The Black Hole Horizon

A. Strominger

Strings '98

Outline

1. General comments
2. Recent work on AdS_3
   black hole entropy
[3. Work in progress on AdS_2]*
4. Conclusions

Emphasis on things
I don't understand.

* time permitting

The quest to understand quantum properties of black holes is 25 yrs. old. An amazing amount has been learned along the way, esp. in the last few years. But we still do not have an answer to the fundamental question

What is a quantum black hole?

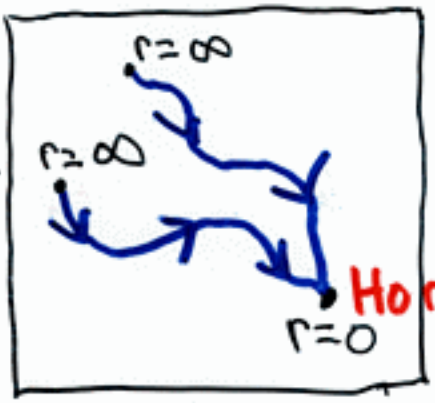
An early feature evident in the Bekenstein-Hawking area-entropy law

$$S_{BH} = \frac{1}{4}\,\text{Area}$$

is the universal character of the black hole horizon. String theory has yet to explain this, but some light has been shed.

At the classical level, the horizons of <u>supersymmetric</u> black holes are universal attractors in the space of geometries/moduli

Geometry/Moduli space



Horizon ~ $AdS_n \times S^n \times M$

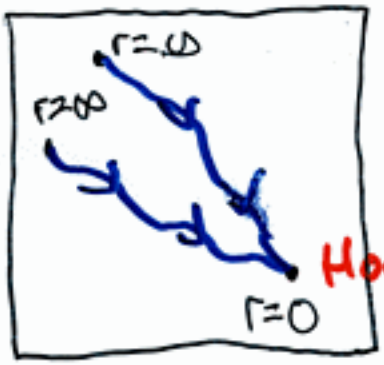
Moduli-independence of the fixed point is <u>required</u> by the entropy-area law.

Gibbons & Townsend
Larsen & Wilczek
Ferrara, Kallosh & AS
Moore

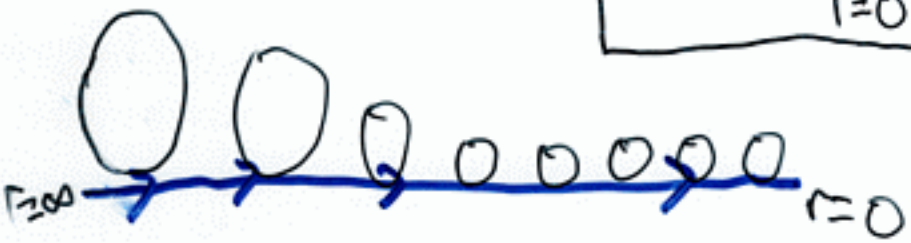
In string theory, the extremal horizon has been described as RG fixed points in the **worldsheet** sigma model

Banks Tseytlin
Giddings Harvey
Polchinski Ashok
Shenker Witten
Cvetic

worldsheet
σ-model
coupling
constant space



Horizon ~ WZW CFT

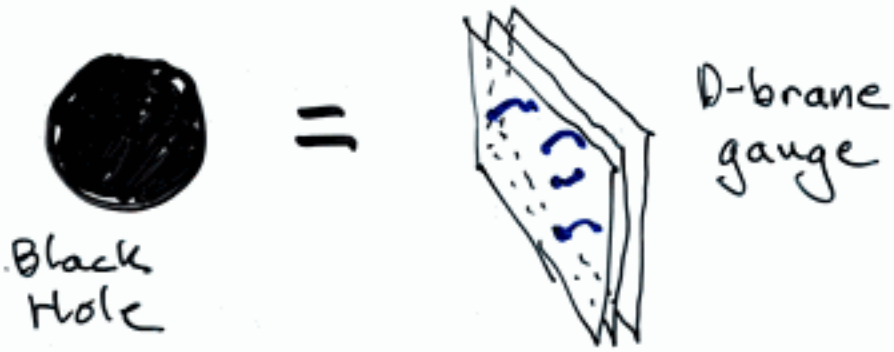


The non-conformal σ-model away from the horizon is "dressed" by the Liouville field ρ to give a $\hat{c}=10$ CFT.

Radial transformations = WS RG flow

Near-Horizon Geometry = Fixed point CFT

More recently the quantum states of certain black holes have been represented by D-brane field theories



Low energy dynamics of the black hole were found to have a universal descriptions by the conformal limit of the brane field theory.

Das Mathur
Maldacena & AS
Gubser & Klebanov

These suspicious behaviors are elegantly related by Maldacena's dictionary

String theory on near-horizon AdS	$\equiv$	Field theory in IR CFT
Black hole radial coordinate	$\equiv$	Energy scale of brane field theory (~energy scale of string worldsheet)

The successful D-brane computations of BH entropy, by the very nature of the dual description, do not give us a picture of spacetime information flow, or explain the universality of the area law. Can we use Maldacena's duality to address this in the AdS picture? In particular, can we enumerate the states responsible for $S_{BH} = \frac{1}{4}\text{Area}$?

(not yet, but maybe....)

Parenthetical Comment

A truly universal explanation of

$$S_{BH} = \frac{1}{4}\,\text{AREA}$$

would cover Schwarzchild as well as supersymmetric black holes.

Near-horizon supersymmetric black holes ~ $\Lambda < 0$ AdS $\Leftrightarrow$ Conformal Field Theory

Near-Horizon Schwarzchild Black Holes ~ Rindler Space ~ $\Lambda = 0$ Minkowski Space $\Leftrightarrow$?

Understanding Area Law ~ Understanding non-perturbative string theory in Minkowski Space

The special case

$$AdS_3 \longleftrightarrow 1{+}1 \text{ CFT}$$

was discovered in the more general setting

$$S_3 = \frac{1}{2\pi}\int d^3x\,\sqrt{-g}\left(R + \frac{2}{\ell^2}\right) + \text{anything}$$

(including KK or string modes)

The "Poincare group" of asymptotic symmetries of the $r=\infty$ cylinder is the 1+1 conformal group

$$\boxed{\text{Therefore } S_3 \text{ is a conformal field theory!!}}$$

't Hooft
Susskind
holography

Brown & Henneaux

Using

$$\{T(\xi_m), T(\xi_n)\} = i(m-n)\,T([\xi_m, \xi_n]) + \frac{ic}{12}(m^3 - m)\,\delta_{m+n}$$

B&H conclude from classical Poisson brackets

$$c = 12\,\ell \gg 1.$$

2+1 $\Lambda < 0$ gravity has black holes with mass M and angular momentum J. They obey

$$S_{BH} = 2\pi\left(\sqrt{M\ell^2 + J\ell} + \sqrt{M\ell^2 - J\ell}\right)$$

$$M = \frac{1}{\ell}\left(L_0 + \bar{L}_0\right)$$

$$J = L_0 - \bar{L}_0$$

Banados, Teitelboim & Zanelli

On the other hand from Cardy's formula.

$$S_{CFT} = 2\pi\left(\sqrt{\frac{cL_0}{6}} + \sqrt{\frac{c\bar{L}_0}{6}}\right)$$

using $c = 12\ell$, $L_0 = \frac{1}{2}(\ell M + J)$, $\bar{L}_0 = \frac{1}{2}(\ell M - J)$

$$= S_{BH}$$

This provides a statistical derivation of S_{BH} for any consistent, unitary theory of 2+1, $\Lambda < 0$ gravity.

Examples?

A) Pure 2+1 gravity w/ $\Lambda < 0$

B) String theory on
$AdS_3 \times S^3 \times M^4$
(= near-horizon geometry
of D1-D5 blackhole)

(A) is a sector of (B).

Pure 2+1 gravity

$$S = \int d^3x \sqrt{-g}\left(R + \frac{2}{\ell^2}\right)$$

$$= \ell \int \left(A_L \, dA_L + \tfrac{1}{3} A_L^3\right) - \ell \int A_R \, dA_R + \tfrac{1}{3} A_R^3$$

$$A_{L,R} = \omega \pm \frac{e}{\ell}$$

This is a

$$SL(2,R)_k \otimes SL(2,R)_{-k}$$

Chern-Simon theory on the disc with $k = 2\ell$.

Witten

The Chern-Simon theory has only boundary dynamics

2+1 $SL(2,R)^2$ CS Theory

boundary
$\rightarrow$ 1+1 $SL(2,R)$ WZW theory w/ constraints

$\rightarrow$ 1+1 Liouville theory with $c = 12\ell$ as predicted

Liouville field ~ boundary metric

Are these boundary conditions physically appropriate?

Banados
Coussaert Henneaux
& Van Driel
- - - - -

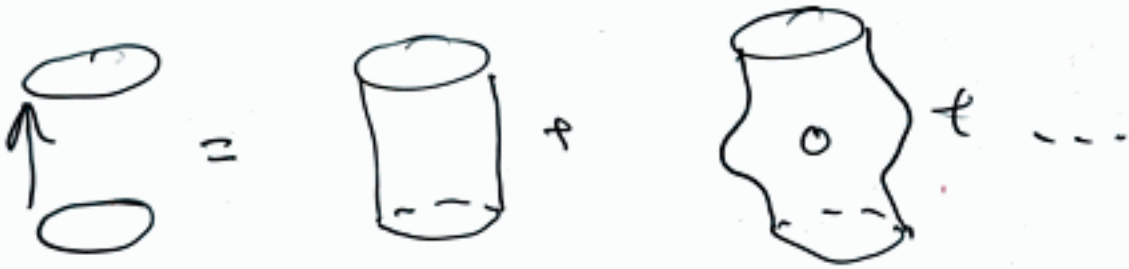
Problems

0) Nature of Hilbert space - even vacuum - unclear.

1) Can't apply Cardy formula because of nonunitarity.

2) Do not seem to be black holes in theory on disc, which have nontrivial $SL(2,R)$ holonomy.

Perhaps there is a "pure gravity" theory w/ dynamical topology change & black holes



If so, it should be described by $c = 12\ell$ CFT on boundary.

<u>Speculation</u>

The CFT in question $SL(K,R)_2$, related by rank-level duality to $SL(2,R)_K$.

Vafa-Carlip-AS

on to string theory....

String Theory

Q_1-D1 + Q_5-D5 black string on K3 $\doteq$ Higgs branch D-brane gauge theory

$\downarrow$ near-horizon $\qquad\qquad$ $\downarrow$ IR limit

$AdS_3 \times S^3 \times K3$ $=$ 1+1 CFT on (deformation of) $Sym_{Q_1 Q_5}(K3)$

After careful normalization

$$c = 12\ell = 6Q_1Q_5$$

c might have been computed this way on l.h.s. rather than counting dimensions on r.h.s.

$AdS_3 \Leftrightarrow$ N-S N-S vacuum of CFT
(spinors antiperiodic)

Note $M(AdS_3) = -1$

$= \frac{1}{\ell}(L_0 + \bar{L}_0)$

$\Rightarrow L_0 = -\frac{c}{24}$

Coussaert & Henneaux

Excitations of $AdS_3 \Leftrightarrow$ Excitations of N-S CFT

$=$ Operators on the plane

The RR sector is complicated on both sides. Best understood through spectral flow from NS-NS.

CFT Chiral Primaries	=	BPS Particle States in AdS_3
$\mathcal{O} =$		Mode of scalar field Φ
$\mathrm{Sym}_{Q_1 Q_5}(\omega_{a\bar{b}} \psi^a(z) \psi^{\bar{b}}(\bar{z}))$ ← K3 harmonic form		
$L_0 = \bar{L}_0 = \frac{1}{2} = J = \bar{J}$		$L_0 = \bar{L}_0 = \frac{1}{2} = J = \bar{J}$
$[L_1, \mathcal{O}] = 0$		$[L_1, \Phi] = 0$
Complete set of $\mathcal{O}$s derived from cohomology of $\mathrm{Symm}_{Q_1 Q_5}(K3)$		$\Rightarrow \Box \Phi = -\frac{L^2 + \bar{L}^2}{J Q_1 Q_5} \Phi = -m^2 \Phi$
Perfect, intricate match, almost...		using $L^2 = -L_0(L_0 - 1) + L_{-1} L_1$ $\Rightarrow m^2 = -\frac{1}{4\sqrt{Q_1 Q_5}}$
Maldacena & AS, Seiberg & Sezgin, Sundell, de Boer		Complete set from KK supergravity reduction

The Stringy Exclusion Principle

The operator $\mathcal{O}$ corresponds to a single boson in a particular mode of the field Φ.

$$\mathcal{O}^2$$

is a new chiral primary. It corresponds to two particles in the same mode. Since

$$\mathcal{O}^n = 0, \quad n > 2Q_1Q_5 \sim \frac{1}{g^2}$$

there is an exclusion principle which cannot be seen in string/sugra perturbation theory. Similar phenomena have been encountered elsewhere.

Gross Gubser Polchinski

Where is the entropy?

Under spectral flow RR → NS-NS the states responsible for BH entropy

$$S_{BH} = 2\pi \sqrt{n Q_1 Q_5}$$

become right-chiral primaries.

In general for levels $n > Q_1 Q_5$ supergravity cannot be trusted, as seen in the exclusion principle. So apriori we might not find these states, and they are just described as a black hole in AdS picture. But maybe supersymmetry can help us ....

Conjectural AdS Description of Black Hole Entropy

The maximal number of particles in a right-left chiral primary is $Q_1 Q_5$:

$$\mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_{Q_1 Q_5} |0\rangle$$

where $h_0 = \bar{h}_0 = \frac{1}{2}$ for each $\mathcal{O}_i$. Consider the general level n right-chiral state

$$\prod_{i=1}^{Q_1 Q_5} (\bar{L}_{-n_i} \mathcal{O}_i) |0\rangle$$

$$\sum n_i = n$$

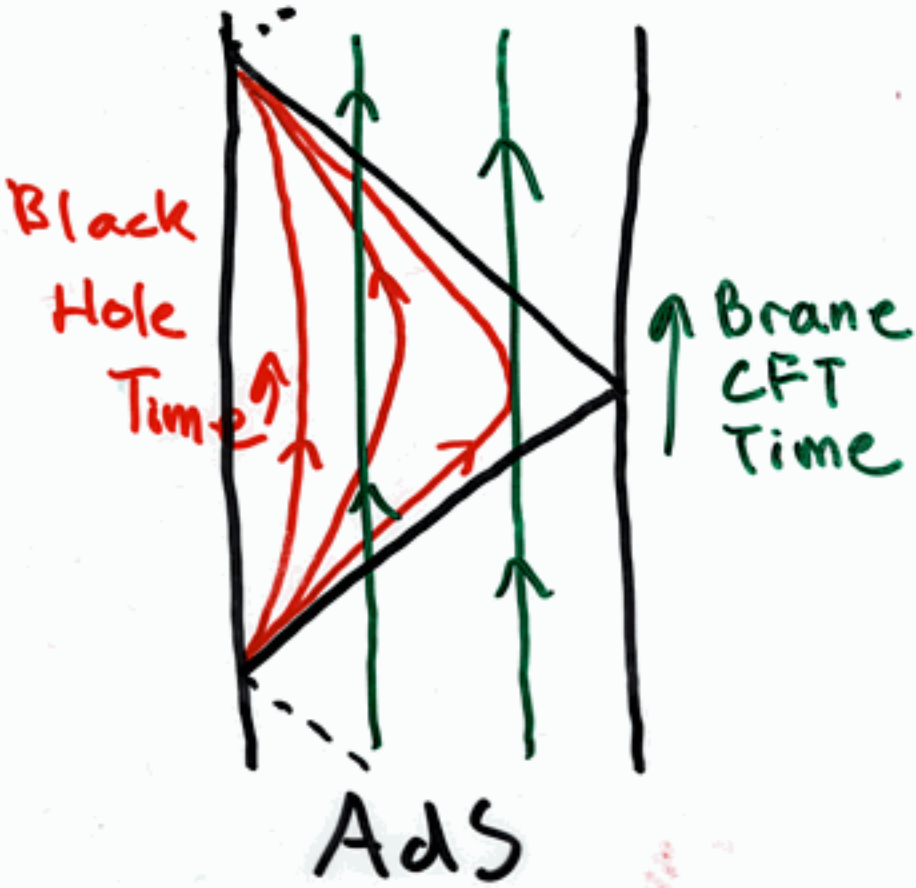
where $\bar{L}$ is a left super-Virasoro generator. The number of such states is

$$S = 2\pi \sqrt{Q_1 Q_5 n} = S_{BH}$$

BUT PROBLEMS W/ THIS PICTURE Maldacena & AS

Concluding Challenge

Find what's wrong with this picture:



and why black hole evolution in **time** is unitary!