



Planckian Scattering as a Holographic Field Theory

**The Scattering Matrix Approach to the
Quantum Black Hole**

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What is the S-matrix Ansatz about ?

- In-falling particles interact gravitationally with Hawking radiation [t Hooft '96]
- The interaction takes place by means of a shock-wave
- Outgoing trajectories get shifted by a factor proportional to the in-going momentum
- This provides a mechanism to recover the information
- From this one can construct a microscopical S-matrix, which equals the Veneziano amplitude!

$$S = \langle \text{out} | \text{in} \rangle = \int \mathcal{D}X \exp iI[X]$$

where

$$I[X] = \int d^2\sigma \frac{1}{8\pi G} (\partial_i X_\mu \partial^i X^\mu + B_{\mu\nu} W^{\mu\nu})$$

Quantisation

- Quantum consistency requires [de Haro '97]:

$$[X(\tilde{\sigma}), X(\tilde{\sigma}')] = -4\pi i G F^{\mu\nu}(\sigma - \sigma')$$

where

$$F^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} \epsilon^{ij} \partial_i X_\alpha \partial_j X_\beta f(\tilde{\sigma} - \tilde{\sigma}')$$

together with

$$[X(\tilde{\sigma}), P(\tilde{\sigma}')] = i(g^{\mu\nu} + A^{\mu\nu}) \delta(\tilde{\sigma} - \tilde{\sigma}')$$

$$A^{\mu\nu} = O\left(\frac{\ell^2_{\text{Pl}} p_{\text{in}}}{b}\right)$$

This gives a generalised Heisenberg uncertainty principle due to the gravitational interaction (momentum transfer) [de Haro '98]

- This model goes beyond the eikonal approximation

Holographic field theory

- We have a **two-dimensional field theory**
- The **Hilbert space is determined by the state**

$$|X^\mu(\tilde{\sigma})\rangle \quad \text{or} \quad |P^\mu(\tilde{\sigma})\rangle$$

- The **in- and out-Hilbert spaces are identical**

$$|P^\mu_{\text{in}}\rangle = |P^\mu_{\text{out}}\rangle$$

Holographic field theory!

In components, e.g.:

$$[u(\tilde{\sigma}), p_i(\tilde{\sigma}')]=i\partial_i u \delta(\tilde{\sigma}-\tilde{\sigma}')$$

Latin: transverse index (WS)

Greek: longitudinal index (TS)

$$[p_\alpha(\tilde{\sigma}), p_i(\tilde{\sigma}')]=ip_\alpha(\tilde{\sigma}')\partial_i \delta(\tilde{\sigma}-\tilde{\sigma}')$$

$$[p_i(\tilde{\sigma}), p_j(\tilde{\sigma}')]=ip_i(\tilde{\sigma}')\partial_j \delta(\tilde{\sigma}-\tilde{\sigma}') + ip_j(\tilde{\sigma})\partial_i \delta(\tilde{\sigma}-\tilde{\sigma}')$$

$$[p^{\text{in}}_i(\tilde{\sigma}), P^{\text{out}}_j]=i\partial_j p^{\text{in}}_i(\tilde{\sigma}) \quad \text{etc.}$$

$$P_\mu = \int d^2\tilde{\sigma} \, p_\mu(\tilde{\sigma})$$

SO(2,1) algebra for 2+1 scattering

- Reduction to 2+1 dimensions

Define "link variables"

$$a^\mu{}_A \equiv \int_A d\sigma \partial_\sigma x^\mu = x^\mu(A_l) - x^\mu(A_0)$$

They satisfy

$$G_3 R_3 \equiv G_4$$

$$[a^\mu{}_A, a^\nu{}_B] = i R_3 G_3 \epsilon^{\mu\nu\alpha} a^\alpha{}_A \delta_{AB}$$

SO(2,1) algebra

- Generalised Heisenberg relation:

$$[a^\mu, p^\nu] = i(g^{\mu\nu} - R_3 G_3 \epsilon^{\mu\nu\alpha} p_\alpha)$$

Conclusions

- There is, at least at this very simple level, a way to recover the information and construct an S-matrix
- The S-matrix is related to string theory. The amplitude of the scattering is the Veneziano amplitude
- Quantum mechanical consistency requires the coordinates to satisfy a non-commutative algebra. Heisenberg's uncertainty principle is modified by a factor proportional to Newton's constant
- The model is a holographic field theory. The in- and out-Hilbert spaces are identical
- In 2+1 dimensions, the algebra between the coordinates reduces to the $SO(2,1)$ algebra

References

- [t Hooft 96] Gerard 't Hooft, *Int. J. Mod. Phys. A* 11 (1996) 4623, gr-qc/9607022
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