Quantum Gravity in the Bad Old Days

\[ Z = \int d[g_{\mu\nu}] \exp i \int \mathcal{S} g R \]

Divergences

Topology Change

Information Loss
THE HOLOGRAPHIC PRINCIPLE

\[ S_{\text{max}} = \frac{\text{Area}}{4G\hbar} \leq \text{Volume} \]

NUMBER OF D.O.F. \approx S_{\text{max}}
One bit per unit transverse area in L.C.F.
Matrix Theory (SYM) = DLCQ of M-T

Holography must be a property of SYM.

\begin{align*}
\text{3-Torus} & \\
\text{M-theory Torus} & \\
\text{SYM Torus} & \\
S &= N^2 \Sigma^3 T^3 \\
\langle X^2 \rangle &= \frac{L^3}{L} N \Sigma^2 T^2 = G_8^{\text{th}} N \Sigma^2 T^2 \\
\text{Area} = \langle X^6 \rangle &= G_8 N^3 \Sigma^6 T^6 \\
G_8 \frac{S}{A} &= \frac{1}{T^3 \Sigma^3 N}
\end{align*}
\[ \frac{G_0 S}{A} = \frac{1}{T^3 \Sigma^3 N} \]

Recall \( \Sigma_{\text{eff}} \sim N^{1/3} \Sigma \)

\[ T_{\text{crit}} = \frac{1}{\Sigma N^{1/3}} \]

Phase transition at \( \frac{G_0 S}{A} \approx 1 \)

\begin{align*}
N &> S \\
N &\sim S & T = T_{\text{crit}} \\
N &< S
\end{align*}
The case of $\text{AdS}_5 \times S_5$

A spatial section of AdS

$$dS^2 = R^2 \left[ \frac{4 d\Omega^2}{(1-r^2)^2} - \frac{(1+r^2)^2}{(1-r^2)^2} dt^2 \right]$$
Supergravity/IIb string theory in $AdS_5 \times S_5$
is equivalent to 3+1 dim super Yang Mills
on the boundary $(S^3 \times R) \times S_5$

\[ (g_{\text{YM}}^2 N)^{1/4} \ell_5 = R \text{ (radius of } AdS_5 \times S_5) \]

In some sense holographic but
in what sense is the number of
D.O.F. = 70 \text{ per Planck area?}
**Number of S.Y.M Degrees of Freedom**

is \( \infty \) for 2 reasons

1. Continuum QFT
2. \( N \to \infty \)

That's OK because boundary sphere has \( \infty \) area.

Can we regulate these \( \infty \)'s?

The U.V., I.R. connection

\( E.W. \)

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**Diagram:**

- **SYM (radius = 1)**
  - Regulator length = \( \delta \)
- **SUGRA/String**
  - \( R = 1 - \delta \)
An Example of UV, IR

\[ \text{Area} = \Delta t \int \sqrt{g_{00} g_{rr}} \, dr \]

\[ \text{Energy} = \int \sqrt{g_{00} g_{rr}} \, dr = \int \frac{1 + r^2}{(1 - r^2)^2} \, dr \times R^{d-1} \]

This is linearly divergent at \( r = 1 \)
It reflects the \( \infty \) U.V. energy of a point charge.

Integrating to \( r = 1 - \delta \) gives

\[ \text{Energy} \sim \frac{1}{\delta} \]

As would be expected from U.V. cutoff.
Now Count

\[ N_{DOR} = \frac{N^2}{g^3} \]

SYM SIDE

**Area of AdS**

\[ \text{Boundary} = \frac{R^3}{g^3} \]

Gravity Side

\[ N_{DOR} = \frac{N^2}{R^3} \text{ Area} \]

*Now use:*

\[ g_{ym}^2 N = R^4 \lambda_5^4 \]

\[ g_{ym}^2 = g_{st} \]

\[ \frac{g_{st} \lambda_5}{R^5} = \frac{1}{G_5} \]

\[ N_{DOR} = \frac{\text{Area}}{G_5} \]
THE RADIAL DIRECTION IS SCALE SIZE

UV → IR → UV → IR
Holography + Cosmology

\[ ds^2 = a(t)^2 dx^i dx^i \]

\[ a = t^p \]

\[ t = 0 \]

\[ S_H \leq A_H \]

Today

\[ S_H \sim 10^{50} - 10^{100} \]

\[ A_H \sim 10^{120} \]

Tomorrow

\[ X_H \sim t^{1-p} \]

\[ S_H \sim X_H^d = t^{d-dp} \]

\[ A_H \sim (aX_H)^{d-1} \sim t^{d-1} \]

\[ d-1 \geq d-dp \]

or

\[ p \geq \frac{1}{d} \]

Bound on expansion rate
EoS of state:

\[ \text{Pressure} = \gamma (\text{energy density}) \]

\[ a(t) \sim t^{2 \over 3(1+\gamma)} = t^p \]

Bound \( \gamma \leq 1 \)

Standard Causal Bound

Yesterday:

Extrapolating backward one finds

\[ \frac{S_H}{A_H} \approx 10^{-28} \left( \frac{t_d}{t} \right)^{1/2} \]

\( t_d = \text{decoupling time} \)

At Planck time

\[ \frac{S_H}{A_H} \sim 1 \]