

Duality Symmetry in the Schwang. Sen Model

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Consider the BRST formulation of Maxwell theory. The effective action is given by

$$S_{eff} = \int d^4x (\pi_\mu \dot{A}^\mu + \dot{\rho} \bar{c} + \dot{c} \bar{\rho} - H_0 - \{ \Psi, Q \}),$$

where π_μ is the conjugate momentum of A_μ , c and \bar{c} are ghosts, and ρ and $\bar{\rho}$ their conjugate momenta. The BRST charge Q and Hamiltonian H_0 are given by

$$Q = \int d^3x (\partial_i \pi^i c - i \rho \pi_0),$$

$$H_0 = - \frac{1}{2} (\pi^i \pi_i + B^i B_i)$$

The gauge fixing function which selects the Coulomb gauge is

$$\Psi = \frac{i \bar{c} \partial_i A^i}{\epsilon} + \bar{\rho} A_0$$

where we let $\epsilon \rightarrow 0$ after the redefinitions $\pi_0 \rightarrow \epsilon \pi_0$ and $\bar{c} \rightarrow \epsilon \bar{c}$.

We then get

$$S_{eff} = \int d^4x (\pi_i \dot{A}^i + A_0 \partial_i \pi^i + \pi_0 \partial_i A^i - H_0 + \dot{c} \bar{\rho} + i \bar{\rho} \dot{\rho} - i \bar{c} \nabla^2 c).$$

At this level there is a duality transformation of the fields given by ⁽²⁾

$$\delta A_i = \theta \nabla^{-2} \epsilon_{ijk} \partial^j \Pi^k, \quad (1)$$

$$\delta \Pi_i = \theta \epsilon_{ijk} \partial^j A^k,$$

and the variations of the remaining fields being zero. This is the non-local duality transformation introduced by Deser and Teitelboim [1].

The generating functional for the Maxwell theory is

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\Pi^i \mathcal{D}c \mathcal{D}\bar{c} \mathcal{D}P \mathcal{D}\bar{P} e^{iS_{\text{eff}}}.$$

Integrating in Π_0 and A^0 produces the usual delta functions which characterize the Coulomb gauge. The integrations over P and \bar{P} are trivial and at the end we get

$$Z = \int \mathcal{D}A_i \mathcal{D}\Pi_i \mathcal{D}c \mathcal{D}\bar{c} \delta(\nabla \cdot A^i) \delta(\nabla \cdot \Pi^i) e^{iS_{\text{eff}}},$$

$$S_{\text{eff}} = \int d^3x (\Pi_i \dot{A}^i - H_0 - i\bar{c} \nabla^2 c). \quad (2)$$

We will now show that the Schwinger-Sen formulation of Maxwell theory [2] can be obtained by requiring that the duality transformation (2) be local. To this end we introduce two auxiliary fields C_i and P_i so that

$$\delta A_i = \theta C_{iT}, \quad \delta \Pi_i = \theta P_{iT}$$

where the index T indicates the transversal part of the field as required by the Coulomb gauge condition on A_i and Π_i . These fields are fixed through the equations

$$\nabla^2 C_{iT} = \epsilon_{ijk} \partial^j \Pi^k, \quad P_{iT} = \epsilon_{ijk} \partial^j \Lambda^k$$

and the transversality conditions $\partial^i C_i = \partial^i P_i = 0$. We then get

$$\delta C_i = -\theta \Lambda_i, \quad \delta P_i = -\theta \Pi_i$$

which allow us to identify Λ_i and C_i with the Schwarz-Sen potentials $A_i^a, a=1,2$.

The Schwarz-Sen action can be obtained if we notice that

$$P^i \dot{C}_i = -\Lambda^i \dot{\Pi}_i, \\ \epsilon^{ijk} \nabla^2 \partial_j C_{kT} = -\nabla^2 \Pi^i$$

Then (2) can be written as

$$S_{eff} = \int d^3x \left(\frac{1}{2} \Pi_i \dot{\Lambda}^i + \frac{1}{2} P_i \dot{C}^i - H_0 - i\bar{c} \nabla^2 c \right) \quad (3)$$

where now

$$H_0 = \frac{1}{2} \left[(\vec{\nabla}_x \vec{c})^2 + (\vec{\nabla}_x \vec{\Lambda})^2 \right].$$

Up to the ghost term, eq. (3) is the Schwarz-Sen action in Hamiltonian form. Although our discussion already demonstrates the equivalence of the two approaches it is possible to prove that the generating functional Z is equal to the corresponding functional obtained using the BV formalism for the Schwarz-Sen action. [5].

After the introduction of the auxiliary fields C_i and P_i , Z can be written as

$$Z = \int \mathcal{D}\Lambda_i \mathcal{D}\Pi_i \mathcal{D}C_i \mathcal{D}P_i \mathcal{D}c \mathcal{D}\bar{c} \delta(C_{iT} - \nabla^{-2} \epsilon_{ijk} \partial^j \Pi^k) \times \\ \times \delta(P_{iT} - \epsilon_{ijk} \partial^j \Lambda^k) \delta(\partial_i \Lambda^i) \delta(\partial_i \Pi^i) \delta(\partial_i C^i) \delta(\partial_i P^i) e^{iS_{eff}}$$

Now we can write

$$\delta(\epsilon_{i\tau} - \nabla^{-2} \epsilon_{ijk} \partial^j \pi^k) = \det^{-1}(\nabla^{-2} \epsilon_{ijk} \partial^j) \delta(\pi_i + \epsilon_{ijk} \partial^j C^k)$$

$$= \det(\epsilon_{ijk} \partial^j) \delta(\pi_i + \epsilon_{ijk} \partial^j C^k)$$

and insert it in the previous equation for Z.

On the other side, the Schwarz-Sen action has first and second class constraints, given by

$$\Omega_0^a = \pi_0^a, \quad \Omega^a = \partial_i \pi^{ia}$$

$$\Omega_{i\tau}^a = \pi_{i\tau}^a + \frac{1}{2} \epsilon_{ab} \epsilon_{ijk} \partial^j A_{k\tau}^b$$

respectively. In the gauge $A_0^a = \partial_i A^{ia} = 0$ the generating functional for the Schwarz-Sen action is given by

$$\bar{Z} = \int DA_i^a D\pi_i^a \delta(\partial^i A_i^a) \delta(\partial^i \pi_i^a) \det(\nabla^2) \det^{1/2}(\epsilon_{ab} \epsilon_{ijk} \partial^j) \times$$

$$\times \delta(\pi_{i\tau}^a + \frac{1}{2} \epsilon_{ab} \epsilon_{ijk} \partial^j A_{k\tau}^b) e^{i\bar{S}_{eff}}$$

$$\bar{S}_{eff} = \int d^4x \left(\pi_i^a \dot{A}_a^i - \frac{1}{2} (\vec{\nabla} \times \vec{A})^2 \right),$$

where the trivial sector A_0^a and π_0^a has already been integrated out. We see that the $\det \nabla^2$ factor in \bar{Z} also arises in Z due to the ghost contribution. Moreover, as $\det(\epsilon_{ab} \epsilon_{ijk} \partial^j) = \det^2(\epsilon_{ijk} \partial^j)$ we see that Z and \bar{Z} are identical.

We have thus shown the equivalence, at the quantum level, of Schwarz-Sen and Maxwell theories. This was expected as both theories are free and classically equivalent. The important point is that the Schwarz-Sen model can be understood as a way to implement duality as a local symmetry in Maxwell theory.

References

[1] S. Deser and C. Teitelboim, Phys. Rev. D 13 (1976) 1592.
 [2] J.H. Schwarz and A. Sen, Nucl. Phys. B 411 (1994) 35.
 [3] H.O. Ginotti, M. Gomes, V.O. Rivello and A.J da Silva, Phys. Rev. D 56 (1997) 6615.