High energy scattering of $D0$ branes in SUGRA.[1]

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Abstract

We study scattering of $D0$ branes in eleven dimensional supergravity (SUGRA) using the tree level four point amplitude. The range of validity of SUGRA allows us to make reliable calculations of relativistic brane scattering, inaccessible in string theory because of halo effects. We also compare the rate of annihilation of $D0-\bar{D}0$ branes to the rate of elastic $D0-\bar{D}0$ scattering and find that the former is always smaller than the latter. We also find a pole in $\sigma_{D0-\bar{D}0}$. We exploit the analogy with the positronium to argue that the brane anti-brane pairs form branium atoms in 3 spatial dimensions. We also derive a long range effective potential for interacting branes which explicitly depends on their polarizations. We compare two approaches to large impact parameter brane scattering: in GR, polarizations of the branes are kept constant during the interaction, while in QFT, one sums over all possible polarizations. We show that the GR and QFT approaches give the same answer.

1 Introduction

The purpose of this paper is threefold: Firstly, we use Witten's idea [1] that $D0$ branes are just Kaluza-Klein gravitons of the eleven dimensional supergravity to study the brane interaction as a problem of tree level quantum gravity. We explicitly include brane polarizations in our calculations.

Secondly, we scatter $D0-\bar{D}0$ for different values of momenta. At small momentum we probe distances much larger than the Plank scale and cross

[1]This is a short version of hep-th/9805202

sections vanish. As we increase the momentum the brane annihilation catches up with elastic scattering and we expect it to become dominant near the Planck scale where SUGRA description breaks down. Thirdly we study long range brane scattering and find that the functional form of the amplitude remains the same for $D0 - D0$ brane scattering for small and for large incoming transverse momenta. As expected, the $D0 - \bar{D}0$ interaction levels off with the $D0 - D0$ interaction for large momenta.

2 Historical digression on Quantum Gravity.

Our task is greatly simplified by Sannan's [2] calculation of four-point amplitude in quantum gravity which is independent of the number of dimensions. Since the success of any given quantum gravity calculation is limited, it is hard to trace earlier attempts to quantize gravity. The father of "modern" quantum gravity, Richard Feynman, [3] refused to publish his results for 10 years. It had been 7 years and 30 citations after Bryce DeWitt [4] published three and four graviton vertex functions when Berends and Gastmans [5] found an error in his 360 term four graviton vertex function. Berends and Gastmans, as well as Feynman, used computers to make algebraic calculations. It is curious to note that Sigurd Sannan, 20 years and 200 citations later, found one more mistake in DeWitt's original calculation. Sannan's graviton - graviton scattering amplitude (calculated by hand) contains 150 terms (it would take four pages size of Physical Review to write it down)

Before we undertake the study of graviton scattering, we should try to understand how it may depend on the polarization. The graviton polarization itself is not very well defined in the literature. Killing symmetry becomes a gauge symmetry for linearized GR. Graviton self interaction terms are not gauge invariant. Also the whole concept of perturbation theory is not defined since the Minkowski metric around which we perturb is a pure gauge from the point of view of gauge theory. However a good argument can be made from the GR point of view. Let us consider an approximate solution to the Einstein's equation

$$g_{\mu\nu} = \eta_{\mu\nu} + e_{\mu\nu} e^{ikx} \tag{1}$$

where

$$e_{\mu\nu} e^{\mu\nu} << 1 \tag{2}$$

The energy density of this configuration is $T_{00} \sim E^2 e^2 / l_p^8$. Now let us impose a self-consistency condition on (2): we want this energy density when put on the right hand side of the Einstein's equation to create gravitons with polarization of the same magnitude as the original one. The "mean field" equation is

$$e \sim e^2/(Er)^2 \tag{3}$$

From which it follows that $rE >> 1$. The limit of linearized GR where any graviton interactions, including that of the spin flip , are suppressed is just Born's approximation.

3 Long Range Brane Interactions.

In this section we find physical limits of Sannan's 150 term amplitude which reduce it to 6 terms. A way to proceed is to remember that the completeness relation for polarizations of the photon in QED gets drastically simplified once we choose its space momentum along one of the coordinate axes. Since the completeness relation is proportional to a photon propagator, one finds great simplifications in the photon scattering amplitudes as well. A natural way to single out a space direction is to consider scattering of branes with small transverse momentum q. In this limit the natural dimensionless parameter is qR_{10}. For small qR_{10}, we are dealing with forward scattering in the compact direction. For small $\frac{1}{qR_{10}}$ it is a forward scattering in a non-compact direction. Since in both limits the momentum exchange is infinitesimal, it is not surprising that we get the same functional form for the amplitude. The details of the construction are as follows:

The graviton has $d(d-3)/2$ different polarizations in d dimensions. In d dimensions it is exactly the number of independent components in the traceless symmetric matrix $d-2$ by $d-2$. For a $D0$ or $\bar{D}0$ brane "moving" only in the compact 10-th direction, the 11-momentum is

$$k^m = (E, 0, 0, ..0, \pm 1/R_{10}) \tag{4}$$

We can choose the polarization as the following matrix

$$\begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & & & & 0 \\ \vdots & & & & \vdots \\ 0 & & & & 0 \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix} \tag{5}$$

, where all the empty spaces inside of (5) correspond to some arbitrary numbers. At this stage, if we have four different gravitons, they may all have different polarizations, corresponding to different matrix elements of (5). However, the 11-momentum (4) of i-th graviton is transverse to the polarization of the j-th graviton.

$$k^i_m e^j_{mn} = 0 \tag{6}$$

This observation is technically very important since it reduces Sannan's amplitude (formula (3.16) of [2]) to only six terms. Now we can make a rotation in the $x_{10}x_9$ plane and allow "a little bit of momentum" in the non-compact directions. The matrix (5) will receive corrections of order $\sim R_{10}q$ where q is the magnitude of non-compact momentum. As long as qR_{10} is small the leading order terms of the amplitude expansion in terms of qR_{10} will contain only six terms as mentioned above. In other words, for small qR_{10}, we can approximately take the 11-momentum of i-th graviton to be transverse to the polarization of the j-th graviton. This corresponds to non-relativistic brane scattering. Just before we list the advertised 6-term amplitude, we should mention yet another possibility. Let us start with four gravitons which have momentum in some non-compact direction and none in the compact. These gravitons will have polarization matrices of the form (5), satisfying mutual transversality condition (6). Now, let us make a rotation to allow a little bit of compact momentum $1/R_{10}$. It corresponds to having small corrections to the amplitude of order $1/(qR_{10})$. Our branes are now fully relativistic. The convenient choice of polarization has allowed us to get the following amplitude for small and large values of non-compact momenta:

$$T = -1/4\kappa^2\left(\frac{st}{u}(e_1e_3)(e_2e_4) + \frac{su}{t}(e_1e_4)(e_2e_3) + \frac{tu}{s}(e_1e_2)(e_3e_4)\right) - \\ 1/2\kappa^2\left(s(e_1e_4e_2e_3) + t(e_1e_2e_4e_3) + u(e_1e_2e_3e_4)\right) \tag{7}$$

We use Sannan's notation with

$$S = \frac{2}{\kappa^2}\int d^{11}x\sqrt{-g}R \tag{8}$$

The reason why for obtaining the same functional form of the amplitude for ultra relativistic and non-relativistic is a simple one: Choice of polarization (5) allows us to treat gravitons as particles of mass $1/R_{10}$ in one limit and of mass q in the other.

For the large impact parameter scattering, the t channel dominates and in the limit $t \to 0$, we get the the following amplitude

$$T_{longrange} = -1/4\kappa^2 \frac{su}{t}(e_1e_4)(e_2e_3) \tag{9}$$

For large impact parameter, the θ-deflection angle, is small and the amplitudes become

$$T_{D0-D0} = -\frac{4\kappa^2 q^4}{q^2\theta^2}(e_1e_4)(e_2e_3) \tag{10}$$

$$T_{D0-\bar{D}0} = -\frac{4\kappa^2 E^4}{q^2\theta^2}(e_1e_4)(e_2e_3) \tag{11}$$

For small momenta q the $D0 - \bar{D}0$ scattering dominates that of $D0 - D0$ and for large momenta they are equal. This is what we could have expected since fast moving branes which have an impact parameter so large that they almost do not deflect are charge blind. As discussed in previous chapters, from GR point of view, polarizations of a brane should be set equal before and after the scattering. Alternatively, one can think that polarizations of branes might change during the interaction but our picture of their interaction is a bit fuzzy which corresponds to averaging over all polarizations. In this slightly rotated reference frame, the completeness relation is very simple:

$$\sum_{p=1}^{44} e^p_{ij} e^p_{kl} = \frac{1}{88}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{9}\delta_{ij}\delta_{kl}) \tag{12}$$

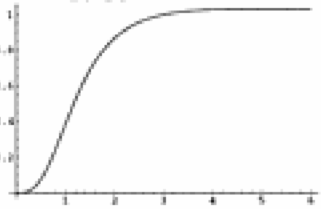
, where indices $i, j = 1, 2, ...9$ run over the "inside" of the matrix (5). The averaging is done with the aid of the following formula

$$\langle (e_1e_4)(e_2e_3)\rangle = \sqrt{(e_1e_4)^2(e_2e_3)^2}/2 \tag{13}$$

where factor of two appears to compensate for over counting, since nothing is changed if we interchange polarizations of first and fourth branes with that of the second and third. With the use of (12) we find that the right hand side of (13) is 1. Therefore averaging over polarizations gives the same result as keeping the polarization of a given brane the same before and after the interaction.

Let us note that one can easily derive higher order corrections to the effective potential from the all-channel amplitude which would give, in general, higher powers of $1/r$ with appropriate powers of momenta. Let us stress

Figure 1: $\frac{\sigma_{annihilation}}{\sigma_{D0-D0}}$ versus q for $R_{10} = 1$



again that loop corrections to the effective potential are suppressed as long as r is larger than the Planck scale. To be more precise let us consider the quantum gravity action

$$S \sim \int d^{11}x\sqrt{-g}(R + l_p^2 R^2 + \ldots) \tag{14}$$

For long distance interactions, the t channel dominates and the amplitude from the first term goes as $T \sim \frac{1}{k^2}$, where $k = 2q\sin\theta/2$ is (a very small) exchanged momentum. Since loop corrections to the GR actions go roughly as $\int \frac{dk^{10}}{k^2+t}$ the quantum correction to the amplitude is $l_p^2 + k^2 + O(k^4)\ldots$ Therefore in the limit of forward scattering $Er >> 1$, $E << m_P$, quantum gravity corrections are totally negligible.

4 Conclusion

Let us give an overview of our results. We started the paper by studying large impact parameter brane scattering. The amplitude for the arbitrary polarization is extremely complicated and we limited ourselves to almost forward scattering which is the only limit where quantum gravity can be treated perturbatively. We found that in the limits of very fast and very slow moving branes the amplitude took the same functional form. We also explained why polarization of the branes should not change during the interaction, at least to leading order in the large impact parameter expansion. On the other hand one can get the same answer by considering repeated brane scattering for different polarizations and then taking the average. We derived an effective potential for brane interactions with the aid of the retarded Green's function and matching of the QFT and Quantum Mechanical scattering amplitudes.

We also discussed the possibility for the formation of the brane -anti-brane atom and by analogy with positronium we suggested that it might be possible only in 3 spatial dimensions. We have derived cross sections for brane -anti-brane pair creation from two gravitons as well as for the annihilation of the brane anti-brane pair. Consistent with the understanding that supergravity is a long distance limit of M theory, the cross-section for brane creation is of the same order of magnitude as that for elastic brane scattering, which essentially means that we are in the regime of mostly elastic scattering.

References

[1] E. Witten, Nucl. Phys. B443 (1995) 85

[2] S. Sannan, Phys. Rev. D34 (1986) 1749

[3] R. Feynman, Acta Physica Polonica 24 (1963) 697; J.R. Klaudered., Magic without magic, Freeman and Co (1972),pp 355 and 377; C. Sykes, No ordinary genius: the illustrated Richard Feynman, Norton (1994)

[4] B. DeWitt, Phys. Rev. 162 (1967) 1239

[5] F. Berends and R. Gastmans, Nucl. Phys. B88 (1975) 99

[6] V. Berestetskii, E. Lifshitz, L. Pitaevskii, Quantum Electrodynamics, Pergamon Press (1982)