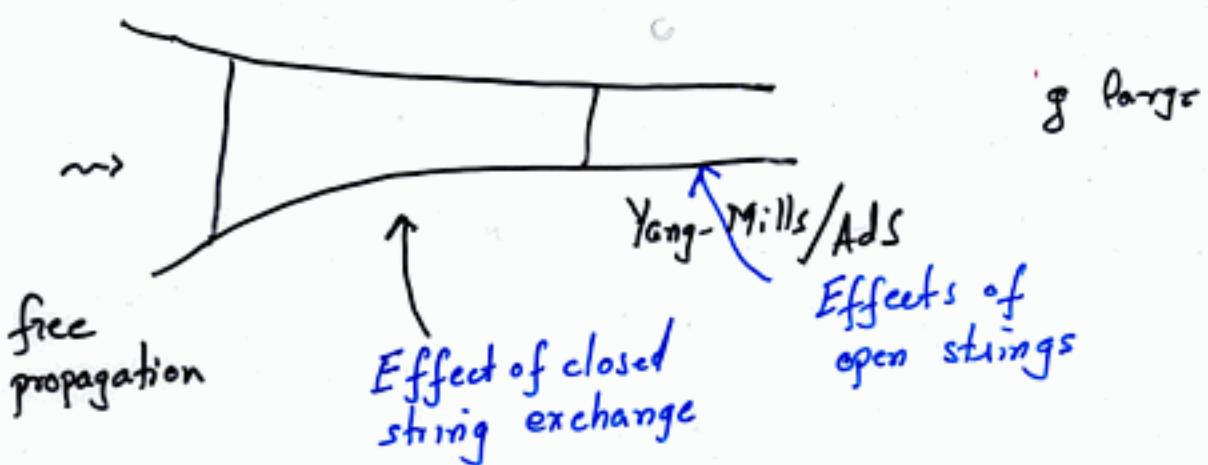
[S.D.M. '97]

## Remarks

Question: What is  $\sigma$  for particles thrown from infinity onto D-Branes?



$$g \rightarrow \infty$$



String world sheet moduli space  
= Closed String part + Open string part.



May not be able to take into account by perturbative ~~effects~~  
corrections to Yang-Mills