

"String Junctions and the BPS
Spectrum of $N=2$ Theories"

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hep-th/9803142

Outline

- I BPS States in $N=2$ Theories
- II $N=2$ Theories on Probes
- III String Junctions

Related Work:

- O. Bergman & A. Fayyazuddin, hep-th/9802033
- Y. Imamura, hep-th/9802189
- M. Gaberdiel, T. Hauer, B. Zwiebach,
hep-th/9803205
- O. DeWolfe, T. Hauer, A. Iqbal, B. Zwiebach
hep-th/9805220

I BPS States in N=2 Theories

A. General Review

- i) Break 1/2 SUSIES
- ii) Saturate a bound $M = |\Delta|$
 - △ central charge (complex)

Example: Gauge theory

Coulomb branch $G_N \rightarrow U(1)^N$

BPS states are electrically / magnetically charged under some $U(1)$ factors.

$$\Delta = \sqrt{2} \sum_{i=1}^N (n_e^i a_i(u) + n_m^i \bar{a}_i(u)) + \sum_j S_j m_j$$

S_j abelian flavor symmetry charges
 m_j bare masses

(n_e, n_m) electric / magnetic charges

BPS states can appear in

hypermultiplets — quarks, dyons ...

vector multiplets — vector bosons, ...

In general, there are BPS particles for only some of the possible charges (n_e, n_m, s).

The BPS spectrum can (and does) jump as the moduli are varied.

Curves of Marginal Stability (CMS)

Generically,

$$|\Delta| \geq \sum_k |\Delta_k|$$

$$\sum n_e^k = n_e$$

$$\sum n_m^k = n_m$$

$$\sum s^k = s$$

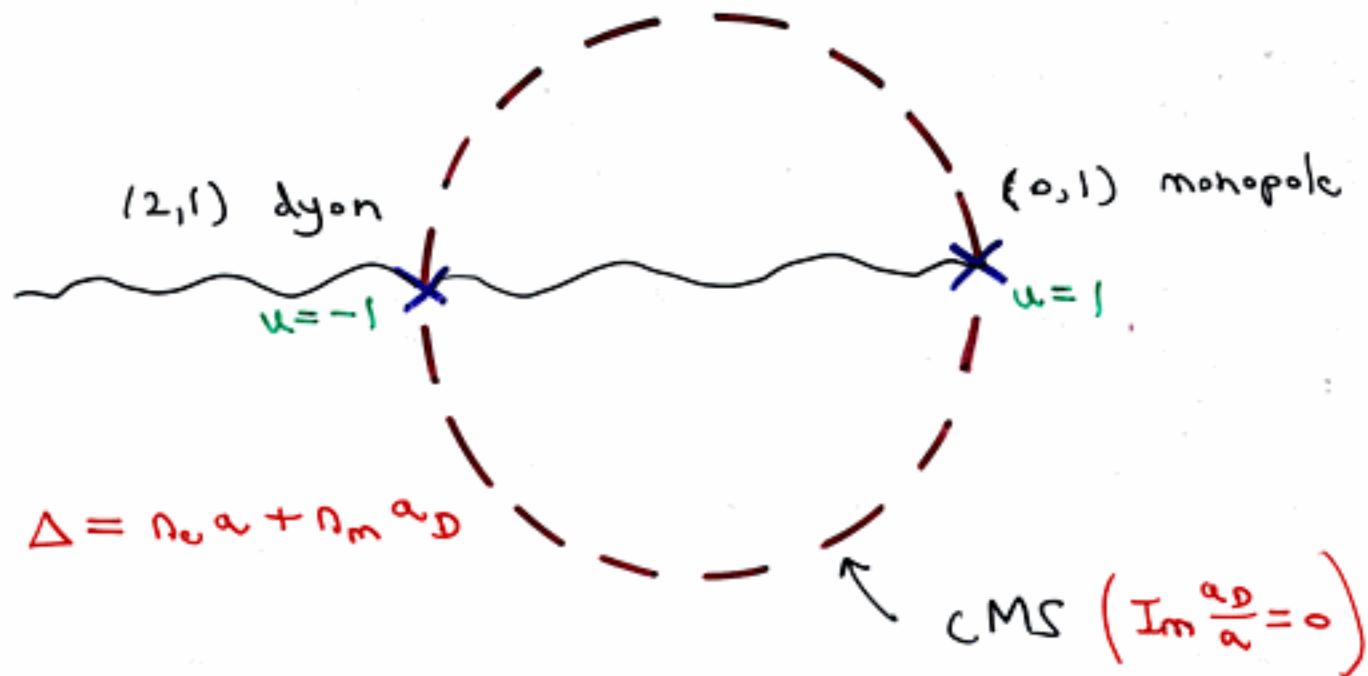
Equality when all phases align.



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B. $SU(2)$ Yang-Mills (Seiberg-Witten)

u plane



outside : $(2n, \pm 1)$ dyons
 $(\pm 2, 0)$ W bosons

Inside : $(2,1)$ dyon
 $(0,1)$ monopole

Particles decay on crossing the CMS.

Simple rule ?

(6)

Various techniques for determining the
BPS spectrum.

Field theory:

- i) Semi-classical theory (^{Sen}
S.S., Stern, Zaslow
Harvey, Gauntlett
Henningson)
- ii) Consistency with the singularity structure
(Bilal, Ferrari)

M / string techniques:

- i) IIB on $CY_3 \rightarrow$ self-dual strings
(Kleemann, Larkoski, Mayr, Vafa, Warner)
- ii) M theory \rightarrow BPS membrane configurations
(Witten
Yi, Henningson
Mikhailov
Fayyazuddin & Spalinski)
- iii) BPS spectrum on D3-brane probes
in F theory

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II Probe Realizations of $N=2$ Theories

A. Seven-branes in IIB

Two-dimensional transverse space (u)

$(1,0)$ 7-brane at $u=u_0$

$$\gamma \sim \frac{1}{2\pi i} \ln(u-u_0)$$

$SL(2, \mathbb{Z}) \quad (1,0) \rightarrow (\rho_1 \nu) \text{ 7-brane}$

$$\gamma \rightarrow \frac{a\gamma + b}{c\gamma + d}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1+\rho\nu & \rho^2 \\ -\rho^2 & 1-\rho\nu \end{pmatrix}$$

$(\rho_1 \nu)$ 7-brane

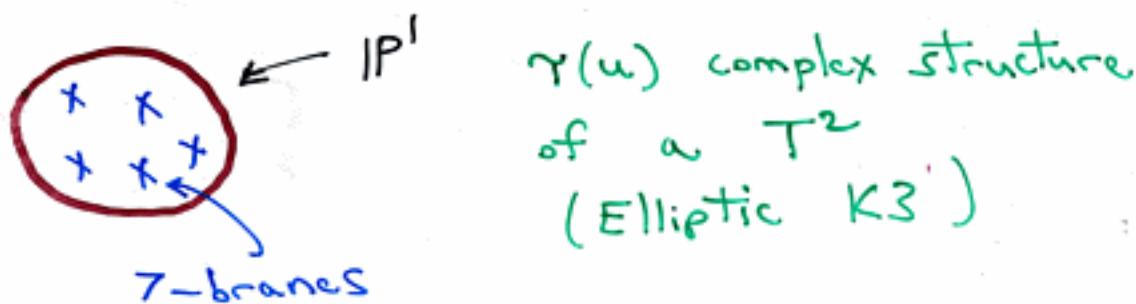
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$(\rho_1 \nu)$ string

B. F theory on K3

IIB string theory on \mathbb{P}^1 (coordinate u)
 + 24 7-branes (18 mutually local)



Singularities: $H_0, H_1, H_2, D_4, E_6, E_7, E_8$

A D3-brane on the singularity has
 a global symmetry appropriate to
 the singularity.

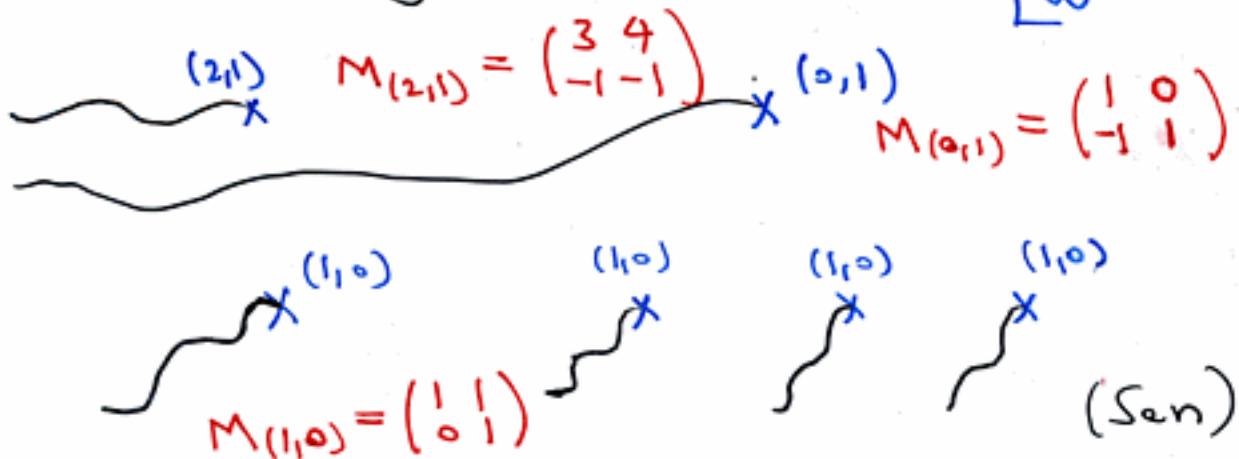
D, H: Symmetries realized by conventional
 gauge theories

E: 'tensionless string theories'

(Banks, Douglas, Seiberg)

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D₄ Take a limit 18 7-branes $\rightarrow \infty$
leaving 6 7-branes.

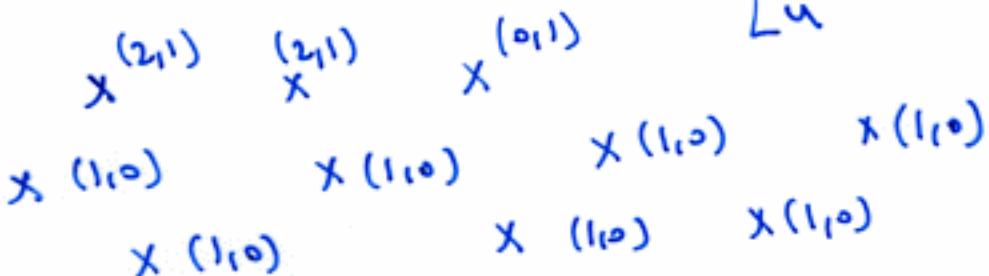


Moduli space of $SU(2) + 4$ fund. hypers.

D3-brane at $u=u_0$ realizes this gauge theory.

N D3-branes $\rightarrow Sp(N) + \frac{1}{2}$ antisym. (Douglas, Lowe, Schwarz)
+ 4 fund.

E₈ 10 7-branes (Dasgupta, Mukhi)



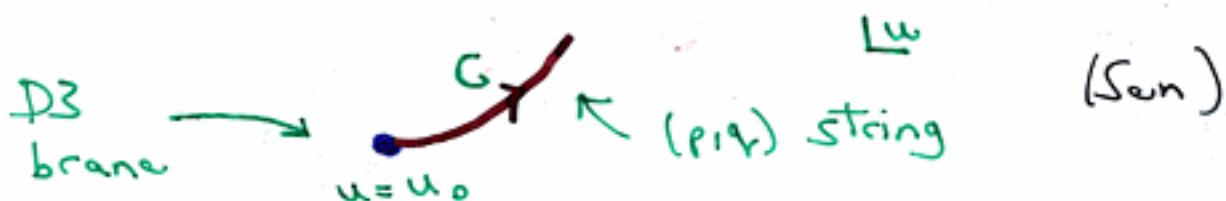
No known microscopic description of theory on
D3-brane.

$$T = e^{\frac{i\pi}{3}} \text{Monodromy}_{(\infty)} \quad TS = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

C. Puzzles with the BPS spectrum

Seiberg-Witten theory (pure $SU(2)$)

Does a (p_{1q}) dyon exist at $u=u_0$?



$$M_{p_{1q}} = \int_C |dw_{p_{1q}}|$$

$$|dw_{p_{1q}}|^2 = |\rho + q\gamma|^2 |dw_{1..}|^2$$

BPS configurations \leftrightarrow geodesics in the metric
 $|dw_{p_{1q}}|^2$

$$M_{p_{1q}} = \int_C |dw_{p_{1q}}| = \left| \int_C dw_{p_{1q}} \right|$$

$$dw_{p_{1q}}(u_0) \sim p da(u_0) + q da_D(u_0)$$

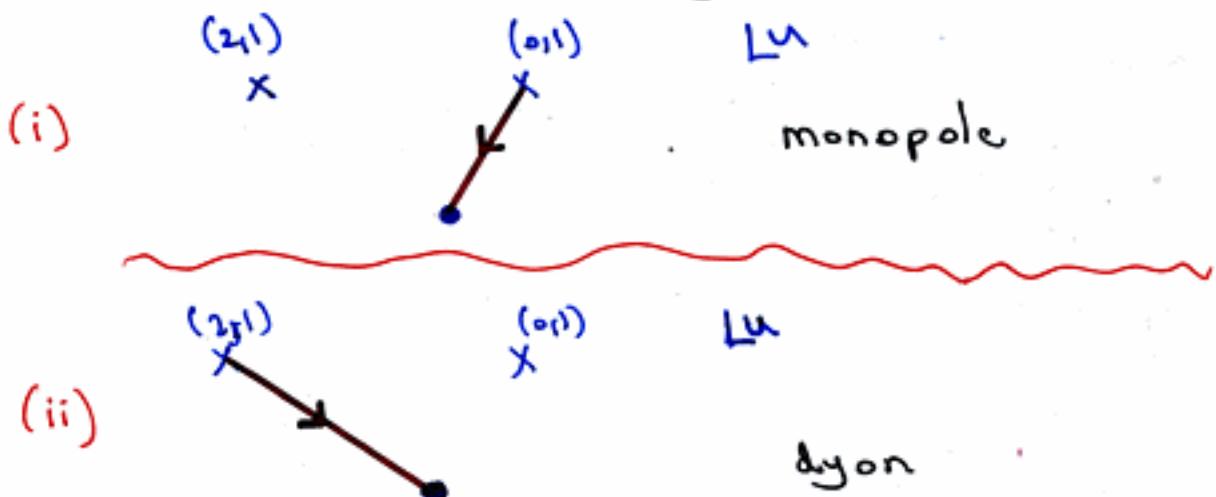
Phase of $w_{p_{1q}}$ is constant along a geodesic C .

Fixes tangent vector of C at $u=u_0$.

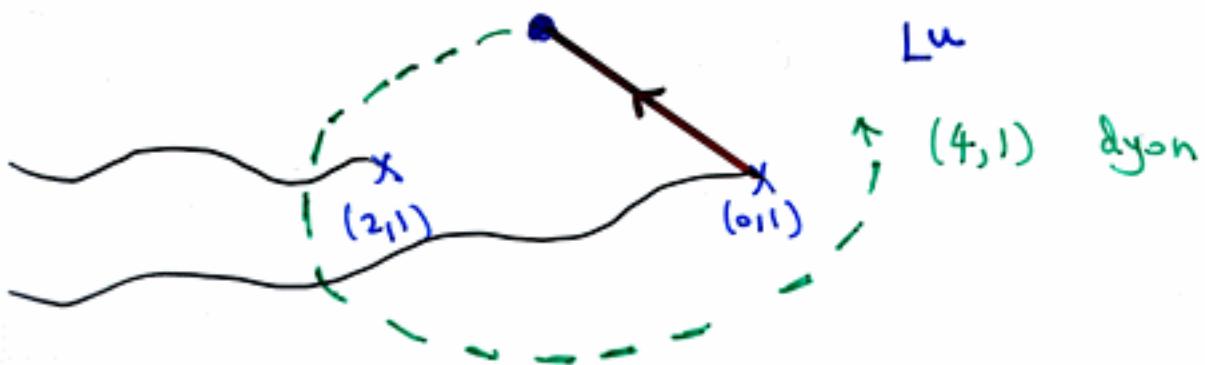


Cannot be BPS

Now we have a puzzle.



Strings must end somewhere. Where are the dyons and ω -bosons?



What happens under monodromy?

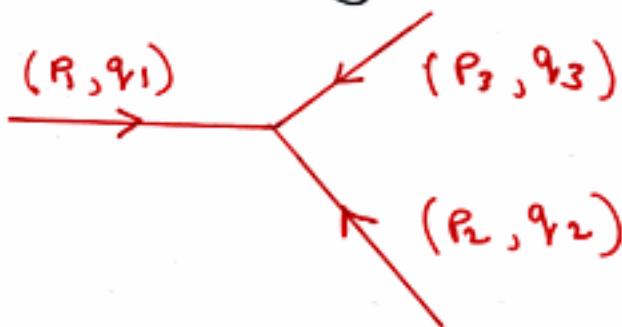
Unusual dynamics in the strong coupling region.

III String Junctions

A. Flat Space

(i) Break $1/4$ strings

(ii) Three string junctions

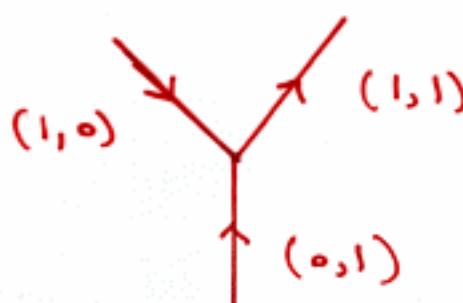


$$\sum p_i = 0 \quad \sum q_i = 0$$

$$\vec{T} = p + q \vec{\gamma} \quad (\text{planar case})$$

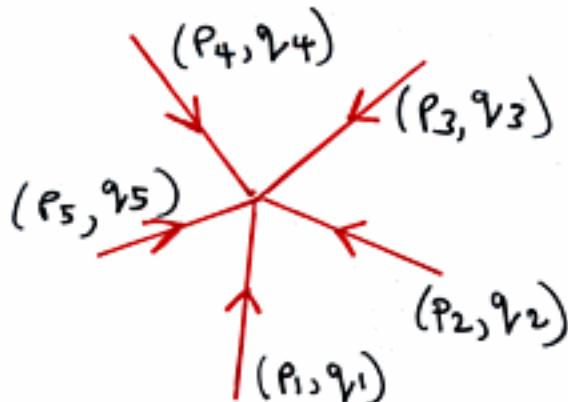
$$\sum \vec{T}_{p_i, q_i} = 0$$

Gaberdiel, Zwirbach
 Schwarz
 Aharony, Sonnenschein, Yankielowicz
 Dasgupta, Mukhi
 Sen
 ...

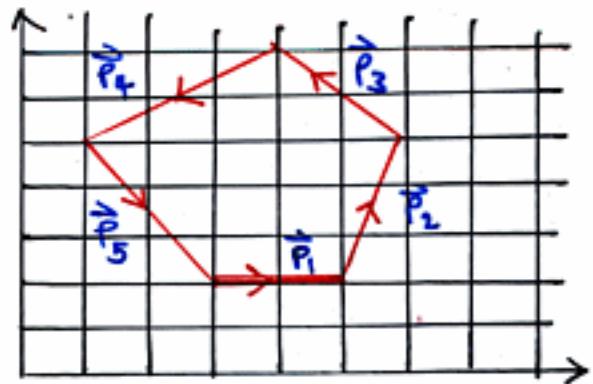


Simpler case

(iii) Many string junctions

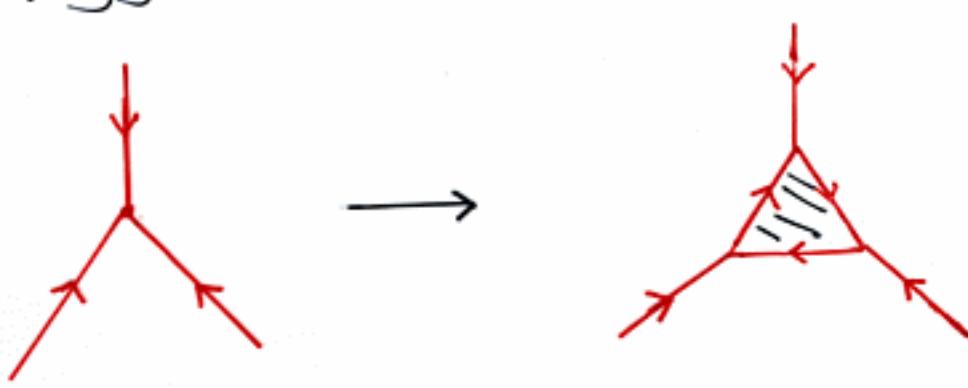


Associated polygon



Junction exists iff polygon is convex.

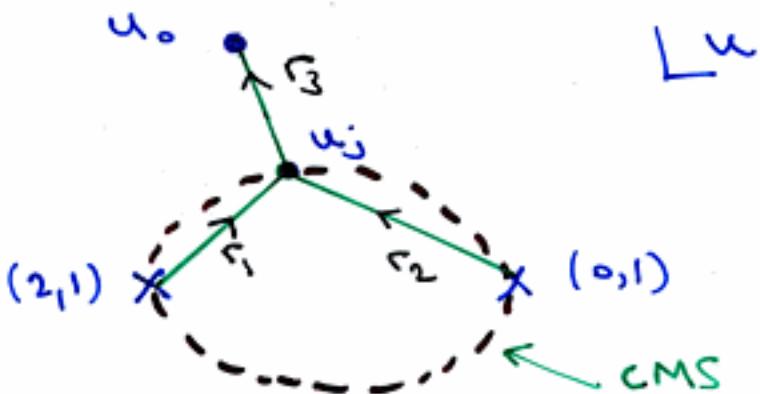
The junction has ‘breathing modes’. These moduli correspond to integer points in the polygon.



One modulus case.

B. Applications in F theory.

Our missing states are realized by string junctions.



Realizes a particle $(2r_1, r_1 + r_2)$

When is this classically BPS?

$$\begin{aligned} m(u_0) &= m_3 + m_2 + m_1 \\ &= |\omega_3(u_0) - \omega_3(u_j)| + \\ &\quad |\omega_1(u_j)| + |\omega_2(u_j)| \\ &\geq |\omega_3(u_0)| \end{aligned}$$

Equality iff

$$\text{Arg}(\omega_1(u_j)) = \text{Arg}(\omega_2(u_j)) = \text{Arg}(\omega_3(u_j))$$

But,

$$\omega_1(u_j) = c_1 (2\alpha(u_j) + \alpha_D(u_j))$$

$$\omega_2(u_j) = c_2 \alpha_D(u_j)$$

$\Rightarrow c_1$ and c_2 have opposite sign.

$$\text{Im}(\alpha_D/\alpha) = 0$$

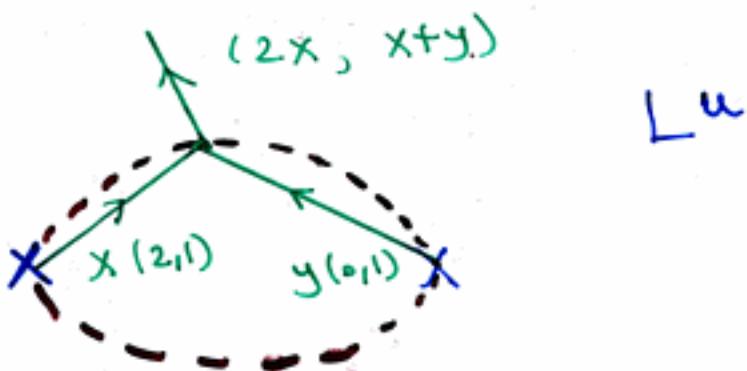
So u_j is on the CMS.

Conclude:

Inside CMS, we only have two particles, $(2,1)$ dyon and the $(0,1)$ monopole.

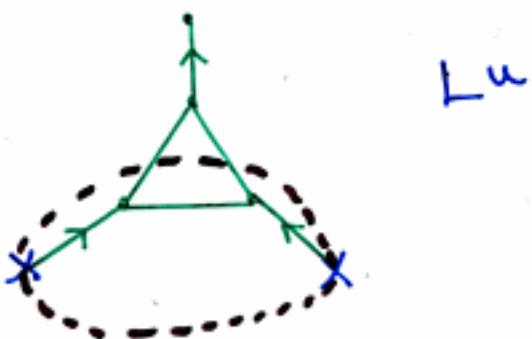
C. A Selection Rule

(i) There are too many possible states.



We only want $x+y = \pm 1, 0$

(ii) Moduli



These moduli are spurious (note the reduced supersymmetry). In some cases, these junctions are smoothly connected to single string configurations.

Sketch the argument:

(Schwarz
Aspinwall)

$$M\text{ theory on } T^2 \simeq IIB \text{ on } S^1$$

$$R_B \sim \frac{\ell_P^3}{R_1 R_2} \quad A = R_1 R_2 \text{ area of the torus}$$

F theory is a limit of M theory on an elliptically-fibered space ($A \rightarrow 0$).

M theory:

Degenerations
of the fiber

M5 wrapped
on the fiber

M2 wrapped
on a (p_{19}) cycle
of the fiber

IIB :

T -branes

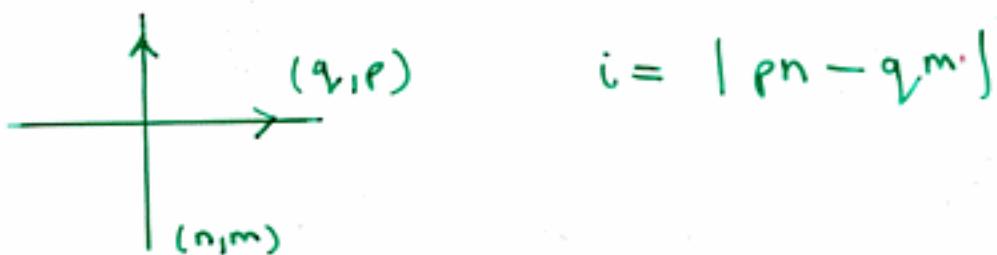
D3-brane

(p_{19}) string

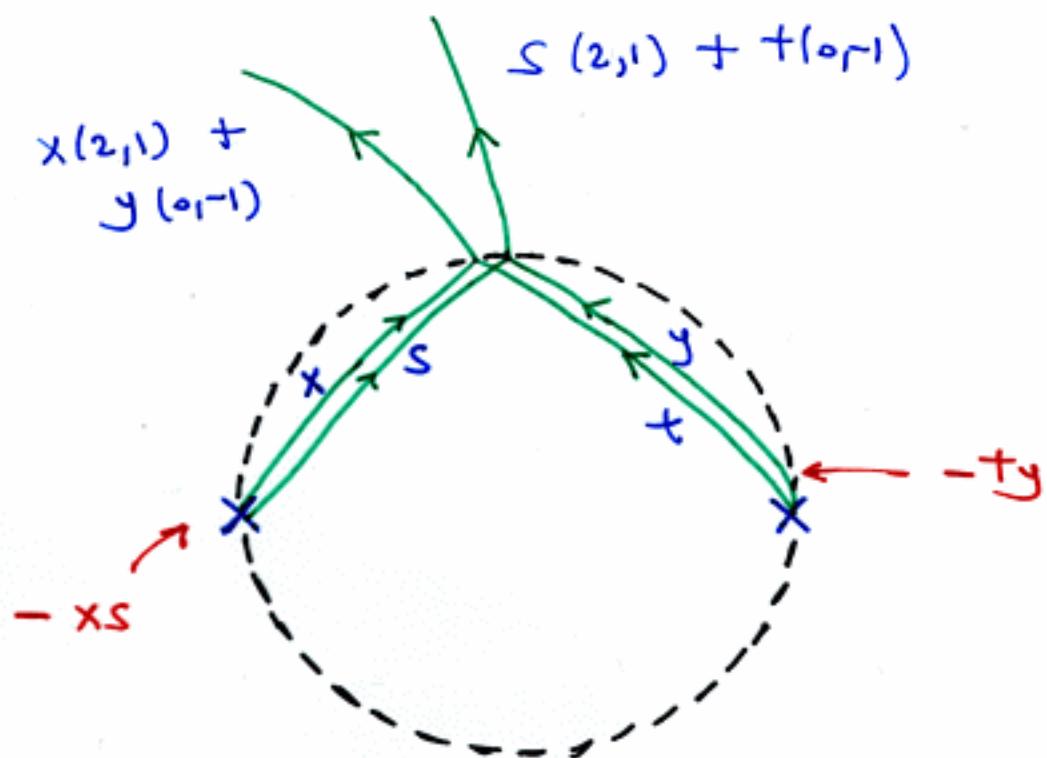
BPS junction lifts to a membrane

wrapped on a holomorphic curve. (Krogh, Lee
Matsuo, Okuyama)

We will compare a given junction to a known BPS junction by computing the intersection number, i..
 i must be non-negative.



$$i = |pn - qm|$$



Intersection number

$$i((2x, x-y), (2s, s-t)) = -(s-t)(x-y) + |sy-tx|$$

Monodromy invariant

If there is a state with charge

$$(2x, b) \quad b > 1$$

then

$$i((2x, b), (2[\frac{x}{b}], 1)) < 0$$

Conclude:

$$(2x, b) \quad |b| > 1 \quad \text{not allowed.}$$

Leaves $b=0$, W -bosons
 $b=\pm 1$, dyons

as desired.

We have computed the BPS spectrum!