The Black Hole Horizon

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Outline

1. General comments
2. Recent work on AdS$_3$
   black hole entropy
3. Work in progress on AdS$_3$ [x]
4. Conclusions

Emphasis on things I don't understand.

# Time permitting
The quest to understand quantum properties of black holes is 25 yrs. old. An amazing amount has been learned along the way, esp. in the last few years. But we still do not have an answer to the fundamental question

What is a quantum black hole?
An early feature evident in the Bekenstein-Hawking area-entropy law

\[ S_{BH} = \frac{1}{4} \text{Area} \]

is the universal character of the black hole horizon. String theory has yet to explain this, but some light has been shed.
At the classical level, the horizons of supersymmetric black holes are universal attractors in the space of geometries/moduli.

\[ \text{Horizon} \sim \text{AdS}_n \times S^m \times M \]

Moduli-independence of the fixed point is required by the entropy-area law.

Gibbons & Townsend
Larson & Wilczek
Ferrara, Kallosh
AS
Moore
In string theory, the extremal horizon has been described as RG fixed points in the \textit{worldsheet sigma model}.

\begin{itemize}
  \item \textit{Worldsheet 6-model coupling constant space}
  \item \textit{Horizon} = WZW CFT
\end{itemize}

The non-conformal 6-model away from the horizon is "dressed" by the Liouville field $\phi$ to give $c=10$ CFT.

Radial transformations = W3 RG-flow

Near-Horizon Geometry = Fixed point CFT
More recently the quantum states of certain black holes have been represented by D-brane field theories.

\[ \text{Black Hole} = \text{D-brane gauge} \]

Low energy dynamics of the black hole were found to have a universal description by the conformal limit of the brane field theory.

Das Nathur, Maldacena & TS Gubser & Klebanov
These suspicious behaviors are elegantly related by Maldacena's dictionary:

\[
\begin{align*}
\text{String theory on near-horizon AAdS} & \quad \equiv \quad \text{Field theory in IR CFP} \\
\text{Black hole radial coordinate} & \quad \equiv \quad \text{Energy scale of brane field theory} \\
& \quad \equiv \quad \text{(energy scale of string worldsheet)}
\end{align*}
\]
The successful computations of BH entropy, by the very nature of the dual description, do not give us a picture of spacetime information flow, or explain the universality of the area law. Can we use Maldacena's duality to address this in the AdS picture? In particular, can we enumerate the states responsible for

\[ S_{\text{BH}} = \frac{1}{4} \text{Area} \]

(not yet, but maybe... )
A truly universal explanation of
$S_{BH} = \frac{1}{4} \text{ AREA}$

would cover Schwarzschild as well as
supersymmetric black holes.

Near-horizon
supersymmetric black holes
$\sim$ $\mathcal{N}$ = 0 AdS

 ↔

Conformal
Field Theory

Near-Horizon
Schwarzschild
Black Holes
$\sim$ Rindler Space
$\sim$ $\Lambda$ = 0 Minkowski Space

Understanding
non-perturbative
string theory
in Minkowski space
The special case

\[ \text{AdS}_3 \leftrightarrow 1+1 \text{ CFT} \]

was discovered in the more general setting

\[ S_3 = \frac{1}{2\pi} \int \mathrm{d}^3x \sqrt{-g} \left( R + \frac{2}{\ell^2} \right) + \text{anything} \]

(including KK or string modes)

The "Poincare group" of asymptotic symmetries of the \( r=0 \) cylinder is the I\!I\!I conformal group

Therefore \( S_3 \) is a conformal field theory!!
Using
\[ \epsilon T(S_m), T(S_n)^3 = i(m-n) T([S_m, S_n]) + \frac{ic}{12} (m^3 - m) \delta_{m+n} \]

B&H conclude from classical Poisson brackets
\[ c = 12 \gamma \gg 1. \]
2+1 \Lambda \geq 0 \text{ gravity has } \text{black holes with mass } M \text{ and angular momentum } J. \\
\text{They obey} \\
S_{BH} = 2\pi \left( \sqrt{MR^2 + J^2} + \sqrt{MR^2 - J^2} \right) \\
M = \frac{1}{8} \left( l_0 + \bar{l}_0 \right) \\
J = l_0 - \bar{l}_0
On the other hand from Cardy's formula:

\[ S_{\text{cft}} = 2\pi \left( \sqrt{c-L_0} + \sqrt{c-L_0} \right) \]

using \( c=2k \), \( L_0 = \frac{1}{2}(8M+5) \), \( \hat{L}_0 = \frac{1}{2}(8M-5) \)

\[ = S_{\text{BH}} \]

This provides a statistical derivation of \( S_{\text{BH}} \) for any consistent, unitary theory of 2+1, \( \Lambda < 0 \) gravity.
Examples?

A) Pure 2-tl gravity w/ Λ < 0

B) String theory on $\text{AdS}_3 \times S^3 \times M_4$

(= near-horizon geometry of 0l-05 black hole)

(A) is a sector of (B).
Pure 2d gravity

\[ S = \int d^3x \sqrt{-g} \left( R + \frac{2}{\ell^2} \right) \]

\[ = \kappa \int \left( A_L \wedge dA_L + \frac{1}{3} A_L^3 \right) \]
\[ - \kappa \int A_R \wedge dA_R + \frac{1}{3} A_R^3 \]

\[ A_{L,R} = \omega \pm \frac{e}{\ell} \]

This is a

$SL(2,\mathbb{R})_K \otimes SL(2,\mathbb{R})_{-K}$

Chern-Simons theory on
the disc with $K = 2\kappa$.  

Witten
The Chern-Simons theory has only boundary dynamics

\[ \mathcal{L} \subset \mathcal{L} (2, \mathbb{R})^2 \text{ CS Theory} \]

\[ \Rightarrow \text{ boundary} \]

\[ \Rightarrow \text{ \textit{It}H SL(2, \mathbb{R}) \text{ WZW theory w/ constraints}} \]

\[ \Rightarrow \text{ \textit{It}H Liouville theory with } \]

\[ c = 128 \text{ as predicted} \]

\[ \text{Liouville field } z \text{ boundary metric} \]

Are these boundary conditions physically appropriate?

Banados, Coussaert, Hennaux & Van Driel
Problems

0) Nature of Hilbert space - even vacuum - unclear.

1) Can't apply Cardy formula because of nonunitarity.

2) Do not seem to be black holes in theory on disc, which have nontrivial $SU(2,\mathbb{R})$ holonomy.
Perhaps there is a "pure gravity" theory w/ dynamical topology change & black holes

\[
\begin{array}{c}
\text{\rotatebox[origin=c]{90}{$\uparrow$}} \\
\text{\rotatebox{90}{$\circ$}}
\end{array}
\quad = \quad \begin{array}{c}
\text{\rotatebox{90}{$\circ$}}
\end{array}
\quad + \quad \begin{array}{c}
\text{\rotatebox{90}{$\circ$}}
\end{array}
\]

If so, it should be described by \( c = 12 \) CFT on boundary.

**Speculation**

The CFT in question \( SL(K,R)_2 \) is related by rank-level duality to \( SL(2,R)_K \).

Vafa-Carlip-AS

...on to string theory...
String Theory

\( Q_1 \cdot D1 + Q_5 \cdot D5 \)
black string on K3
\[ \quad \downarrow \text{near-horizon} \]
\( \text{AdS}_3 \times S^3 \times K3 \)

\( = \)

Higgs branch
D-brane gauge theory
\[ \text{IR} \]
\[ \Delta \text{limit} \]

1/1 CFT
on (deformation of)
\( \text{Sym}_{Q_1,Q_5}(K3) \)

After careful normalization
\( c = 12 g = 6 Q_1 Q_5 \)
\( c \) might have been computed this way on l.h.s. rather than counting dimensions on r.h.s.
$\text{AdS}_3 \leftrightarrow N\text{-}S \text{ N}\text{-}S \text{ vacuum of CFT}$

(spincors anti-periodic)

Note $M(\text{AdS}_3) = -1$

$= \frac{t}{2} (\ell_0 + \bar{\ell}_0)$

$\Rightarrow \ell_0 = -\frac{c}{24}$

*Cousset & Henneaux*

Excitations of $\text{AdS}_3 \leftrightarrow$ Excitations of

$N\text{-}S \text{ CFT}$

= Operators on the plane

The RR sector is complicated on both sides. Best understood through spectral flow from NSNS.
CFT

Chiral primaries

BPS Particles

States in AdS3

\[ \Psi \quad \Omega \]

\[ = \]

\[ \text{Modes of scalar field} \]

\[ \text{Maldacena, A. S.} \]

Bagger, Kay, Sezgin, Sundell

**Perfect intricate match, almost...**

Complete set of $\mathbf{\mathcal{G}}$

derived from cohomology of symmetries

\[ \mathcal{L}_1 = 0 \]

\[ \mathcal{L}_2 = 0 = J \]

\[ \mathcal{L}_3 = 0 = \mathcal{J} \]

\[ \text{K3 harmonic form} \]

\[ \text{Symmetry group (Z) \times (Z)} \]

\[ \text{AdS3} \]

\[ \text{Particle} \]

\[ \text{Reduction on N=1 supergravity} \]

\[ \text{Complete set from} \]

\[ L^2 = -l(l+1) \]

\[ m_l = \frac{-1}{\sqrt{6} \Omega} \]

\[ \Omega \]

\[ \begin{cases} 
\text{L} = 2l + \frac{1}{2} \\
\text{L} = 2l + 1 \\
\text{L} = 2l \\
\text{L} = 2l + \frac{3}{2} \\
\text{L} = 2l + \frac{1}{2} \\
\text{L} = 2l - 1 \\
\text{L} = 2l - \frac{1}{2} \\
\text{L} = 2l - 2 \\
\text{L} = 2l - 3 \\
\text{L} = 2l - \frac{3}{2} \\
\text{L} = 2l - 1 \\
\end{cases} \]
The Stringy Exclusion Principle

The operator $O$ corresponds to a single boson in a particular mode of the field $\Phi$.

$O^2$ is a new chiral primary. It corresponds to two particles in the same mode. Since

$O^{n=0}, n > 2 \Phi, Q_5 \sim \frac{1}{g^2}$

there is an exclusion principle which cannot be seen in string/sugra perturbation theory. Similar phenomena have been encountered elsewhere.

Gross, 't Hooft, Polchinski.
Where is the entropy?

Under spectral flow $RR=NS-NS$ the states responsible for BH entropy

$$S_{BH} = 2\pi \sqrt{N} Q_1 Q_5$$

become right-chiral primaries.

In general for $N$ levels in 17 $Q_1 Q_5$ supergravity cannot be trusted, as seen in the exclusion principle. So a priori we might not find these states, and they are just described as a black hole in AdS picture. But maybe supersymmetry can help us ....
Conjectural Ads Description of Black Hole Entropy

The maximal number of particles in a right-left chiral primary is \( q_1 q_5 \):

\[
0, 0, 0, \ldots, 0, q_1 q_5 1 0 7
\]

where \( \omega = \xi = \frac{1}{2} \) for each \( \Theta_i \). Consider the general level \( n \) right-chiral state

\[
\prod_{i=1}^{q_1 q_5} (\xi - n_i \Theta_i) 1 0 7
\]

\[
\sum n_i = n
\]

where \( \xi \) is a super-Virasoro generator.

The number of such states is

\[
S = 2 \pi \sqrt{q_1 q_5 n'} = S_{\text{BH}}
\]

But problems w/ this picture (Maldacena & A5)
Concluding Challenge

Find what's wrong with this picture:

and why black hole evolution in time is unitary!