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- HOLOGRAPHY VS. EXTENSIVITY IN THE CONTEXT OF THE AdS/CFT CORRESPONDENCE

- REVIEW OF $O(N^2)$ RESULTS.

- STUDY OF $O(N^0)$ CASE.

  ISSUES AND METHOD

- SEARCH FOR STRINGY INFORMATION – HAGEDORN ISSUES.

MALDACENA,
{ GUBSER, KLEBANOV, PEET, TSEYTLIN
GUBSER, POLYAKOV, KLEBANOV
WITTEN
DOUGLAS, POLCHINSKI, STROMINGER

FIND $d_{effective}$ BY HEATING THE SYSTEM

$$\underset{m_{KK}}{M} \ll T \ll \underset{m_s}{\Lambda_{UV}}$$

$$VT^{D-1} \gg 1$$

$$\frac{F}{TV} \xrightarrow[T \text{ LARGE}]{} -n_{eff}\, T^{d_e - 1}$$

A CONDITION FOR HOLOGRAPHY!

$$d_e < D$$

$$S = \sum_{i=0}^{\infty} (N^2)^{1-i} f(\lambda_s N)$$

$$\lambda_s N \gg 1$$

ENTROPY AS A FUNCTION OF T^{d_c-1}

$(AdS)_{p+2} \times E_{D-p}$ $\qquad$ D/p

ORDER N^2 $\quad$ $\xrightarrow[T_0]{0 \quad T^p}$ T $\qquad$ $\xrightarrow[T_0]{0 \quad GW \quad HCFT}$ T $\quad T^p$

CLASSICAL $\qquad$ QUANTUM

ORDER N^0 $\quad$ $\xrightarrow[T_0]{T^?}$ T $\qquad$ T^p

QUANTUM

EXAMINE:

1. THE ROLE OF AdS
2. THE ROLE OF GRAVITY
3. VALIDITY OF THE CORRESPONDENCE TO ORDER N^0

A POSSIBILITY OF SEVERAL MASTERFIELDS

A ROLE FOR GRAVITATIONAL "DYNAMICS"

∂X

X_1

$\vdots$

X_n

$$Z_{CFT} \cong \sum_{i=1}^{n} \exp\left(-\hat{I}_{SUGRA}(x_i)\right)$$

SEVERAL STRING BACKGROUNDS MIXED ?

WITTEN
HAWKING-PAGE

$O(N^2)$ - REVIEW

CLASSICAL SUGRA / PLANAR CFT

CFT ON

A $p+1$ WORLD VOLUME $S^1_\beta \times S^p = \partial X_i$

CORRESPONDS TO SUGRA ON:

$$X_1 = \frac{(AdS)^+_{p+2}}{Z} \times M_{D-p-2} \; ; \; X_2 = (AdS)^{BH}_{p+2} \times M_{D-p-2}$$

TOPOLOGY: $S^1 \times R^{p+1} \times M_{D-p-2}$ $\qquad$ $R^2 \times S^p \times M_{D-p-2}$

$t \equiv t + \beta_0$

ANY β_0

$$T_{min} = \frac{\sqrt{(p+1)(p-1)}}{2\pi b}$$

$b = c\, l_p\, N^\delta$

$\delta = \frac{1}{4}$ D3

$\delta = \frac{1}{3}$ (2,0)

$\delta = \frac{1}{6}$ M2

$$ds^2(X_1) = \left(1+\frac{r^2}{b^2}\right)dt^2 + \frac{dr^2}{1+\frac{r^2}{b^2}} + r^2 d\Omega_k + b^2 ds^2(M)$$

THE SAME b

$U(1), O(1), SU(2) \ldots$

RED-SHIFTS

$$\beta(r) \xrightarrow[r\to\infty]{} \frac{\beta_0 r}{b} \qquad R_{sh}(r) \xrightarrow[r\to\infty]{} r$$

$$\frac{R}{\beta} \xrightarrow[r\to\infty]{} \frac{b}{\beta_0}$$

$$ds^2(X_2) = \mu(r) + \frac{dr^2}{\mu(r)} + r^2 d\Omega_k + b^2 ds^2(M)$$

$$g_{tt} = \mu(r) = \frac{r^2}{b^2} + 1 - \frac{r_0^{d-2}}{r^{d-2}}\left(\frac{r_0^2}{b^2}+1\right)$$

$$\beta_0 = \frac{4\pi b^2 r_0}{r_0^2(k+1)+b^2(k-1)}$$

$$r_0^{min} = b\sqrt{1-\frac{2}{k+1}} \qquad \beta^{max} = \frac{2\pi b}{\sqrt{(k+1)(k-1)}}$$

$T\to\infty \quad k \text{ "}\to\text{" } \infty$

REGULATE

$$I(x_i)_{cl} = N^{\delta(D-2)} \frac{2C^{D-2}}{(2\pi)^{D-3}} \underbrace{|\Omega_k|}_{S^k} \underbrace{|\Omega_M|}_{S^{D-k-2}} \frac{\beta_0}{f^{k+2}} \left(r_{max}^{k+1} - r_{min}^{k+1} \right)$$

?

$\beta_0 > \beta_{max}(k)$

ONLY x_1 EXISTS

$r_{min}^{cl.}(x_1) = 0$

$I(x_1) \sim \beta_0$ "VACUUM ENERGY"

SUBTRACT

$S = \beta_0 \frac{\partial}{\partial \beta_0} I - I = 0$ $O(N^2)$

GW PHASE ON THE CFT SIDE

$\beta_0 < \beta_{max}$

BOTH X_1 AND X_2 CONTRIBUTE

$$I(X_2) - I(X_1) = N^{\delta(D-2)} \frac{C^{D-2}}{(2\pi)^{D-3}} \Omega_k \Omega_M b^{-k-2} \beta_0$$

$$\left(b^2 r_0^{k-1} - r_0^{k+1} \right)$$

$(r_0^{min} \triangleq) r_0 < b$

$\frac{2\pi b}{k-1} < \beta_0 < \beta_{max}$

X_1 DOMINATES

$S \sim 0$

$r_0 > b$

$\beta_0 < \frac{2\pi b}{k-1}$

X_2 DOMINATES

HIGH TEMPERATURES AS WELL

FROM X_2 DOMINANCE

$$\beta F(\beta)_{PLANAR} = -N^{\delta(D-2)} C_{k+1} \left(\frac{R}{\beta}\right)^k$$

$r_{min} = r_0$ IS β - DEPENDENT

THUS $S \neq 0$

$$S(X_2)_{\substack{\text{PLANAR}\\ \text{CLASSICAL}}} = N^{\delta(D-2)} \, c_{p+1} \left(\frac{R}{\beta}\right)^{p}$$

$$\sim N^{\delta(D-2)/(p+1)} \left((E-E_{vac})R\right)^{\frac{p}{p+1}}$$

$p=3 \quad D=10 \quad \delta = \frac{1}{4}$

$$S(X_2) \sim N^2 V T^3$$

HERE WAS HOLOGRAPHIC VS. O
MICRO COUNTING

NEXT THE ONE LOOP QUANTUM CORRECTION

ONE LOOP SUGRA – $\frac{1}{N}$ CFT CORRECTIONS

WORLD VOLUME EXTENSIVE

$$\beta F(\beta, X) = I(X)_{1\text{-loop}} = \frac{1}{2} \mathrm{Tr} \log (I'')_X$$

$(I'')_X$ – THE COMPLETE FLUCTUATIONS KERNEL

EXTENSIVE D-1 VS. HOLOGRAPHIC p

WILL ONE OBSERVE D AT HIGH T ?

N DEPENDENCE ON SUGRA SIDE INCLUDED IN

THE BACKGROUND – NOT IN THE NUMBER OF

SPECIES. (QUARKS GLUONS VS. HADRONS)

HOROWITZ

OOGURI

GIBBONS, PERRY ; DOWKER, KENNEDY

THE METHOD OF EVALUATING βF

FOR HIGH T THERE IS A WKB LIKE APPROXIMATION FOR THE FREE ENERGY.

$$\beta F(\beta)_{WKB} = \int d^D x \sqrt{g}\, f(X_i, \beta_{local})$$

$$\beta_{local} = \beta_0 \sqrt{g_{00}}$$

LOCAL RED SHIFTED TEMPERATURE

CELLS OF LOCAL T

LOCAL FLAT

THE FLAT-SPACE FREE ENERGY DENSITY FOR A FIELD OF MASS m

$$f(\beta, m)_D = -n_{D.O.F} \frac{\Gamma(D/2)}{\Gamma(D)\pi^{D/2}} \beta^{-D} \int_{\beta m}^{\infty} dz \frac{(z^2 - \beta^2 m^2)^{\frac{D-1}{2}}}{e^z - (-1)^F}$$

$$f(\beta, m)_D \underset{m\beta \gg 1}{\sim} -\beta^{-D} (\beta m)^{\frac{D-1}{2}} \exp(-\beta m) + O(e^{-2m\beta})$$

m IS SOME MASS-GAP

$$m_s \sim \frac{1}{\ell_s} \quad ; \quad m_{KK} \sim \frac{1}{R_{KK}}$$

A KK THRESHOLD IS REACHED WHEN:

$$\beta_{local}\, m^{KK}_{local} < 1$$

OTHERWISE $D_{effective}$ DROPS.

WHAT REMAINS IS THE "MASSLESS"

FREE ENERGY IN AN EFFECTIVE VOLUME

$$\beta F(\beta)_{WKB} = -C_{D_{eff}} \int d^{D_{eff}} x \sqrt{g} \left(\beta \sqrt{g_{00}}\right)^{-D_{eff}}$$

$$= -C_{D_{eff}} \beta^{-D_{eff}} \tilde{Vol}(X_{eff})$$

$\tilde{Vol}(X_{eff})$ IS THE VOLUME OF X IN

THE "OPTICAL" METRIC

$$\tilde{g}_{\mu\nu} = \frac{g_{\mu\nu}}{g_{00}}$$

ONE LOOP IN X_1

THE OPTICAL METRIC OF X_1 IS:

$$\tilde{d}s^2(X_1) = dt^2 + \frac{dr^2}{\left(1+\frac{r^2}{\ell^2}\right)^2} + \frac{r^2}{\left(1+\frac{r^2}{\ell^2}\right)} d\Omega_p^2 + \frac{\ell^2}{\left(1+\frac{r^2}{\ell^2}\right)} ds^2(M)$$

$$\tilde{V}ol(X_1)\Big|_{r_{min}}^{r_{max}} = \Omega_p \Omega_M \beta_0 \ell^{p+1} \int_{z_{min}}^{z_{max}} \frac{dz\, z^p}{(1+z^2)^{\frac{D}{2}}}$$

$$z = \frac{r}{\ell}$$

$$\tilde{V}(Ads_{p+2})\Big|_{r_{min}}^{r_{max}} = \Omega_p \beta_0 \ell^{p+1} \int_{z_{min}}^{z_{max}} \frac{dz\, z^p}{(1+z^2)^{\frac{p+2}{2}}}$$

THE LOCAL TEMPERATURE IS LARGER THAN THE KK THRESHOLD FOR M_{D-p-2} WHEN:

$$\left(1+\frac{r^2}{\ell^2}\right)\beta_0^2 < \ell^2$$

$T_{LOCAL} > KK$ THRESHOLD FOR M_{D-p-2} WHEN

$$\left(1+\frac{r^2}{\ell^2}\right)\beta_0^2 < \ell^2$$

FOR LARGE LOCAL TEMERATURE THAT IS:

$$r < \frac{\ell^2}{\beta_0}$$

SO FOR LARGE r; $r > \frac{\ell^2}{\beta_0}$

D_{eff} DROPS, NO KK EXCITATIONS ON

M_{D-p-2}.

ONE LOOP ON X_1:

$$I(X_1)_{1\text{-LOOP}} \simeq$$

$$-n_D\, a_D\, \beta_0^{-D}\left[\tilde{V}_{eff}(X_1)\right]_{z=0}^{z=\frac{\ell}{\beta_0}} \qquad -\left(\frac{\ell}{\beta_0}\right)^{D-1}$$

$$-n_{p+2}\, a_{p+2}\, \beta_0^{-p-2}\left[\tilde{V}_{eff}(AdS_{p+2})\right]_{z=\frac{\ell}{\beta_0}}^{z=0} \qquad -\left(\frac{\ell}{\beta_0}\right)^{p}$$

FOR LARGE T $-\left(\frac{\ell}{\beta_0}\right)^{D-1}$ DOMINATES.

A CLEAR VIOLATION OF HOLOGRAPHY?

FOR LARGE $r > \frac{\ell^2}{\beta_0}$ THE OPTICAL VOLUME IS FINITE. ASYMTOTICALLY X_1-AdS DOES NOT RECEIVE EXTENSIVE CONTRIBUTIONS.

X_1 IS EXTENSIVE FROM $0 < r < \frac{\ell^2}{\beta_0}$ IR GAUGE CONTRIBUTIONS.

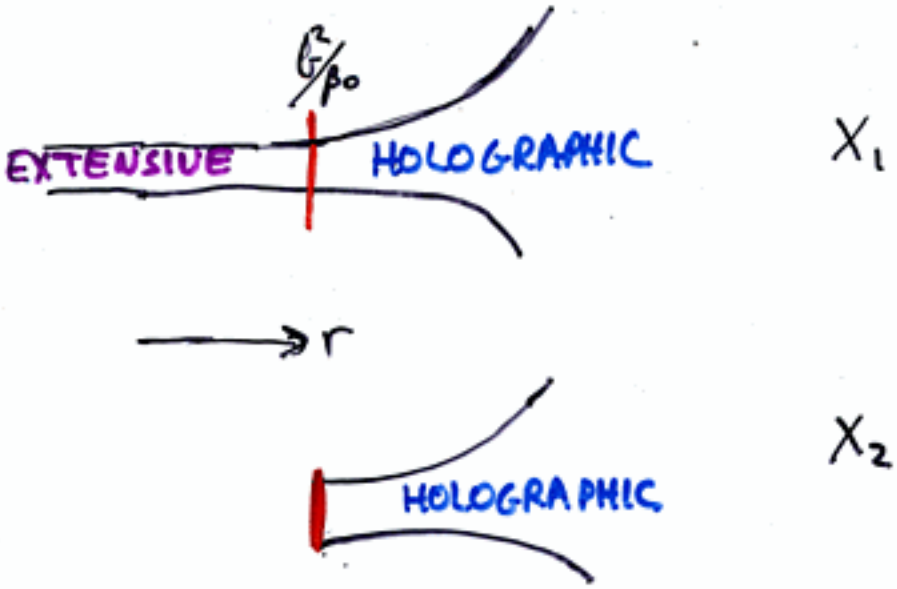
THUS AdS-X_1 LEFT ON ITS OWN IS EXTENSIVE.

HOWEVER FOR LARGE T $(T > T_0)$ IT IS X_2 (AdS BLACK HOLE) WHICH DOMINATES.

GRAVITATIONAL ASPECTS OF THE CORRESPONDENCE

ALMOST SUMMARY

ONE LOOP IN X_2



$U(N)$
$SU(N)$
$O(N)$

$$r_0^{\text{BH-HORIZON}} \sim r^{\text{KK CUTOFF}} \sim \frac{\ell^2}{\beta_0}$$

THUS THE THEORY DOES OBEY A HOLOGRAPHY CONDITION TO ORDER N^0 DUE TO GRAVITATIONAL EFFECTS.

- ANALOG IN STRINGY PICTURE.
  $c_{eff} = d_{eff}$ FLOWS? MIXED BACKGROUNDS?
- GRAVITATIONAL DYNAMICS FOR OTHER VALUES OF Λ

- CROSS-OVER

BOTH X_1 AND X_2 CONTRIBUTE

X_1'S CONTRIBUTION IS SUPRESSED

BY :

$$\exp(-N^2 \Delta f)$$

THE LIMITS ARE FIRST $N \to \infty$, NEXT $\frac{R}{\beta} \to \infty$

NEVERTHELESS ESTIMATE THE CROSSOVER

TEMPERATURE β_0^{CO}

$$\beta_0^{CO} \sim l_p N^{\frac{\delta(1-\cancel{\lambda})}{(D-\cancel{\lambda}-1)}}$$

THUS FOR $p \geq 1$

$\beta_0^{CO} \sim l_p$ OUTSIDE SUGRA REGIME.

l_s

- SHORT DISTANCE EFFETS

SHORT DISTANCE EFFECTS:

LEAD ALSO FOR HIGH T TO CONTRIBUTIONS

OF ORDER β^{-p}.

THEY ARE LINEARLY DEPENDENT ON $\mathrm{Vol}(E_{p+2})$

Θ_{2n} HAS $2n$ DERIVATIVES OF g

$$\Theta_{2n} \sim \partial^{2n} g \sim R^n$$

$$\langle \Theta_{2n} \rangle_X \sim \int_X \frac{\Theta_{2n}(x)}{\ell_p^{D-2n}} \sim \frac{\mathrm{Vol}(X)}{\ell^{2n} \ell_p^{D-2n}}$$

$$\sim N^{\delta(D-2)} \frac{\mathrm{Vol}(E_{p+2})}{\ell^{p+2}} \left(\frac{1}{N^{\delta}}\right)^{n-1}$$

NO Λ_D

WEAKLY COUPLED STRING $\ell_p = \lambda^{1/4} \ell_s$; $\lambda = c\alpha'/(\ell_{10})$

$$\langle \Theta_{2n,g} \rangle_X \sim \int_X \lambda^{2g-2} \frac{\Theta_{2n}(x)}{\ell_s^{10-2n}} \sim \frac{\mathrm{Vol}(E_{p+2})}{\ell^{p+2}} N^2 \left(\frac{1}{\sqrt{\lambda N}}\right)^{n-1} \left(\frac{(\lambda N)^2}{N^2}\right)^{2g}$$

A SEARCH FOR HAGEDORN EFFECTS

- FOR $\partial X = S^1 \times S^3$

X_1 MAY SUPPORT LOOPS OF ORDER ℓ_s

ONLY FOR

$$T_{gauge} \sim \frac{(\lambda N)^{1/4}}{\ell} \sim \frac{1}{\ell_s}$$

- YM NOT VALID BACK TO STRING

- X_2 TAKES OVER – LENGTH ℓ_s LOOPS ARE SHIELDED BY THE HORIZON.

HAGEDORN CENSORSHIP

- FOR $\partial X = S^1 \times T^3$ $\quad X = \tilde{X}_1, \tilde{X}_2$ MORE SINGULAR

LOOPS OF ORDER ℓ_s ON X^1 FOR ANY T_{gauge}

X_2 EXISTS FOR ANY β_0

FOR $(\lambda N)^{1/4} \gg 1$ CENSORSHIP PERSISTS.

FOR $(\lambda N)^{1/4} \ll 1$ APPROXIMATION BREAKS DOWN AND HAGEDORN EFFECTS COULD BE EXPOSED - SPECULATE?

MORE
LESS THAN MEETS THE EYE

BREAK DOWN OF HOLOGRAPHY

FOR NON CFT THEORIES ON D/p

AND $p \geq 5$

THE OPTICAL VOLUME IS NOT FINITE —

NO DECOUPLING.