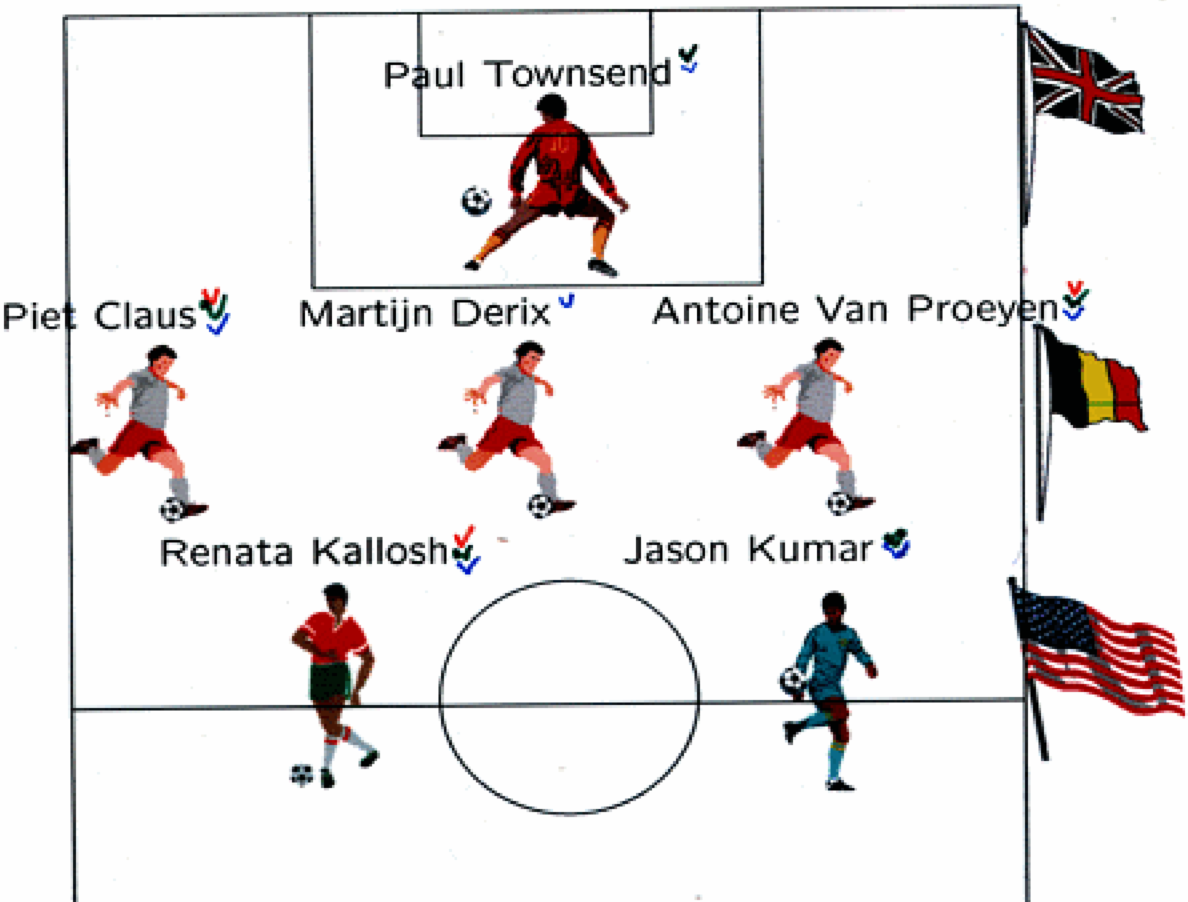
From anti-de Sitter background to superconformal world sheet symmetry



M5: hep-th/9711161

general bosonic: hep-th/9801206

black holes and superconformal mechanics: hep-th/9804177

supersymmetric theory: ongoing

In short

- Solution of the supergravity theory
  $\rightarrow$ Killing vectors and spinors $\rightarrow$ supergroup
- World-sheet theory where the supergroup gives rigid symmetries
- local GCT and κ-symmetry.
- gauge fixing leads to
  a conformal world-sheet theory.
- Application:
  $d = 4$, $N = 2$ supergravity solution
  (near horizon Reissner-Nordström black hole)
  leads to world-line theory which is
  '*relativistic superconformal mechanics*'.

Plan

1. Bosonic theory
   - Brane solutions
   - Killing vector adS algebra
   - world-volume action
   - conformal algebra

2. Supersymmetric world-volume theory
   - solutions of supergravity and Killing vectors/spinors
   - supergroups
   - world-volume action
   - κ-symmetry
   - gauge fixing leads to superconformal symmetry

3. Conclusions

'relativistic superconformal mechanics': example developped in parallel to the general presentation.

1. Brane solutions with adS Killing vector algebra

$(m = 0, 1, \ldots p;\ m' = 1, \ldots d - p - 1)$: $\quad d = 4$, 10 or 11

$$
\begin{aligned}
ds^2 &= H_{\text{brane}}^{-\frac{2}{p+1}} dx_m^2 + H_{\text{brane}}^{\frac{2}{d-p-3}} dX_{m'}^2 \\
H_{\text{brane}} &= 1 + \left(\frac{R}{r}\right)^{d-p-3} \quad ; \qquad r^2 = X^{m'} X^{m'} \\
H_{\text{hor}} &= \lim_{r \to 0} \left[1 + \left(\frac{R}{r}\right)^{d-p-3} \right] = \left(\frac{R}{r}\right)^{d-p-3}
\end{aligned}
$$

- brane solution interpolates beween asymptotically flat and near horizon anti-de Sitter geometry

  Gibbons, Townsend

- large N (many branes solution) $(R^x \propto N)$

  Maldacena

- there is a special duality transformation that removes the constant

  Hyun

  Boonstra, Peeters, Skenderis

  Cremmer, Lavrinenko, Lü, Pope, Stelle, Tran anti-de Sitter Killing vector algebra

$$ds^2_{hor} = \left(\frac{r}{R}\right)^{\frac{2(d-p-3)}{p+1}} dx^2_m + \left(\frac{R}{r}\right)^2 dr^2 + R^2 d^2\Omega$$

is $adS_{p+2} \times S^{d-p-2}$ metric.

Killing vectors $SO(p+1,2) \times SO(d-p-1)$

Example: $p = 0,\ d = 4.$ Rename $R \to M$

$$\begin{aligned} ds^2 &= -H^{-2}_{\text{brane}} dt^2 + H^2_{\text{brane}} dX^2_{m'} \\ H_{\text{brane}} &= 1 + \frac{M}{r} \end{aligned}$$

Is Reissner-Nordstrom (RN) black hole.

Near horizon geometry

$$ds^2_{hor} = -\left(\frac{r}{M}\right)^2 dt^2 + \left(\frac{M}{r}\right)^2 dr^2 + M^2 d^2\Omega$$

which is the Bertotti–Robinson (BR) metric.

$adS_2 \times S^2$.

$\delta t = a + b\,t + c\,t^2 + c(M^4/r^2)$; $\qquad \delta r = -r\,(\partial_t \delta t)$

$SO(1,2)$ and $SO(3)$ isometries of $d^2\Omega$.

This scheme works in various theories:

	d	p
M5	11	5
M2	11	2
D3	10	3
Self-dual string (D1+D5)	6	1
Magnetic string	5	1
Tangerlini black hole	5	0
Reissner-Nordström black hole	4	0

All have $adS_{p+2} \times S^{d-p-2}$ geometry.

E.g. self-dual string can also be obtained from 10 dimensions with $D1 + D5$, having $adS_3 \times S^3 \times E_4$.

More generalizations, with products of spheres, ...

Boonstra, Peeters, Skenderis, hep-th/9803231

Bosonic world-volume theory

$$
\begin{aligned}
S_{cl} &= S_{Born-Infeld} + S' + S_{Wess-Zumino} \\
S_{BI} &= -\int d^{p+1}\sigma \sqrt{-\det \mathcal{G}_{\mu\nu}} \\
\mathcal{G}_{\mu\nu} &= g^{\text{ind}}_{\mu\nu} + T_{\mu\nu} \\
g^{\text{ind}}_{\mu\nu} &= \partial_\mu X^M \partial_\nu X^N G_{MN}
\end{aligned}
$$

with

RN, $M2$)	$T_{\mu\nu} = 0$	$S' = 0$
$D3$	$T_{\mu\nu} = F_{\mu\nu}$	$S' = 0$
$M5$	$T_{\mu\nu} = i\mathcal{H}^*_{\mu\nu}$	$S' = \int d^6\sigma\, \mathcal{H}^{*\mu\nu}\mathcal{H}_{\mu\nu}$

a: auxiliary field of Pasti-Sorokin-Tonin appears in

$$
u_\mu = \partial_\mu a\ ; \qquad \mathcal{H}_{\mu\nu\rho} = 3\partial_{[\mu} B_{\nu\rho]}
$$

$$
\mathcal{H}_{\mu\nu} = \frac{u^\rho}{\sqrt{u^2}}\mathcal{H}_{\mu\nu\rho}\ ; \qquad \mathcal{H}^*_{\mu\nu} = \frac{u^\rho}{\sqrt{u^2}}\mathcal{H}^*_{\mu\nu\rho}
$$

Geometric input:

- G_{MN}, solution of supergravity
- Wess-Zumino term

BR metric:

$$ds^2 = -\left(\frac{r}{M}\right)^2 dt^2 + \left(\frac{M}{r}\right)^2 \left(dr^2 + r^2 d^2\Omega\right)$$

$r = 0$ is coordinate singularity. Define ρ

$$\frac{r}{M} = \left(\frac{2M}{\rho}\right)^2$$

$$ds^2 = -\left(\frac{2M}{\rho}\right)^4 dt^2 + \left(\frac{2M}{\rho}\right)^2 d\rho^2 + M^2 d^2\Omega$$

Bosonic part of WZ:

$$S_{WZ} = \int A \; ; \qquad A = \frac{r}{M} \dot{t}\, d\tau$$

World-line action

$$
\begin{aligned}
S &= mS_{BI} + qS_{WZ} \\
&= -m\int d\tau \sqrt{-g_{00}^{ind}} + q\int A \\
g_{00}^{ind} &= \left(\frac{2M}{\rho}\right)^4 \left[-(\dot{t})^2 + \left(\frac{\rho}{2M}\dot{\rho}\right)^2\right] \\
&\quad + M^2\left[\dot{\theta}^2 + \sin^2\theta\dot{\phi}\right]
\end{aligned}
$$

Gauge of time reparametrizations: $t = \tau$.

In Hamiltonian language

$$
H = \left(\frac{2M}{\rho}\right)^2 \left[\sqrt{m^2 + \frac{\rho^2 p_\rho^2 + 4L^2}{4M^2}} - q\right],
$$

$(L^2 = p_\theta^2 + \sin^{-2}\theta p_\phi^2)$

is $\boxed{H = -p_0}$ solving

$$
(p_0 - qA)^2 G^{00} + p_{m'} G^{m'n'} p_{n'} + m^2 = 0 .
$$

charged particle in BR background.

$$\boxed{H = \frac{p_\rho^2}{2f} + \frac{mg}{\rho^2 f}}$$

$$f = \frac{1}{2}\left[\sqrt{m^2 + (\rho^2 p_\rho^2 + 4L^2)/4M^2} + q\right]$$

$$m\,g = 2M^2(m^2 - q^2) + 2L^2$$

Limit

$M \to \infty$; $(m-q) \to 0$; $M^2(m-q)$ fixed

gives $f \to m$, and is conformal mechanics of de Alfaro, Fubini and Furlan, 1976.

'non-relativistic conformal mechanics' (large black hole mass).

The full solution we get from the 'brane-like' procedure: *'relativistic conformal mechanics'*.

With $L = 0$, force vanishes when $m = q$.

Combination of symmetries $\rightarrow$

conformal symmetry

- Rigid symmetries

  - $SO(3)$ rotations on θ, ϕ

  - anti-de Sitter

$$\begin{aligned} \delta t &= a + b\,t + c\,t^2 + c\frac{M^4}{r^2} \\ \delta r &= -r\,(b + 2c\,t) \end{aligned}$$

- Local symmetry: time reparametrizations

$$\delta t = \xi(\tau)\dot{t}\ ; \qquad \delta r = \xi(\tau)\dot{r}\ ; \qquad \ldots$$

Gauge fixing $t = \tau$

$$0 = a + b\tau + c\tau^2 + c\frac{M^4}{r^2} + \xi(\tau)$$

$a =$ translations
$b =$ dilatations
$c =$ special conformal transformations

$SO(2,1)$ finite conformal group in 1 dimension

PS: Notes about infinite dimensional groups in case $p = 1$

1. Brown and Henneaux, 1986:
consider also other geometries which have adS_3 as near-horizon limit. Symmetries between such asymptotic geometries form Virasoro algebra of which $SO(2,2) = SU(1,1) \times SU(1,1)$ is finite dimensional subgroup.

2. F. Brandt, J. Gomis and J. Simón, hep-th/9707063 and 9803196:
There are extra symmetries of $\int d^2\sigma \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})}$

$$\begin{aligned} \delta X^M &= h^M(X)\,\lambda(\mathcal{F}) \\ \delta V_\mu &= -\lambda'(\mathcal{F})\sqrt{g}\,(1+\mathcal{F}^2)\epsilon_{\mu\nu}\left(\partial^\nu X^M\right) h_M(X) \end{aligned}$$

where

$$h^M(X) \quad : \quad \text{Killing vectors of the metric } G_{MN}$$

$$\mathcal{F} = -\frac{\epsilon^{\mu\nu}F_{\mu\nu}}{2\sqrt{g}}$$

The arbitrary function λ thus provides a sort of Kač-Moody extension of the isometry group.

2. Supersymmetric theory

In terms of $Z^\Lambda = \{X^M, \theta^A\}$,

superspace coordinates in $d = 11, 10, 4, \ldots$

and possibly other forms.

$$g^{\text{ind}}_{\mu\nu} = \left(\partial_\mu Z^\Lambda E_\Lambda{}^{\underline{M}}\right)\left(\partial_\nu Z^\Sigma E_\Sigma{}^{\underline{N}}\right)\eta_{\underline{MN}}$$

(underline is flat index)

$d = 4$, $p = 1$ example:

comes from $N = 2$, $d = 4$ supergravity.

$A \leftarrow (\alpha i)$; $i = 1, 2$, α spinor index

$$S_{WZ} = \int d\tau\, \dot{Z}^\Lambda A_\Lambda$$

E.g. flat superspace

$$E_M{}^{\underline{M}} = \delta_M{}^{\underline{M}}\ ; \qquad E_{\alpha i}{}^{\underline{M}} = \tfrac{1}{2}\left(\gamma^{\underline{M}}\theta_i\right)_\alpha$$

$$A_M = 0\ ; \qquad A_{\alpha i} = \tfrac{1}{2}\varepsilon_{ij}\theta^j_\alpha$$

leads to

$$g^{\text{ind}}_{\tau\tau} = \left(\dot{X}^M - \tfrac{1}{2}\bar{\theta}\Gamma^M\dot{\theta}\right)\left(\dot{X}^N - \tfrac{1}{2}\bar{\theta}\Gamma^N\dot{\theta}\right)\eta_{MN}$$

$$S_{WZ} = \frac{1}{\sqrt{2}}\int d\tau\, \dot{\bar{\theta}}^i \varepsilon_{ij}\theta^j + h.c.$$

Start from solutions of supergravity

ds^2 as BR with mass M (near-horizon)

Soln. with electric charge Q and magnetic P :

$$F_{0r} = -\frac{Q}{M^2}\ ; \qquad F_{\theta\phi} = -P\sin\theta$$

with $P^2 + Q^2 = M^2$.

For Killing spinors, define 2×4 real spinors

$$(\mathcal{P}_\pm)_A{}^B = \frac{1}{2}\left(\delta_A^B \pm \frac{1}{M}\left(Q + i\gamma_5 P\right)\varepsilon^{ij}\gamma_0\right)$$

Killing spinors are

$$(\mathcal{P}_+\epsilon)^i = \left(\frac{M}{r}\right)^{1/2}\eta_+^i$$

$$(\mathcal{P}_-\epsilon)^i = \left(\frac{r}{M}\right)^{1/2}\left(\eta_-^i - \frac{t}{M}\gamma_{0r}\eta_+^i\right)$$

where $\eta_\pm^i$ are Killing spinors of sphere $\hat{m} = \theta, \phi$

$$\nabla_{\hat{m}}\eta_\pm^i(\theta,\phi) = \mp\frac{1}{2M}\gamma_r\gamma_{\hat{m}}\eta_\pm^i(\theta,\phi)$$

4 solutions for each sign.

Lü, Pope, Rahmfeld, hep-th/9805151

Algebra of Killing spinors and vectors: $SU(1,1|2)$

Commutators give adS and $SO(3)$ of sphere.

$$
\begin{aligned}
[\eta_+, \eta_+] &= P \\
[\eta_+, \eta_-] &= D + SO(3) \\
[\eta_-, \eta_-] &= K
\end{aligned}
$$

or

$$
\begin{pmatrix} SU(1,1) & \eta^i_\pm \\ \eta^i_\pm & SU(2) \end{pmatrix}
$$

Should always be of a similar form, with

1. $SO(p+1,2)$ should appear as factor in bosonic part of the superalgebra.

2. fermionic generators in a spinorial representation of that group.

$\rightarrow$ more bosonic symmetries (R)
should appear as symmetries of non-adS part of the target space (here S^2)

First see isomorphisms of conformal groups

$$\begin{array}{llll} p=0 & SO(1,2) & \sim & SU(1,1) \sim Sp(2) \\ p=1 & SO(2,2) & \sim & SO(1,2) \times SO(1,2) \sim SU(1,1) \times SU(1,1) \\ p=2 & SO(3,2) & \sim & Sp(4) \\ p=3 & SO(4,2) & \sim & SU(2,2) \end{array}$$

Results for $p \geq 2$:

p	superalgebra	R	nr.ferm.
2	$OSp(N\|4)$	$SO(N)$	$4N$
3	$SU(2,2\|N)$	$U(N)$ for $N \neq 4$	$8N$
		$SU(4)$ for $N = 4$	
4	$F(4)$	$SU(2)$	16
5	$OSp(6,2\|2N)$	$USp(2N)$	$16N$

$p=0$ (or 2 factors for $p=1$)

superalg.	R	nr.ferm.
$OSp(N\|2)$	$O(N)$	$2N$
$SU(N\|1,1)$ $(N \neq 2)$	$U(N)$	$4N$
$SU(2\|1,1)$	$SU(2)$	8
$OSp(4^*\|2N)$	$SU(2) \times USp(2N)$	$8N$
$G(3)$	G_2	14
$F(4)$	$SO(7)$	16
$D^1(2,1,\alpha)$	$SU(2) \times SU(2)$	8

Examples

	sAlg.	$\underline{G}$	
M5	$OSp(6,2\|4)$	$SO(5)$	$adS_7 \times S^4$
M2	$OSp(8\|4)$	$SO(8)$	$adS_4 \times S^7$
D3	$SU(2,2\|4)$	$SO(6)$	$adS_5 \times S^5$
D1+D5	$(SU(1,1\|2))^2$	$SO(4)$	$adS_3 \times S^3$
BR	$SU(1,1\|2)$	$SO(3)$	$adS_2 \times S^2$

World-volume (world-line) action

BI-term, based on supervielbeins, ...

can be obtained from 'Gauge completion'

Nath, Arnowitt, PL 65B(76)73
Cremmer, Ferrara, PL 91B(80)61
Castellani, van Nieuwenhuizen, Gates, PRD22(80)2364
de Wit, Peeters, Plefka, hep-th/9803209

WZ-term starts now from solution $A_\mu = W_\mu$, supergravity solution

$$W_0 = \frac{Q}{M^2} r \; ; \qquad W_\phi = P \cos\theta$$

Other way: supercosets: here

$$\frac{SU(1,1|2)}{U(1) \times U(1)}$$

Castellani, Ceresole, D'Auria, Ferrara, Frè, Trigiante
hep-th/9803039
Matsaev, Tseytlin, hep-th/9805028 and 06095
Kallosh, Rajaraman, hep-th/9805041

κ-symmetry

World-volume theory has the rigid symmetries

> $N = 4$, $d = 2$ has 8 real supersymmetries, and the solutions which we considered have 8 real Killing spinors (2 complex doublets of $SU(2)$).

But also κ symmetry. This imposes $q = m$

$$\delta_\kappa \theta = (1 + \Gamma)\kappa \; ; \qquad \Gamma = \frac{1}{\sqrt{-g_{00}}} \varepsilon^{ij} \gamma_{\underline{M}} \Pi^{\underline{M}}$$

Γ is complicated matrix, but $\Gamma^2 = 1$.

At 'classical values'

$t = \tau$; $r = r_0$; $\theta = \theta_0$; $\phi = \phi_0$; fermions $= 0$

$$\Gamma_{cl} = \varepsilon^{ij} \gamma_0 \; ; \qquad 1 + \Gamma_{cl} = 2\mathcal{P}_-$$

if $P = 0$ (only electr. charged $Q = M$)

zero force if $P = 0$, thus $Q = \pm M$

Irreducible κ symmetry: $\mathcal{P}_- \kappa = 0$.

$$\delta_\kappa \theta = \mathcal{P}_+ \kappa + ...$$

Local symmetries

- world-volume diffeomorphisms
- Kappa symmetry

Broken by gauge choice:
reformulation as $\boxed{adS_{p+2} \Rightarrow Conf_{p+1}}$

Remember bosonic:

$$\delta_{adS} r = -r\,(b + 2c\,t) \;; \qquad \delta_{gct} r = \xi(\tau)\dot{r}$$

Gauge choice: $\delta t = a + b\,\tau + c\,\tau^2 + c\frac{M^4}{r^2} + \xi(\tau) = 0$
r transforms as a scalar with Weyl weight 1.

Fermionic:

$$\delta \mathcal{P}_+ \theta^i = \left(\frac{M}{r}\right)^{1/2} \eta^i_+ + \kappa^i + \ldots.$$
$$\delta \mathcal{P}_- \theta = \left(\frac{r}{M}\right)^{1/2} \left(\eta^i_- - \frac{t}{M}\gamma_{0r}\eta^i_+\right) + \ldots$$

Gauge fixing

$$\mathcal{P}_+\theta = 0$$

Conformal supersymmetry: where
η_- takes role of Q-supersymmetry
η_+ of S-supersymmetry.

Summary

We establish superconformal symmetry of the gauge-fixed non-gravitational brane actions

Starting from solution of supergravity with background $adS_{p+2} \times S^{d-p-2}$ geometry.

Non-relativistic superconformal mechanics was done by
Akulov and Pashnev, 1983.
Fubini and Rabinovici, 1984.

We obtain

'*relativistic superconformal mechanics*'

having as limit $M \to \infty$ non-relativistic superconformal mechanics.

PS: there is still a simplification, keeping only the radial mode. Based on $OSp(1|2) \subset SU(2|1,1)$.

We qualified for the case

d p

4 - 1