

A Non-Superstring (I)

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preliminary results
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One of our biggest challenges is to understand how $\Lambda \approx 0$ without unbroken supersymmetry

Non supersymmetric string vacua (such as the $O(16) \times O(16)$ Heterotic String) typically perturbatively generate a cosmological constant.

- We will present an example of a non-supersymmetric (below $\frac{L}{l_s}$) string vacuum which appears to have

$\Lambda = 0$ order by order in g_s .

- We will discuss how this example fits into a broader framework relating

String background QFT

$\Lambda(g_s) = 0$ $\beta(g) = 0$
flat dilaton potential fixed line

Consider the following asymmetric orbifold:

generators:

	f	$\frac{g}{f}$
	-1 1	1 -1
	-1 1	1 -1
	-1 1	S ↙
	-1 1	S ↙
left-right symmetric shift by $\frac{1}{2}$ lattice vector ($X \rightarrow X + \frac{R}{2}$)	$\rightarrow S$	1 -1
	$\rightarrow \frac{S}{(-1)^{F_R}}$	$\frac{1-1}{(-1)^{F_L}}$

1
2
3
4
5
6

⇒ no massless twisted states

• $f + g$ don't commute:

$$\text{e.g. } X_s^R \xrightarrow{f} X_s^R + \frac{R}{2} \xrightarrow{g} -X_s^R - \frac{R}{2}$$

$$\qquad\qquad\qquad \xrightarrow{g} -X_s^R \xrightarrow{f} -X_s^R + \frac{R}{2}$$

$$\text{so } fg = gf T_{34}^L T_{56}^R$$

We are at the self-dual radius

$$P_L = \frac{m}{2R} + nR \quad R = \frac{1}{\sqrt{2}} \quad e^{iP_L X_L} = e^{\frac{m+n}{\sqrt{2}} X_L}$$

$$P_R = \frac{m}{2R} - nR \quad X_L \rightarrow X_L + \sqrt{2} = X_L + 2R$$

$$X = \frac{1}{2}(X_L + X_R) \quad \text{leaves } e^{iP_L X_L} \text{ invariant}$$

$$\text{so } (T^L)^2 \approx 1 \text{ on the torus}$$

	f	g	
1	-11	1-1	• Level-matching & all known conditions for higher-loop modular invariance (Freed, Vafa) are satisfied here
2	-11	1-1	
3	-11	S	
4	-11	S	
5	S	1-1	
6	S	1-1	
7	$(-1)^{F_R}$	$(-1)^{F_L}$	kills spacetime SUSY from left-movers
8			kills spacetime SUSY from right-movers

- massless spectrum :

			<u># states</u>
NS NS	$\psi_{-\frac{1}{2}}^{5678}$	$ -\frac{1}{2}\rangle \otimes \psi_{-\frac{1}{2}}^{3478} -\frac{1}{2}\rangle$	16
R R	S_{--}	$\otimes S_{--}$	16
R NS	S_{+-}	$\otimes \psi_{1256}$	16
NS R	ψ_{1234}	$\otimes S_{-+}$	16

- No gravitino \Rightarrow ~~SUSY~~
- Bose-Fermi Degeneracy
- same number of f-invariant and f-antivariants untwisted states in the g-projected theory

- D-brane spectrum: There are invariant combinations like

$$D_0 \xrightarrow{\begin{matrix} f \\ g \end{matrix}} D4[1234] \xrightarrow{g} D4[3456] \xrightarrow{f} D4[1256] \Rightarrow RN \text{ black holes}$$

In fact, using the orbifold group algebra

$$f^2 = T_{56} \quad g^2 = T_{34} \quad fg = gf \quad T_{34}^L T_{56}^R$$

$$f T_{56}^R = T_{56}^R f \quad g T_{34}^L = T_{34}^L g$$

$$f T_{34}^L = (T_{34}^L)^{-1} f \quad g T_{56}^R = (T_{56}^R)^{-1} g$$

T_{34}^R & T_{56}^L commute with everything

one can show that no pair of commuting
orbifold group elements breaks all the SUSY

\Rightarrow 1-loop vacuum energy + untwisted

tadpoles + mass renormalizations vanish

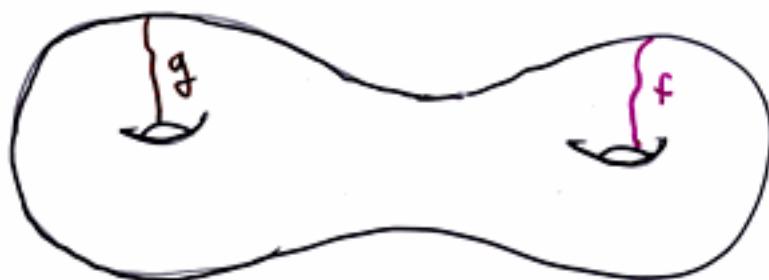
trivially :

$$\sum_{\substack{(h_1, h_2) \\ \text{commuting}}} h_1 \begin{array}{c} \square \\[-1ex] h_2 \end{array} = 0 \quad (\text{Each contribution is effectively supersymmetric.})$$

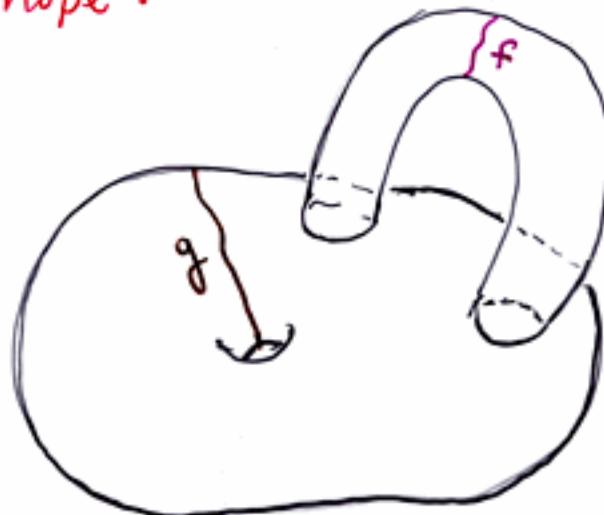
$$1 \begin{array}{c} \square \\[-1ex] 1 \end{array} + f \begin{array}{c} \square \\[-1ex] 1 \end{array} + \dots$$

At higher loops the Bose-Fermi degeneracy is not in itself sufficient to cancel the cosmological constant since the interactions are not supersymmetric.

We run into twist structures like

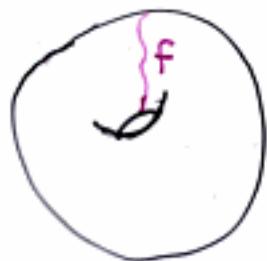


Is there hope?



We have a symmetry in the g -projected theory between states invariant under f and states anti-invariant under f .

Back to 1-loop for a moment:



Hamiltonian description:

$$A = \text{Tr} \left(f^L \bar{f}^R f \right) \quad f = e^{2\pi i T}$$

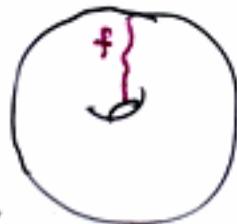
Recall that if we have a fermion twisted by

$$\psi(\sigma_1 + 2\pi, \sigma_2) = -e^{2\pi i a} \psi(\sigma_1, \sigma_2)$$

$$\psi(\sigma_1, \sigma_2 + 2\pi) = -e^{-2\pi i b} \psi(\sigma_1, \sigma_2)$$

its determinant is

$$\frac{\theta \begin{bmatrix} a \\ b \end{bmatrix} (0|r)}{\eta(r)} = e^{\frac{m i \alpha}{2}} f^{\frac{n}{2} - \frac{1}{4}} \prod \left(1 + q^{n-\theta - \frac{k}{2}} e^{2\pi i a} \right) \cdot \left(1 + q^{n-\theta - \frac{k}{2} - m i \alpha} e^{-2\pi i a} \right)$$



$$\begin{matrix} f \\ -1 \\ -1 \\ -1 \\ -1 \\ s \\ s \\ (-1)F_R \end{matrix}$$

$$\begin{matrix} g \\ 1 \\ 1 \\ 1 \\ s \\ s \\ 1 \\ 1 \\ 1 \\ 1 \\ (-1)F_L \end{matrix}$$

GSO phase

$$A = \sum_{a,b} \eta_{ab} \underbrace{\theta^2[b]}_{\psi_{1-4} \text{ determinants}} \underbrace{\theta^2[b+\frac{t}{2}]}_{\psi_{5-8} \text{ determinants}}$$

sum over spin
structures for
left-moving fermions
 ψ_{1-8}

\nwarrow \nearrow
spin structures opposite
around b-cycle due to
f projection

Consider first terms with $a = \frac{t}{2}$ (i.e. all fermions periodic around the a-cycle).

Since $\theta\left[\frac{t}{2}\right] = 0$ these terms in A all vanish. Then we are left with

$$\sum_b \eta_{ab} \theta^2[b] \theta^2[b+\frac{t}{2}]$$

$a=0$ (NS-sector states propagating in the loop)

$$= (1-1) \theta^2[0] \theta^2[\frac{t}{2}] = 0$$

\nwarrow In the NS sector, we project onto states $|s\rangle$ satisfying $(-1)^F |s\rangle = -|s\rangle$, such as $\psi_{-\frac{t}{2}} |\frac{-t}{2}\rangle$

This same mechanism is essentially going to carry over
to the 2-loop diagram (and higher)



To analyze this case carefully we need to go back
to the '80s :

Moduli & Supermoduli

In the Polyakov path integral we integrate
over the metric h and gravitino χ .

After dividing out by diffeomorphisms & local
Supersymmetry transformations, there is a
finite-dimensional space of (super) moduli
left to integrate over:

$$\delta h = \eta \delta T \quad 3g-3 \text{ complex moduli}$$
$$\delta \chi = \eta' \delta \mathcal{S} \quad 2g-2 \text{ complex supermoduli}$$

We can take the worldsheet gravitino to have δ -function support:

$$\chi(v_t, t) = \sum_{a=1}^{2g-2} \delta^a \delta^{(2)}(v_t - v_t(z_a(t)))$$

as long as we satisfy the conditions of

- integration transverse to the gauge slice
- modular invariant choice of points $z_a(t)$.

Then the partition function becomes (after changing variables from (δ^h) to (δ^{γ_a}) and integrating over the fermionic moduli δ^{γ_a})

$$\sum \int dt^{6g-6} [dX][dB][dc] e^{-S_0(\eta, b)} \prod_{a=1}^{2g-1} \delta^a T_F(z_a) \prod_{a=2g-2}^{4g-4} \overline{\delta^a} \overline{T_F}$$

$\underbrace{[dB][dc]}$
sum over spin structures
& orbifold twist structures

from χT_F
coupling in worldsheet
action (T_F is the
worldsheet supercurrent)

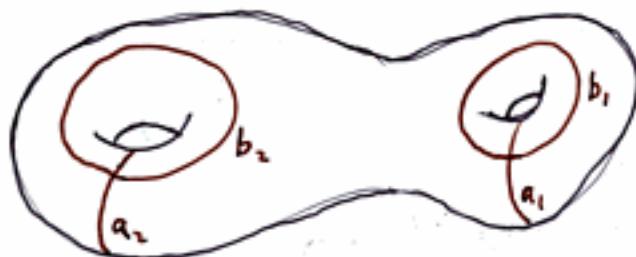
- So basically at higher genus we have to evaluate correlation functions

$$\langle :e^{\alpha} T_F(z_1) : \dots : e^{\alpha} T_F(z_{2g-2}) : \rangle$$

instead of just determinants in order to understand the spin-structure-dependent piece of the partition function.

- Changing the points z_a changes the amplitude by a total derivative on the moduli space. We will address the relevant boundary contributions later.

Following Verlinde & Verlinde '87 we can now write down the spin-structure-dependent factors in our amplitude in terms of θ -functions



$$\pi i(n+\alpha) \tau(n+\gamma) + 2\pi i(n+\gamma)(z+\beta)$$

$$\theta\left[\begin{matrix} \alpha \\ \beta \end{matrix}\right](z|\tau) = \sum_{n \in \mathbb{Z}^2} e$$

$$z \in \mathbb{C}^2 / \mathbb{Z}^2 + \tau \mathbb{Z}^2 \quad z_i(p) = \int_{p_0}^p w_i$$

where w_i is the canonical basis of holomorphic 1-forms satisfying

$$\begin{array}{ll} g_{ij} w_i = \delta_{ij} & g_{ij} w_i = \gamma_{ij} \\ a_i & b_j \end{array}$$

The contributions from $\langle e^{\alpha} \psi^\mu dx^\mu(z_1) e^{\beta} \psi^\nu dx^\nu(z_2) \rangle$

Involves terms of the form

$$\sum_{\alpha, \beta} \frac{\theta^2 \left[\begin{matrix} \alpha \\ \beta \end{matrix} \right] (0|T) \theta \left[\begin{matrix} \alpha \\ \beta + (\zeta, 0) \end{matrix} \right] (0|T) \theta \left[\begin{matrix} \alpha \\ \beta + (\zeta, 0) \end{matrix} \right] (z_1 - z_2 | T)}{\theta \left[\begin{matrix} \alpha \\ \beta \end{matrix} \right] (z_1 + z_2 - 2\Delta)}$$

from

$$e^{\alpha} e^{\alpha} \psi^{1-4} \psi^{1-4}$$

where Δ is the divisor class (i.e. set of zeroes - poles) for a reference spin structure (here at $g=2$ it is just 1 point).

and

$$\sum_{\alpha, \beta} \frac{\theta^2 \left[\begin{matrix} \alpha \\ \beta + (\zeta, 0) \end{matrix} \right] (0|T) \theta^2 \left[\begin{matrix} \alpha \\ \beta \end{matrix} \right] (0|T) \theta \left[\begin{matrix} \alpha \\ \beta \end{matrix} \right] (z_1 - z_2)}{\theta \left[\begin{matrix} \alpha \\ \beta \end{matrix} \right] (z_1 + z_2 - 2\Delta)}$$

from

$$e^{\alpha} e^{\alpha} \psi^{5-10} \psi^{5-10}$$

These expressions will simplify nicely if we choose $z_1 = z_2 = \Delta$

With $z_1 = z_2 = \Delta$ we get

$$\sum_{\alpha, \beta} \eta_{\alpha\beta} \Theta^2 \left[\begin{matrix} \alpha \\ \beta \end{matrix} \right] (0|r) \Theta^2 \left[\begin{matrix} \alpha \\ \beta + (\frac{1}{2}, 0) \end{matrix} \right] (0|r)$$

for both types of contributions.



We have an f-twist around the b_1 cycle

Consider $\alpha_1 = \frac{1}{2}$ terms. In general $\Theta \left[\begin{matrix} a \\ b \end{matrix} \right] (z|r)$ is even or odd in z if $4a \cdot b$ is even or odd. If odd $\Theta \left[\begin{matrix} a \\ b \end{matrix} \right] (0|r) = 0$.

Here $4\alpha \cdot \beta - 4\alpha \cdot [\beta + (\frac{1}{2}, 0)] = -1$

for $\alpha_1 = \frac{1}{2}$. So either $\Theta^2 \left[\begin{matrix} \alpha \\ \beta \end{matrix} \right]$ or $\Theta^2 \left[\begin{matrix} \alpha \\ \beta + (\frac{1}{2}, 0) \end{matrix} \right]$ is odd \Rightarrow $\alpha_1 = \frac{1}{2}$ terms vanish.

So we're left with $\alpha_i = 0$ (NS states propagating).



Summing over β_i then gives

$$\sum_{\alpha_1, \beta_1} (-1) \Theta^2 \left[\begin{smallmatrix} \alpha_1 \\ \beta_1 \end{smallmatrix} \right] \Theta^2 \left[\begin{smallmatrix} \alpha_1 \\ \beta_1 \end{smallmatrix} \right] = 0$$

↑
From $(-1)^F |S\rangle_{NS} = -|S\rangle_{NS}$

So despite the lack of Supersymmetry,
the contribution vanishes !