Strings '98

Three-Point Functions of Chiral Operators in $D = 4$, $\mathcal{N} = 4$ Super-Yang-Mills in the Large N limit.

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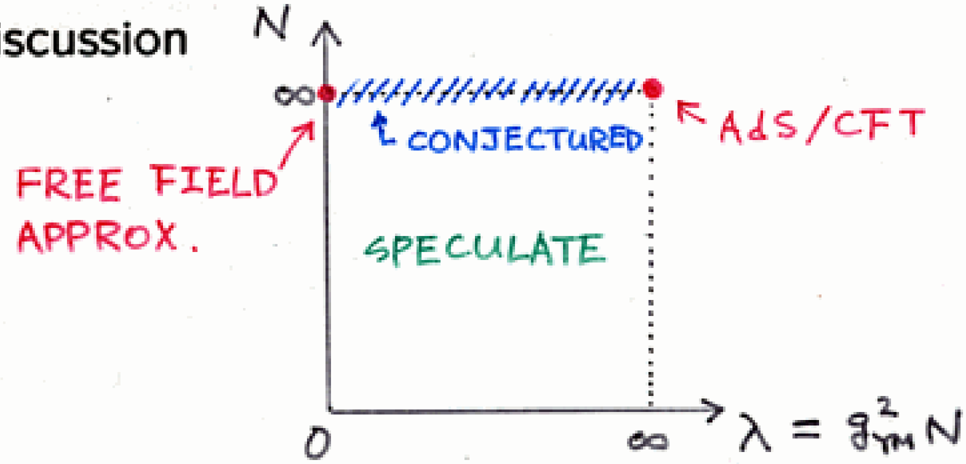
Outline of the Talk

I. SYM_4 and its Chiral opeartors.

II. Weak Coupling result (Free Field Theory)

III. Strong Coupling result (AdS/CFT)

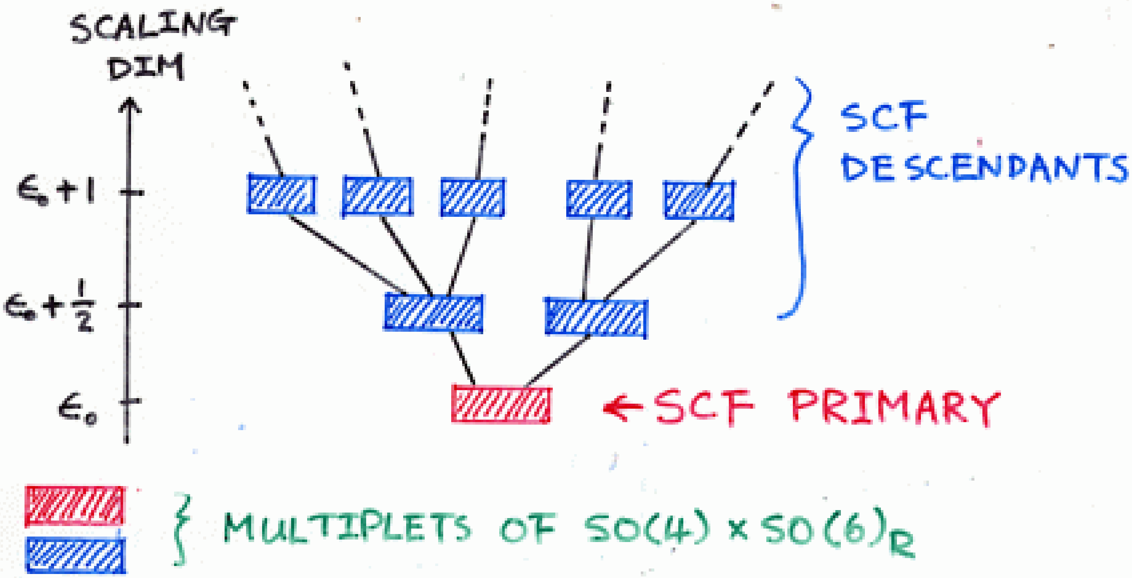
IV. Discussion



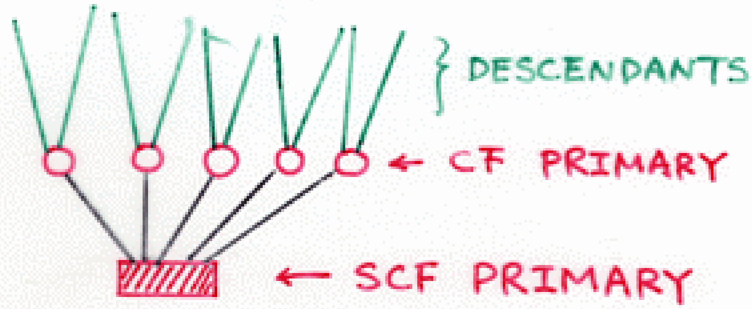
Conjecture: at least in the large N limit, the 3-point functions are independent of λ.

I. SYM$_4$ and its Chiral Operators.

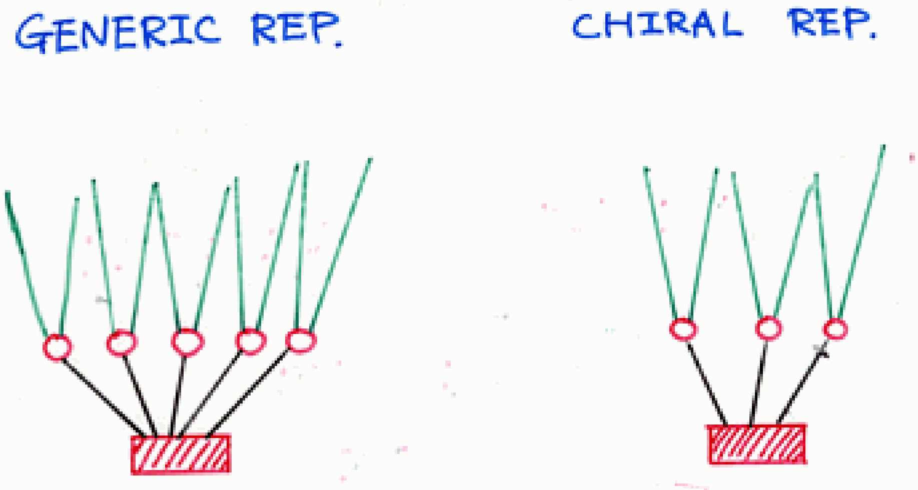
Operators in SYM$_4$ appear in infinite dimensional multiplets under the $d=4$ $\mathcal{N}=4$ SCA.



Representations of the SCA may be decomposed into a finite number of representations of the d=4 conformal algebra.



The $d = 4$, $\mathcal{N} = 4$ algebra has a number of special short representations (chiral) that are composed of a fewer number of conformal representaions. Analogue of BPS particles.



For the $U(N)$ theory study the chiral primaries are which scalars under $SO(4)$, k^{th} traceless symmetric tensor under $SO(6)$, scaling dimension k. (Theory also possesses many nonchiral primaries.)

II. COMPUTATION AT SMALL $\lambda = g_{YM}^2 N$

CPOs of SYM$_4$ are of the form

$$\mathcal{O}^I = \mathcal{C}^I_{i_1..i_k} \mathrm{Tr}(\phi^{i_1}..\phi^{i_k}).$$

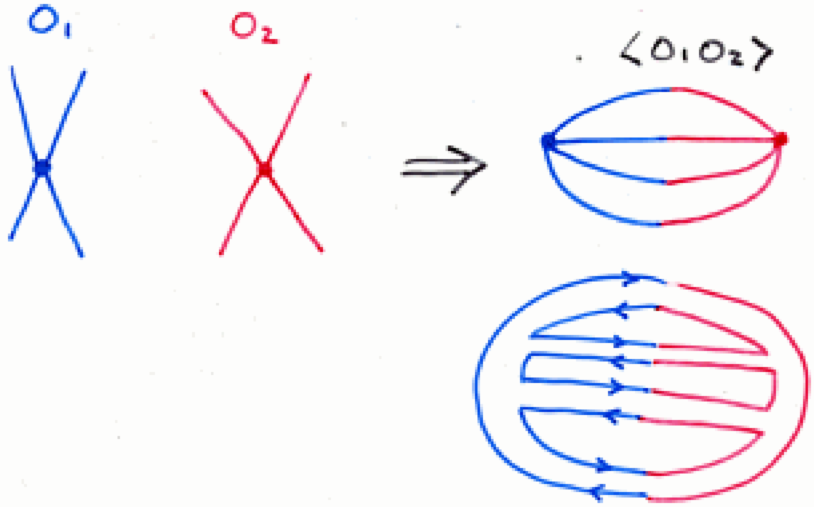
$\phi^i = \phi^i_a T_a$: vector of $SO(6)_R$, adjoint of $U(N)$.
$\mathcal{C}^I$: totally symm. traceless tensor of $SO(6)_R$.

Normalize the SYM action $S = \frac{-1}{2g_{YM}^2} \int TrFF...$

Propagator:

$$\langle \phi^i_a(x) \phi^j_b(y) \rangle = \frac{g_{YM}^2 \delta_{ab} \delta^{ij}}{(2\pi)^2 |x-y|^2}.$$

The 2-point ftn of two CPOs specified by tensors $\mathcal{C}^{I_1}_{i_1 \dots i_{k_1}}$ and $\mathcal{C}^{I_2}_{j_1 \dots j_{k_2}}$.



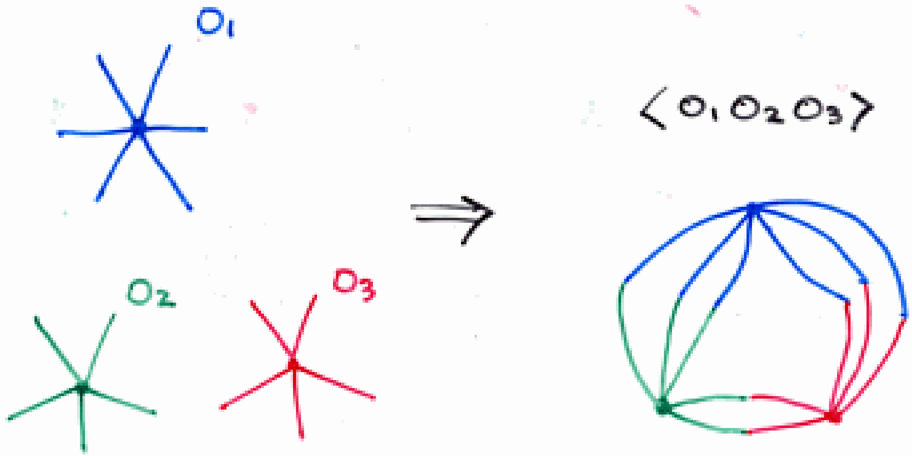
$$\langle \mathcal{O}^{I_1}(x)\mathcal{O}^{I_2}(y)\rangle = \lambda^k \frac{k}{(2\pi)^{2k}|x-y|^{2k}}\langle \mathcal{C}^{I_1}\mathcal{C}^{I_2}\rangle.$$

$$\langle \mathcal{C}^{I_1}\mathcal{C}^{I_2}\rangle \equiv (\mathcal{C}^{I_1})_{i_1\dots i_{k_1}}(\mathcal{C}^{I_2})^{i_1\dots i_{k_2}}.$$

Rescale: $O^I = \frac{\lambda^{\frac{k}{2}}\sqrt{k}}{(2\pi)^k \text{[redacted]}}\mathcal{O}^I$ s.t.

$$\langle O^{I_1}O^{I_2}\rangle = \frac{\delta^{I_1 I_2}}{|x-y|^{2k}}.$$

The 3-point ftn of rescaled CPOs



$\alpha_1 = \frac{1}{2}(k_2 + k_3 - k_1)$, etc.

$$\boxed{\langle O^{I_1}(x) O^{I_2}(y) O^{I_3}(z)\rangle = \frac{1}{N}\frac{\sqrt{k_1 k_2 k_3}\langle C^{I_1} C^{I_2} C^{I_3}\rangle}{|x-y|^{2\alpha_3}|y-z|^{2\alpha_1}|z-x|^{2\alpha_2}}.}$$

Result correct only at $N = \infty$. Nonzero corrections from nonplanar diagrams at finite N.

III. COMPUTATION AT large $\lambda = g_{YM}^2 N$

AdS/CFT correspondence

J. Maldacena

Correlation functions in SYM_4 are related to partition function of IIB on $AdS_5 \times S^5$ as ftn of boundary values of fields.

S. Gubser, I. Klebanov, A. Polyakov ; E. Witten.

$1/N$ correction to the large N limit of SYM
$\Longleftrightarrow$
Quantum correction to the classical limit of String theory.

$1/\lambda$ correction to the large λ limit of SYM $\Longleftrightarrow$ α' correction to the SUGRA limit of string theory.

$\boxed{\text{Procedure}}$

- Identify SUGRA fields coupling to CPOs in S_{int}.

- Compute S_{SUGRA} as fuction of arbitrary boundary values of these fields.

$\boxed{\text{Subtleties}}$

1. S_{int} not known.

H. Kim, L. Romans, P. van Nieuwenhuizen

M. Günaydin, N. Marcus

- Use linearized e.o.m.
  Onshell fluctuations $\rightarrow$ SCA multiplets.
  Match with worldvolume operators.

- Seems insufficient

  - Field identified to linear order.
    However nonlinear field redefinitions in SUGRA $\leftrightarrow$ contact terms in CFT.

  - Field determined up to normalization.
    Consequence: Can compute correlation functions only of normalized operators.

    Note: Distinct from normalization of action.

2. Gravity is a gauge theory.

Need to impose gauge and solve Gauss's law. Some fields physical, others constrained.

Physical Fields

- Represent true physical degrees of freedom. Appear in representations of the SCA.

Compute S_{SUGRA} as ftn of bdry values of physical fields only. Set the value of all other physical fields to zero on boundary.

Physical fields $\leftrightarrow$ CPOs and their Conf descendents denoted by $s^I(x^1, ., x^5)$.

S. Ferrara, A. Zaffaroni

3. IIB SUGRA action is complicated.

- No simple covariant action for IIB SUGRA.
- Even Einstein action nonlocal as function of physical fields.

1. Study linearized e.o.m. to determine s^I

2. Find e.o.m to $O(s^2)$.

3. Determine $S_{eff}(s^I)$ to $O(s^3)$ upto normalization.

4. Fix Normalization of S_{eff} by comparing with $< JJJ >$. Alternatively comparing with Sorokin's action. Same answer.

G. Dall'Agata, K. Lechner, D. Sorokin.

Result of Our Computation

Correctly normalized action for s^I fields to cubic order.

$$S = \frac{4N^2}{(2\pi)^5}\int d^5x\sqrt{-g_1}\{-\sum_I \frac{A_I}{2}[(\nabla s^I)^2 + k(k-4)(s^I)^2] + \sum_{I_1 I_2 I_3}\frac{\mathcal{G}_{I_1 I_2 I_3}}{3} s^{I_1} s^{I_2} s^{I_3}\}.$$

$\mathcal{G}_{I_1 I_2 I_3}$ = Very complicated ftn of I_1, I_2, I_3.

S yields. D. Freedman, S. Mathur, A. Matusis, L. Rastelli

$$\boxed{\langle O^{I_1}(x) O^{I_2}(y) O^{I_3}(z)\rangle = \frac{1}{N}\frac{\sqrt{k_1 k_2 k_3}\langle C^{I_1} C^{I_2} C^{I_3}\rangle}{|x-y|^{2\alpha_3}|y-z|^{2\alpha_1}|z-x|^{2\alpha_2}}}$$

Same formula as in weak coupling limit!!

IV. DISCUSSION

- Conjecture : The 3-point ftns are independent of λ, at least in the large N limit.

Evidence

– True at small and large λ.

– First possible correction from large λ is zero. T. Banks , M. Green.

Suggested Check

Compute the $O(\lambda)$ correction to free field 3-point function at $N = \infty$.

D. Freedman , S. Mathur , A. Matusis , L. Rastelli

hep-th / 9901001

- Not true for 4-point funtions as these receive corrections at $O(\alpha'^3) = O(\lambda^{\frac{-3}{2}})$.

- It may even be that the 3-point function is independent of λ for fixed finite N; ie the free field answer is correct at all N, λ.

$$\langle \mathrm{Tr}_R(F^2)\,\mathrm{Tr}_R(F^2)\,\mathrm{Tr}_R(F^4)\rangle$$

$\mathrm{Tr}(x^2 x^1)$ $\qquad\qquad$ $\mathrm{Tr}_R(x^{2'} \ldots x^3)$