5-branes, KK-monopoles and T-duality

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$$
\begin{tikzcd}
\text{IIB NS-5} \arrow[r, leftrightarrow, dashed] \arrow[d, leftrightarrow] & \text{IIA NS-5} \arrow[d, leftrightarrow] \\
\text{IIA KK} \arrow[r, leftrightarrow, dashed] & \text{IIB KK}
\end{tikzcd}
$$

IIA KK-monopole

- A KK-monopole in D dimensions can be considered an extended object with $D-5$ spatial dimensions and one extra isometry direction transverse to the worldvolume.

- In order to get the right counting of degrees of freedom this isometry is gauged, such that the effective number of embedding scalars is 3, fitting in a $D-4$ dimensional vector multiplet.

- The effective action is described by a $D-4$ dimensional gauged sigma-model

Worldvolume Field	Field Strength	$\sharp$ of d.o.f
X^μ		$10-6-(1)=3$
$\omega^{(0)}$	$\mathcal{K}_i^{(1)}$	1
$\omega_i^{(1)}$	$\mathcal{K}_{ij}^{(2)}$	4
$\omega_{i_1\ldots i_5}^{(5)}$	$\mathcal{K}_{i_1\ldots i_6}^{(6)}$	$-$

Table 1: **Worldvolume field content of the IIA KK-monopole.** It consists on a 1-form $\omega^{(1)}$, a 0-form $\omega^{(0)}$ and 3 embedding scalars X^μ, fitting in a 6 dimensional vector multiplet. One extra degree of freedom has been eliminated through the gauging construction. The 5-form $\omega^{(5)}$ describes the tension of the monopole. We also include the worldvolume solitons and the associated intersection.

The action

$$
\begin{aligned}
S \;=\; & -T_{\mathrm{AKK}} \int d^6\xi \; e^{-2\phi} k^2 \sqrt{1+e^{2\phi}k^{-2}(i_k C^{(1)})^2}\,\times \\
& \times \sqrt{\left|\det\left(\Pi_{ij} - (2\pi\alpha')^2 k^{-2}\mathcal{K}_i^{(1)}\mathcal{K}_j^{(1)} + \frac{(2\pi\alpha')k^{-1}e^{\phi}}{\sqrt{1+e^{2\phi}k^{-2}(i_k C^{(1)})^2}}\mathcal{K}_{ij}^{(2)}\right)\right|} \\
& + \tfrac{1}{6!}(2\pi\alpha')T_{\mathrm{AKK}} \int d^6\xi \; \epsilon^{i_1\ldots i_6}\mathcal{K}_{i_1\ldots i_6}^{(6)} \,.
\end{aligned}
$$

The gauged sigma model metric is given by $\Pi_{ij} = \partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu}$ where

$$
\Pi_{\mu\nu} = g_{\mu\nu} - |k|^{-2} k_\mu k_\nu \,,
$$

with k^μ the Killing vector associated to the transverse target space isometry.

The type IIA KK-monopole couples to a 6-form $(i_k N)$ where N is the Poincaré dual of the Killing vector k_μ.

worldvolume curvatures

$$
\begin{aligned}
\mathcal{K}^{(1)} &= \partial\omega^{(0)} - \frac{1}{2\pi\alpha'}(i_k B) \\
\mathcal{K}^{(2)} &= 2\partial\omega^{(1)} + \frac{1}{2\pi\alpha'}(i_k C^{(3)}) + \ldots \\
\mathcal{K}^{(4)} &= 4\partial\omega^{(3)} + \frac{1}{2\pi\alpha'}(i_k C^{(5)}) + \ldots \\
\mathcal{K}^{(6)} &= 6\partial\omega^{(5)} + \frac{1}{2\pi\alpha'}(i_k N) + \ldots
\end{aligned}
$$

The Poincaré dual of $\omega^{(1)}$ is the 3-form $\omega^{(3)}$ KK-monopole.

$$
\mathcal{K}^{(4)}_{i_1 \ldots i_4} = \frac{1}{2!\sqrt{\det \Pi}\sqrt{1 + e^{2\phi}k^{-2}(i_k C^{(1)\,2})}} \Pi_{i_1 j_1} \ldots \Pi_{i_4 j_4} \epsilon^{j_1 \ldots j_6} \mathcal{K}^{(2)}_{j_5 j_6} \,,
$$

IIB NS-5-brane

Worldvolume Field	Field Strength	$\sharp$ of d.o.f
X^μ		4
$c_i^{(1)}$	$\tilde{\mathcal{F}}_{ij}$	4
$\tilde{c}^{(5)}_{i_1 \ldots i_5}$	$\tilde{\mathcal{G}}^{(6)}_{i_1 \ldots i_6}$	$-$

Table 2: **Worldvolume field content of the IIB NS-5-brane.** In the case of the IIB NS-5-brane, there are four embedding coordinates X^μ and one vector field $c^{(1)}$. The 5-form $\tilde{c}^{(5)}$ describes the tension of the 5-brane.

The action

$$\begin{aligned} S \;=\; & -T_{\mathrm{B5}} \int d^6\xi \; e^{-2\varphi}\sqrt{1 + e^{2\varphi}(C^{(0)})^2}\sqrt{\left|\det\left(g + (2\pi\alpha')\frac{e^{\varphi}}{\sqrt{1+e^{2\varphi}(C^{(0)})^2}}\tilde{\mathcal{F}}\right)\right|} \\ & + \tfrac{1}{6!}(2\pi\alpha')T_{\mathrm{B5}} \int d^6\xi \; \epsilon^{i_1 \cdots i_6}\tilde{\mathcal{G}}^{(6)}_{i_1 \ldots i_6} \,. \end{aligned}$$

The IIB NS-5-brane couples to the 6-form $\tilde{B}$, the Poincaré dual of the NS-NS 2-form B in type IIB.

Worldvolume curvatures

$$\begin{aligned} \tilde{\mathcal{F}} &= 2\partial c^{(1)} + \frac{1}{2\pi\alpha'}C^{(2)} \\ \tilde{\mathcal{G}}^{(4)} &= 4\partial \tilde{c}^{(3)} + \frac{1}{2\pi\alpha'}C^{(4)} + \ldots \\ \tilde{\mathcal{G}}^{(6)} &= 6\partial \tilde{c}^{(5)} + \frac{1}{2\pi\alpha'}\tilde{B} + \ldots \end{aligned}$$

$c^{(1)}$ is dual to a 3-form $\tilde{c}^{(3)}$. In the linear approximation:

$$\tilde{\mathcal{G}}^{(4)}_{i_1 \dots i_4} = \frac{1}{2! \sqrt{|g|} \sqrt{1 + e^{2\varphi} C^{(0)2}}} g_{i_1 j_1} \cdots g_{i_4 j_4} \epsilon^{j_1 \dots j_6} \tilde{\mathcal{F}}_{j_5 j_6} ,$$

S-duality with the D-5-brane is achieved with no extra 3-form field.

S-duality
$c^{(1)} \longrightarrow$ Born-Infeld field
$\tilde{c}^{(5)} \longrightarrow c^{(5)}$ (Dynamical Tension of D5)

IIA NS-5-brane

Worldvolume Field	Field Strength	♯ of d.o.f
X^μ	–	4
$c^{(0)}$	$\mathcal{G}^{(1)}$	1
$a^{(2)}_{ij}$	$\mathcal{H}^{(3)}$	3
$\tilde{b}_{i_1 \ldots i_5}$	$\tilde{\mathcal{F}}^{(6)}$	–

Table 3: **Worldvolume fields of the IIA NS-5-brane.** The IIA NS-5-brane contains four embedding coordinates X^μ, a scalar $c^{(0)}$ and a selfdual 2-form $a^{(2)}$. The 5-form $\tilde{b}$ describes the tension of the 5-brane.

The action

$$\begin{aligned} S \;=\; & -T_{\mathrm{A5}} \int d^6\xi e^{-2\phi} \sqrt{|\det\left(g_{ij} - (2\pi\alpha')^2 e^{2\phi} \mathcal{G}^{(1)} \mathcal{G}^{(1)}\right)|} \times \\ & \times \left\{1 - \tfrac{1}{4\cdot 3!}(2\pi\alpha')^2 e^{2\phi} \mathcal{H}^{(3)\,2} + \ldots\right\} \\ & -(2\pi\alpha')\tfrac{1}{6!} T_{\mathrm{A5}} \int d^6\xi \epsilon^{i_1 \cdots i_6} \tilde{\mathcal{F}}^{(6)}_{i_1 \ldots i_6} \,, \end{aligned}$$

The IIA NS-5-brane couples to the 6-form $\tilde{B}$, the Poincaré dual of the NS-NS 2-form B in type IIA.

Worldvolume curvatures

$$\begin{aligned} \mathcal{G}^{(1)} &= \partial c^{(0)} + \frac{1}{2\pi\alpha'} C^{(1)} \\ \mathcal{H}^{(3)} &= 3\partial a^{(2)} + \frac{1}{2\pi\alpha'} C^{(3)} + \ldots \\ \tilde{\mathcal{F}}^{(6)} &= 6\partial \tilde{b} + \frac{1}{2\pi\alpha'} \tilde{B} + \ldots \end{aligned}$$

Poincaré Self-duality

$$\mathcal{H}^{(3)}_{i_1 i_2 i_3} = \frac{1}{3!\sqrt{|g|}} g_{i_1 j_1} \cdots g_{i_3 j_3} \epsilon^{j_1 \cdots j_6} \mathcal{H}^{(3)}_{j_4 j_5 j_6}$$

IIB KK-monopole

Worldvolume Field	Field Strength	♯ of d.o.f
X^μ	$-$	$10-6-(1)=3$
$\omega^{(0)}$	$\mathcal{K}^{(1)}$	1
$\tilde{\omega}^{(0)}$	$\tilde{\mathcal{K}}^{(1)}$	1
$\omega^{(2)}$	$\mathcal{K}^{(3)}$	3
$\tilde{\omega}^{(5)}$	$\tilde{\mathcal{K}}^{(6)}$	$-$

Table 4: **Worldvolume field content of the IIB KK-monopole.** In the case of the IIB KK-monopole, there are three embedding coordinates X^μ, after gauge fixing the Taub-NUT coordinate, two worldvolume scalars $\omega^{(0)}$, $\tilde{\omega}^{(0)}$, constituting a doublet under S-duality, a self-dual 2-form $\omega^{(2)}$ (S-selfdual) and a 5-form $\tilde{\omega}^{(5)}$, describing the tension of the KK-monopole and also S-selfdual.

The action

$$
\begin{aligned}
S \;=\;& -T_{\rm BKK}\int d^6\xi\; e^{-2\varphi}k^2\sqrt{|\det\left(\Pi_{ij}-(2\pi\alpha')^2k^{-2}e^{\varphi}\mathcal{K}^T M\mathcal{K}\right)|}\times \\
&\times\left\{1-\tfrac{e^{2\varphi}}{4\cdot 3!}(2\pi\alpha')^2k^2\left(\mathcal{K}^{(3)}\right)^2+\ldots\right\} \\
&-(2\pi\alpha')\tfrac{1}{6!}T_{\rm BKK}\int d^6\xi\,\epsilon^{i_1\ldots i_6}\tilde{\mathcal{K}}^{(6)}_{i_1\ldots i_6}\,.
\end{aligned}
$$

The IIB KK-monopole couples to a target space field $i_k\mathcal{N}$, where $\mathcal{N}$ is the Poicaré dual of the Killing vector as a 1-form k_μ.
The gauged sigma-model metric Π_{ij} is defined as for the IIA KK-monopole.
Here $\mathcal{K}^T M\mathcal{K}$ is the $SL(2,\mathbb{R})$-invariant combination:

$$
\mathcal{K}^T M\mathcal{K}=\begin{pmatrix}\mathcal{K}^{(1)} & \tilde{\mathcal{K}}^{(1)}\end{pmatrix}e^{\varphi}\begin{pmatrix}e^{-2\varphi}+C^{(0)2} & C^{(0)}\\ C^{(0)} & 1\end{pmatrix}\begin{pmatrix}\mathcal{K}^{(1)}\\ \tilde{\mathcal{K}}^{(1)}\end{pmatrix},
$$

Worldvolume curvatures

$$
\begin{aligned}
\mathcal{K}^{(1)} &= \partial\omega^{(0)}-\frac{1}{2\pi\alpha'}(i_kB)\\
\tilde{\mathcal{K}}^{(1)} &= \partial\tilde{\omega}^{(0)}+\frac{1}{2\pi\alpha'}(i_kC^{(2)})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{K}^{(3)} &= 3\partial\omega^{(2)} + \frac{1}{2\pi\alpha'}(i_k C^{(4)}) + \ldots \\
\tilde{\mathcal{K}}^{(6)} &= 6\partial\tilde{\omega}^{(5)} - \frac{1}{2\pi\alpha'}(i_k \mathcal{N}) + \ldots
\end{aligned}
$$

The Self-duality of the 2-form $\omega^{(2)}$ in the linear approximation takes the following form:

$$
\mathcal{K}^{(3)}_{i_1 \ldots i_3} = \frac{1}{\sqrt{|\det(\Pi_{ij})|}} \Pi_{i_1 j_1} \ldots \Pi_{i_3 j_3} \epsilon_{j_1 \ldots j_6} \mathcal{K}^{(3)}_{j_4 j_5 j_6} ,
$$

The action of the IIB KK-monopole is invariant under S-duality:

S-duality
$\omega^{(0)} \longrightarrow \tilde{\omega}^{(0)}$
$\tilde{\omega}^{(0)} \longrightarrow \omega^{(0)}$
$\omega^{(2)}$ S-selfdual
$\tilde{\omega}^{(5)}$ S-selfdual

The solitons occurring in this brane fit into representations of the S-duality group.

T-duality: IIB NS-5 $\longleftrightarrow$ IIA KK

Target-space T-duality

$$(i_k N)_{\mu_1 \cdots \mu_6} \longleftrightarrow -\tilde{B}_{\mu_1 \cdots \mu_6}$$

$$(i_k \tilde{B})_{\mu_1 \cdots \mu_5} \longleftrightarrow \tilde{B}_{\mu_1 \cdots \mu_5 z}\,.$$

Worldvolume T-duality

IIB NS-5-brane solitons	T-duality	IIA KK$-$monopole solitons
3-brane* $(2\|KK, NS5)$	$\frac{1}{2\pi\alpha'}Z \longleftrightarrow \omega^{(0)}$	$(3\|NS5, KK)$ 3-brane*
0-brane $(0\|D1, NS5)$	$c^{(1)} \longleftrightarrow \omega^{(1)}$	$(0\|D2, KK)$ 0-brane
4-brane $(4\|D5, NS5)$	$\tilde{c}^{(5)} \longleftrightarrow \omega^{(5)}$	$(4\|D4, KK)$ 4-brane

Table 5: T-duality. In this table we give the T-duality rules for the worldvolume fields. We give aswell the brane solitons that couple to each worldvolume field. The * indicates that they couple to the Poincaré dual of the field. These solitons can be seen as opening of intersecting branes. We also give the corresponding intersection.

T-duality is perfomed along a coordinate Z transverse to the 5-brane. This coordinate appears naturally gauged after the T-duality, so that the number of worldvolume dimensions does not change (this is a difference with respect to the D-brane case).

T-duality: IIA NS-5 $\longleftrightarrow$ IIB KK

Target-space T-duality

$$\bar{B}_{\mu_1 \cdots \mu_6} \longleftrightarrow \mathcal{N}_{\mu_1 \cdots \mu_6 z} \,.$$

Worldvolume T-duality

IIA NS-5-brane solitons	T-duality	IIB KK-monopole solitons
3-brane* $(3\|KK, NS5)$	$\frac{1}{2\pi\alpha'} Z \longleftrightarrow \omega^{(0)}$	$(3\|NS5, KK)$ 3-brane*
3-brane* $(3\|D4, NS5)$	$c^{(0)} \longleftrightarrow \tilde{\omega}^{(0)}$	$(3\|D5, KK)$ 3-brane*
1-brane $(1\|D2, NS5)$	$a^{(2)} \longleftrightarrow \omega^{(2)}$	$(1\|D3, KK)$ 1-brane

Table 6: T-duality. In this table we give the T-duality rules for the worldvolume fields. We give aswell the brane solitons that couple to each worldvolume field. The * indicates that they couple to the Poincaré dual of the field. These solitons can be seen as opening of intersecting branes. We also give the corresponding intersection.

T-duality is performed along a coordinate transverse to the 5-brane.

T-duality IIB NS-5 $\leftrightarrow$ IIA NS-5

T-duality between both type II 5-branes is achieved by a double dimensional reduction of both branes. We call Y the target space direction along which we reduce and we make it coincide with some worldvolume direction σ. The worldvolume directions are thus now split in the form $\mathrm{i} = (i, \sigma)$.

Target space T-duality

$$\tilde{\hat{B}}_{\mu_1 \cdots \mu_5 y} \longleftrightarrow \tilde{\hat{B}}_{\mu_1 \cdots \mu_5 y}$$

Worldvolume T-duality

IIB NS-5-brane solitons	T-duality	IIA NS-5-brane solitons
2-brane* $(2\|D3, NS5)$	$c_\sigma^{(1)} \longleftrightarrow c^{(0)}$	$(3\|D4, NS5)$ 3-brane*
0-brane $(0\|D1, NS5)$	$c_i^{(1)} \longleftrightarrow a_{i\sigma}^{(2)}$	$(1\|D2, NS5)$ 1-brane
2-brane $(2\|D3, NS5)$	$\tilde{c}_{ij\sigma}^{(3)} \longleftrightarrow a_{ij}^{(2)}$	$(1\|D2, NS5)$ 1-brane

Table 7: T-duality. In this table we give the T-duality rules for the worldvolume fields. We give aswell the brane solitons that couple to each worldvolume field. The * indicates that they couple to the Poincaré dual of the field. These solitons can be seen as opening of intersecting branes. We also give the corresponding intersection.

Notice that although the KKA-monopole is T-dual to the IIB 5-brane with one 1-form $c^{(1)}$, the IIA 5-brane is dual to the IIB 5-brane in a "1-3-form" formalism. A Poincaré duality on the 3-form $\tilde{c}^{(3)}$ is needed to recover the IIB 5-brane in the 1-form formalism.

T-duality IIA KK $\leftrightarrow$ IIB KK

T-duality between both type II KK-monopoles is achieved by a double dimensional reduction of both branes. We call Y the target space direction along which we reduce and we make it coincide with some worldvolume direction σ. The worldvolume directions split in the form $\hat{i} = (i, \sigma)$. The gauged isometry direction is kept the same in both monopoles.

Target space T-duality

$$(i_k N)_{\mu_1 \ldots \mu_5 y} \longleftrightarrow (i_k \mathcal{N})_{\mu_1 \ldots \mu_5 y}$$

Worldvolume T-duality

IIA KK-monopole solitons	T-duality	IIB KK-monopole solitons
3-brane* $(3\|NS5, KK)$	$\omega^{(0)} \longleftrightarrow \omega^{(0)}$	$(3\|NS5, KK)$ 3-brane*
2-brane* $(2\|D4, KK)$	$\omega^{(1)}_i \longleftrightarrow \tilde{\omega}^{(0)}$	$(3\|D5, KK)$ 3-brane*
0-brane $(0\|D2, KK)$	$\omega^{(1)}_i \longleftrightarrow \omega^{(2)}_{i\sigma}$	$(1\|D3, KK)$ 1-brane
2-brane $(2\|D4, KK)$	$\omega^{(3)}_{ij\sigma} \longleftrightarrow \omega^{(2)}_{ij}$	$(1\|D3, KK)$ 1-brane

Table 8: T-duality. In this table we give the T-duality rules for the worldvolume fields. We give aswell the brane solitons that couple to each worldvolume field. The * indicates that they couple to the Poincaré dual of the field. These solitons can be seen as opening of intersecting branes. We also give the corresponding intersection.

The worldvolume solitons of the KK-monopoles couple to worldvolume curvatures of the form $\partial\omega + \frac{1}{2\pi\alpha'}(i_k C) + \ldots$, hence these solitons can be interpreted as boundaries of branes that couple to C and have one direction wrapped around the Taub-NUT isometry. T-duality exchange these solitons between the KK-monopoles.