# Domain Walls in Gauged Supergravity

compactified string/M-theory

codimension-one objects

Exploration of gravitational effects

Goals:

(i) ~~Implications for AdS/CFT correspondence (renormalization group flow; bound state spectra of strongly coupled field theories)~~

(ii) Implications for Randall-Sundrum scenario (localization of gravity)

focus

implementation in fundamental theory

(a) W/ K. Behrndt hep-th/9909058, /0001159

parallel effort

Kallosh, Linde & Shmakova hep-th/9910021

Kallosh & Linde hep-th/0001071

massless modes of gauged supergravity

"no-go" theorem for RS type walls

W/ H. Lü & C. Pope hep-th/0001002, /0002054

(b) massive-breathing mode - a candidate field for gravity trapping walls

(c) Earlier work on supergravity walls

($D=4$, $N=1$)

w/ S. Griffies & S.-J. Rey '92

first examples of smooth BPS walls

w/ H. Soleng (generalizations):

Phys. Rept. 289, 159 (1997)

Generalization to thin walls in D-dim:

w/ J. Wang hep-th/9912187

# Outline:

I. Domain walls in supergravity

flat (Minkowski) walls $\Longleftrightarrow$ BPS walls
$\sigma = \sigma_{BPS}$

bent (de Sitter walls $\Longrightarrow$ $\sigma > \sigma_{BPS}$
anti-de Sitter) $\sigma < \sigma_{BPS}$

[illustrate for $D=4$ $N=1$ supergravity]

II. Domain walls in $D=5$ gauged supergravity

(a) $N=2$ gauged supergravity w/vector supermultiplets
["no-go" theorem for RS scenario;
relevant for AdS/CFT correspondence]

(b) massive-breathing mode, parametering the volume of compactified (Einstein) space, e.g. sphere
[a candidate for the gravity trapping wall]

# Illustration: Walls in D=4, N=1 Supergravity

## Bosonic Lagrangian:

$$\mathcal{L} = g_{A\bar{B}}\, \partial_\mu \phi^A \partial^\mu \phi^{\bar{B}} - V + \frac{R}{K}$$

$$V = g^{A\bar{B}}\, \partial_A \hat{W} \partial_{\bar{B}} \hat{W} - 3\frac{1}{M_{Pl}^2}\hat{W}^2$$

$$\hat{W} = \zeta\, |W|\, e^{\frac{K}{2 M_{Pl}^2}}$$

$\zeta = \{+1, -1\}$ (change sign iff $W = 0$)

$W(\phi)$ – superpotential,

$K(\phi, \bar{\phi})$ – Kähler potential

susy extrema $\partial_A \hat{W} = 0 \longrightarrow V_{ext} = 0 \Leftrightarrow \hat{W}_{ext} = 0$

$V_{ext} < 0 \Leftrightarrow \hat{W}_{ext} \neq 0$

Global SUSY

# Illustration: Walls in D=4, N=1 ~~Supergravity~~

$M_{Pl} \to \infty$

## Bosonic Lagrangian:

$$\mathcal{L} = g_{A\bar{B}}\, \partial_\mu \phi^A \partial^\mu \bar{\phi}^{\bar{B}} - V + \frac{R}{K}$$

$$V = g^{A\bar{B}}\, \partial_A \hat{W}\, \partial_{\bar{B}} \hat{W} \cancel{- 3 \frac{1}{M_{Pl}^2} \hat{W}^2}$$

$$\hat{W} = \zeta\, |W|\, e^{\cancel{\frac{K}{2 M_{Pl}^2}}}$$

$\zeta = \{+1, -1\}$ (change sign iff $W = 0$)

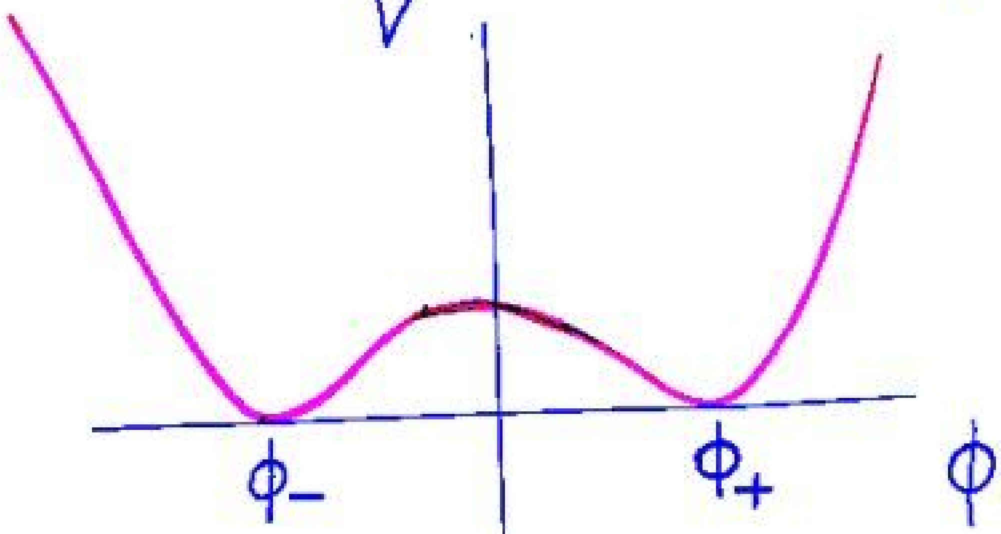
$W(\phi)$ – superpotential,

$K(\phi, \bar{\phi})$ – Kähler potential

susy extrema $\partial_A \hat{W} = 0 \longrightarrow V_{ext} = 0 \Leftrightarrow \hat{W}_{ext} = 0$

$$V_{ext} \overset{=}{\cancel{<}} 0 \Leftrightarrow \hat{W}_{ext} \neq 0$$

Isolated SUSY extrema:- <u>degenerate</u>




$\Downarrow$

(STATIC) DOMAIN WALLS

(supersymetic-BPS)

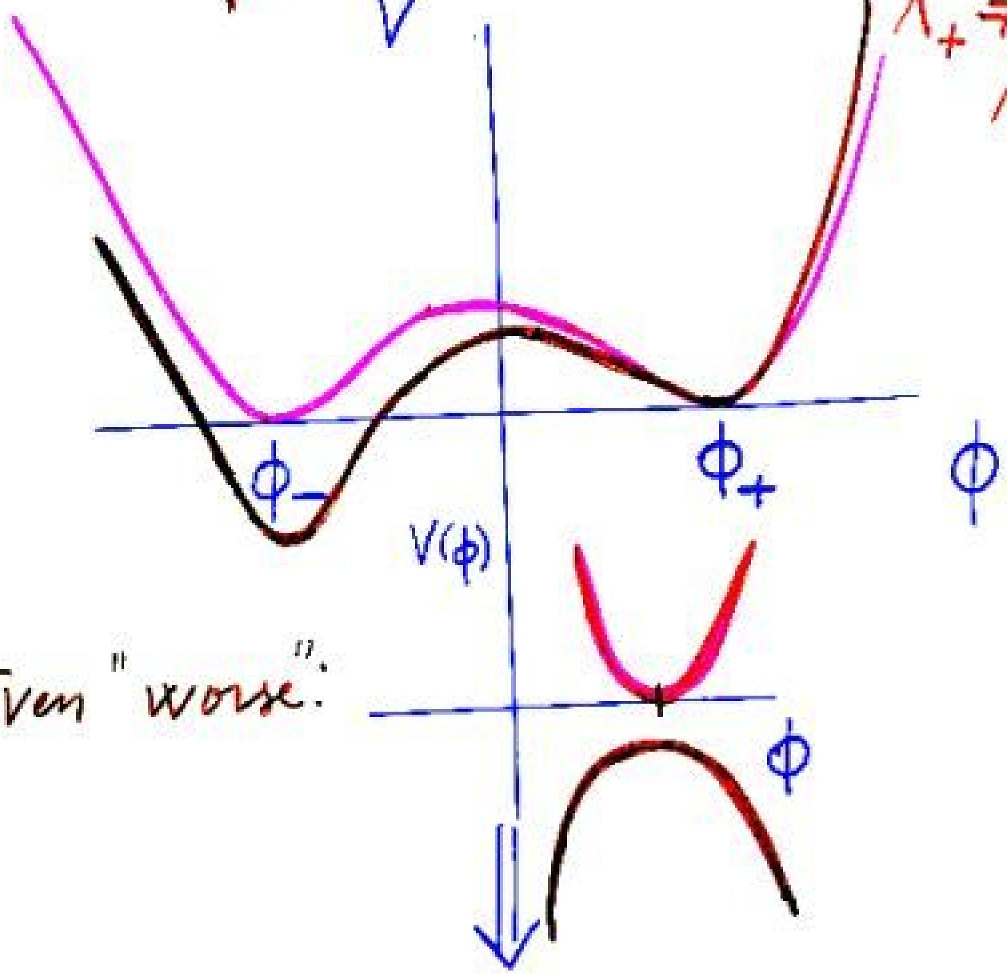
w/ Rey & Quevedo

Abraham & Townsend '91

Isolated local SUSY extrema:- ~~degenerate~~

$\Lambda_+ \neq \Lambda_-$

$\Lambda_\pm \leq 0$




Even "worse":

$\Downarrow$

(STATIC) DOMAIN WALLS

(supersymetic-BPS)

STILL EXIST!

MIRACLE OF SUPERSYMMETRY

w/ Rey & Quevedo

Abraham & Townsend '91

w/ Rey & Griffies '92

# Static (Flat) domain wall solutions beetween supersymmetric vacua

Killing spinor equations:

$\delta \psi^\alpha = 0$ Eq. for $\phi(z)$

$\delta \Psi_\mu^\alpha = 0$ Eq. for $A(z)$ & constraint on $\varepsilon$

gravitino

$$ds^2 = A(z)\left(-dt^2 + dx^2 + dy^2 + dz^2\right)$$

– horospheric coordinates

$$ds^2 = \mathcal{A}(\tilde{z})\left(-dt^2 + dx^2 + dy^2\right) + d\tilde{z}^2$$

– co-moving coordinates

use interchangably

Killing spinor eqs. $(D=4, N=1)$:

- $\partial_z \phi^A = 2 g^{A\bar{B}} \partial_{\phi^{\bar{B}}} \hat{W}$ $\qquad \hat{W} = \zeta |W| e^{\frac{K}{2}}$

* $\partial_z \ln A = -2\hat{W}$ $\qquad \zeta = \{+1, -1\}$

$[\, \varepsilon = \zeta \Gamma_z \varepsilon \,]$

Energy density:

$$\sigma_{BPS} = 2(\hat{W}_+ - \hat{W}_-) = 2\left(\zeta_+ \sqrt{\frac{-\Lambda_+}{3}} - \zeta_- \sqrt{\frac{-\Lambda_-}{3}}\right)$$

Asymptotics:

- $\phi \underset{z\to\pm\infty}{\sim} \phi_\pm + e^{2\partial_\phi^2 \hat{W}_\pm z}$ $\qquad \{\partial_\phi^2 W_- > 0; \partial_\phi^2 W_+ < 0\}$

* $A \underset{z\to\pm\infty}{\sim} e^{-2\hat{W}_\pm z}$

Sign$(\hat{W}_\pm)$ – determines asymptotic behavior of the metric

Necessary condition for exponential fall-off of $A(z)$ & $\sigma_{BPS} > 0$ $\Rightarrow$

$V(\phi_\pm)$ – <u>minimum</u>

# Type II : $Z_2$-symmetric example

$W = \lambda\phi\left(\frac{\phi^2}{3} - \eta^2\right)$

$K = \phi\phi^*$

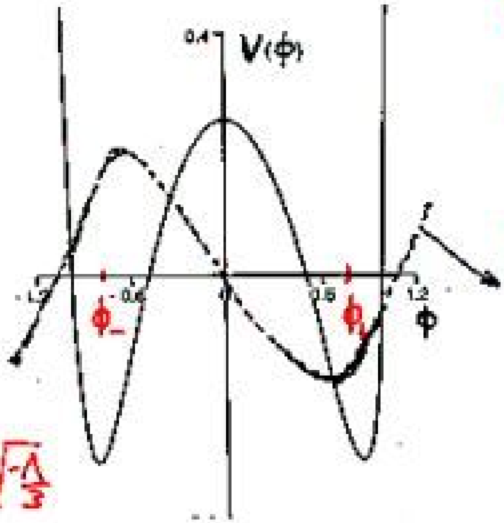
$\hat{W} = \eta |W| e^{\frac{K}{2}}$

$\phi \in \{\text{real}\}$

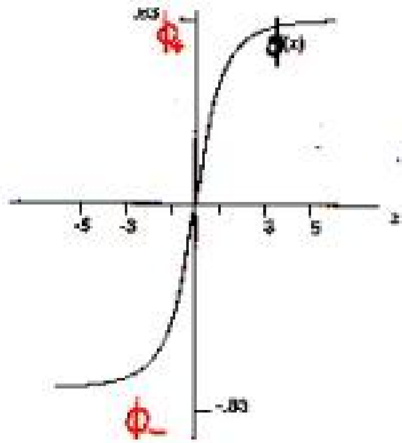
$\partial_\phi \hat{W}(\phi_\pm) = 0$

$\hat{W}(\phi_+) = -\hat{W}(\phi_-) = \sqrt{-\frac{\Lambda}{3}}$

$\sigma_{BPS} = 2\left[\hat{W}(\phi_+) - \hat{W}(\phi_-)\right] = 4\sqrt{-\frac{\Lambda}{3}}$

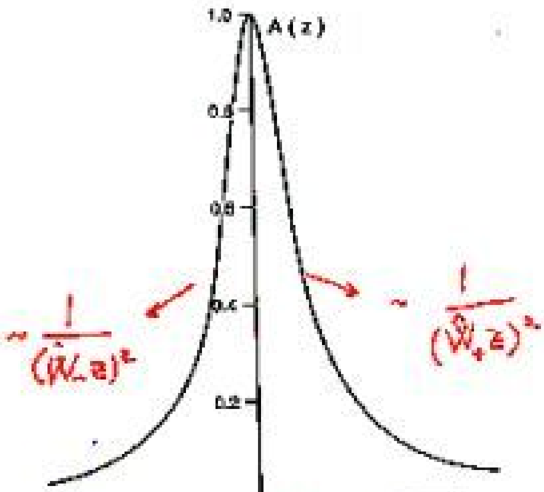



$\partial_z \phi = -2\,\partial_\phi \hat{W}$




$\partial_z\left(\frac{1}{\sqrt{A}}\right) = \hat{W}$

$\left[\epsilon = A^{\frac{1}{2}}(z)\,\epsilon_0\right]$

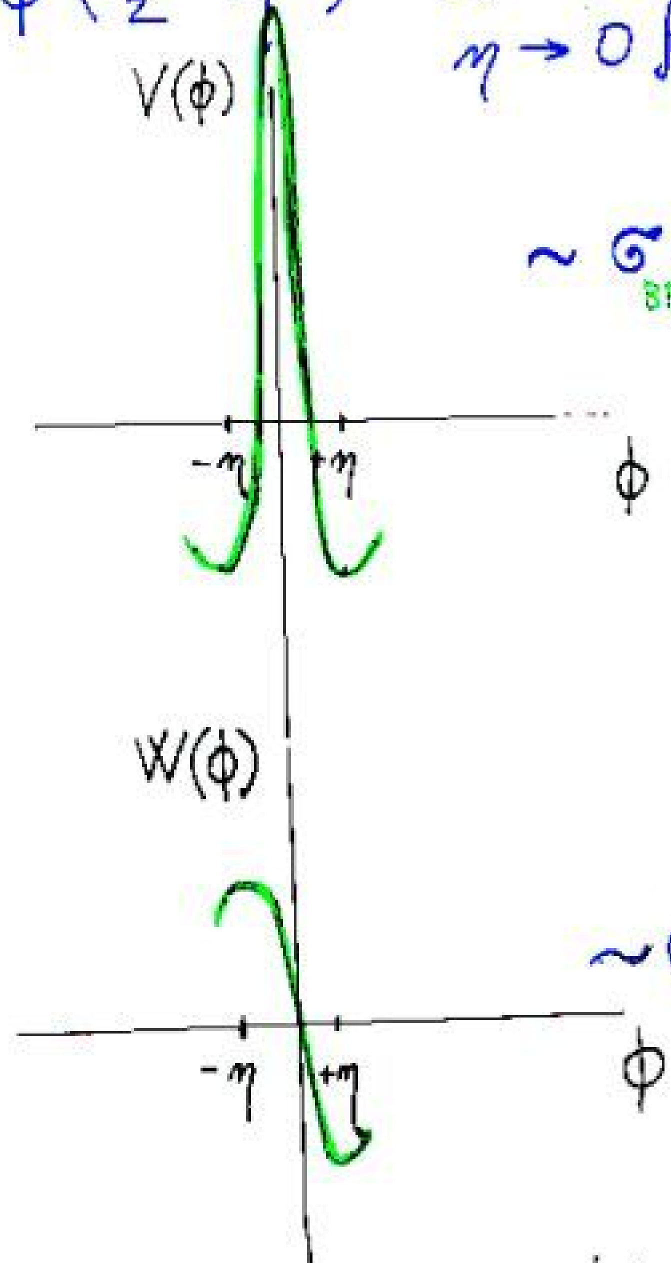



Relevant for RS-scenario

# Thin Wall limit:

$$W = \sqrt{\lambda}\,\phi\left(\frac{\phi^2}{2} - \eta^2\right)$$

$$\left.\begin{array}{l}\lambda \to \infty \\ \eta \to 0\end{array}\right\} \underbrace{\sqrt{\Lambda_\pm}}_{\sigma_{BPS}} \sim \sqrt{\lambda}\,\eta^3 = \text{const.}$$



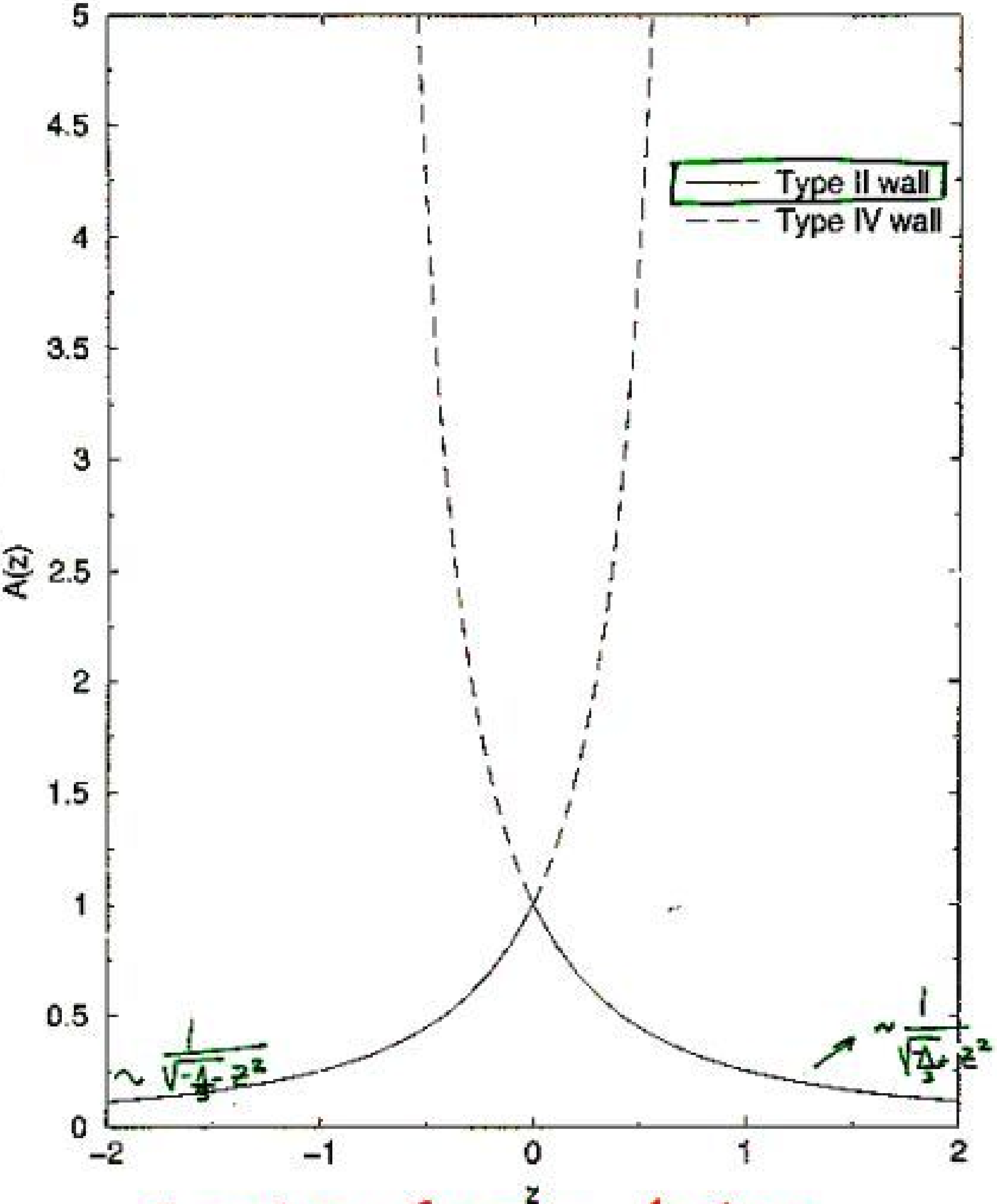

$\sim \sigma_{BPS}\,\delta(z)$

$\sim \sigma_{BPS}\,\Theta(z)$

[Supersymmetry embedding w/singular sources
Altendorfer et al.; Falkowski et al.; Bergshoeff, Kallosh & van Proeyen hep-th/0007044]

Flat (Minkowski) walls – thin limit

$$\sigma_{BPS} = 2\left(\sqrt{\frac{-\Lambda_+}{3}} + \sqrt{\frac{-\Lambda_-}{3}}\right)$$




$z \to \pm\infty$ Cauchy horizon

When supersymmetry is broken?

flat (Minkowski) walls

W/ Griffies & Soleng '93

⋮

Kaloper, Kraus

⋮

W/ Wang /9912187

$$\downarrow$$

bent walls:

Thin limit analysis (Type II):

(a) $\sigma > \sigma_{BPS}$ : $ds^2 = A(z)\,[\text{DeSitter} + dz^2]$

$\Lambda_{wall} = \beta^2 > 0$

$$\sigma = 2\sqrt{-\frac{\Lambda_+}{3}+\beta^2} + 2\sqrt{-\frac{\Lambda_-}{3}+\beta^2} > \sigma_{BPS}$$

$$A(z) = \frac{\beta^2}{\left[\sqrt{-\frac{\Lambda_\pm}{3}}\,\sinh\beta(z+z_\pm)\right]^2}$$

(b) $\sigma < \sigma_{BPS}$ : $ds^2 = A(z)\,[\text{Anti-deSitter} + dz^2]$

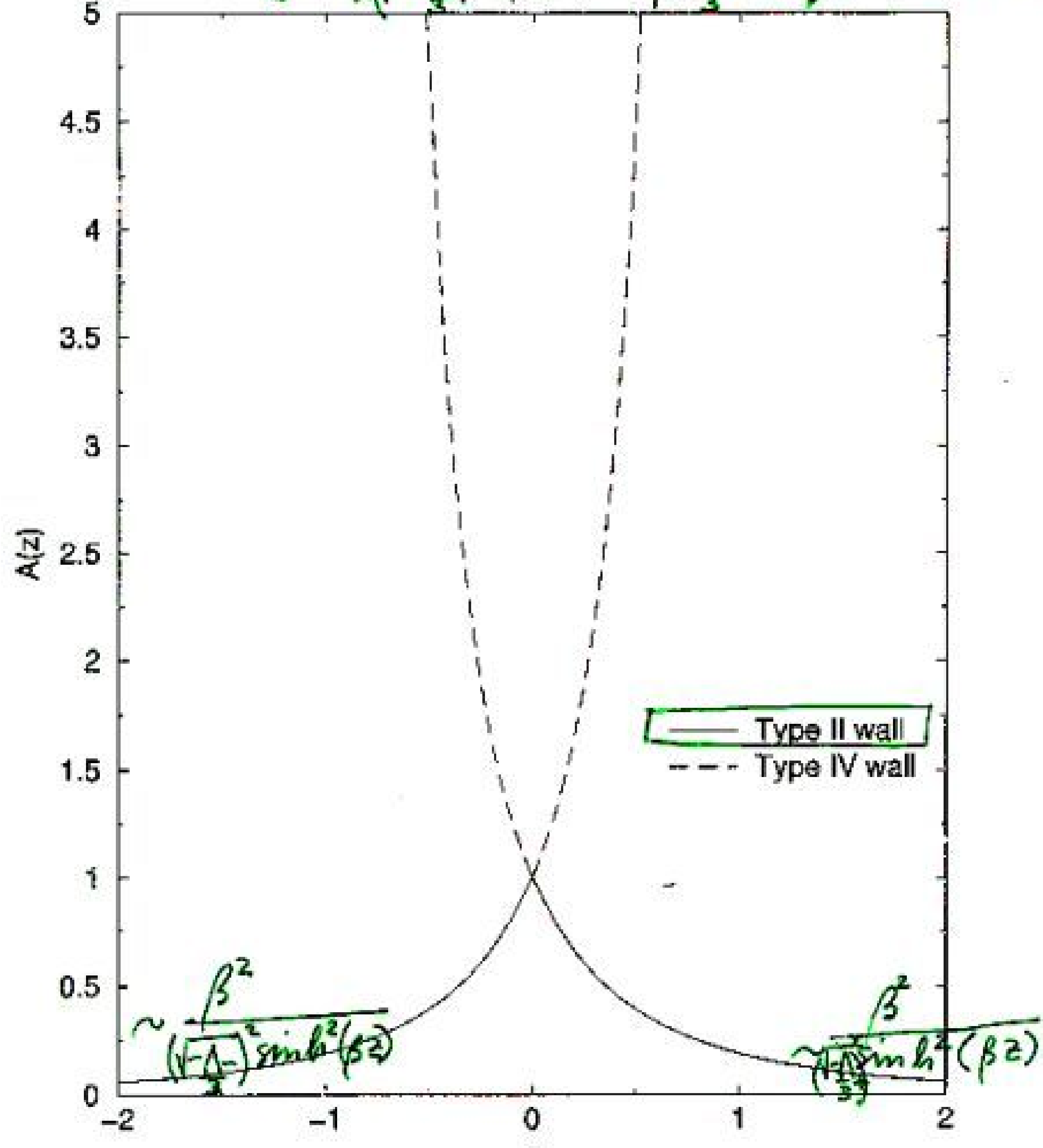
$\Lambda_{wall} = -\beta^2 < 0$

$$\sigma = 2\sqrt{-\frac{\Lambda_+}{3}-\beta^2} + 2\sqrt{-\frac{\Lambda_-}{3}+\beta^2} < \sigma_{BPS}$$

$$A(z) = \frac{\beta^2}{\left[\sqrt{-\frac{\Lambda_\pm}{3}}\,\cos\beta(z+z_\pm)\right]^2}$$

# Bent (de Sitter) walls

$$\sigma = 2\left(\sqrt{-\frac{\Lambda_+}{3}+\beta^2} + \sqrt{-\frac{\Lambda_-}{3}+\beta^2}\right) > \sigma_{BPS}$$

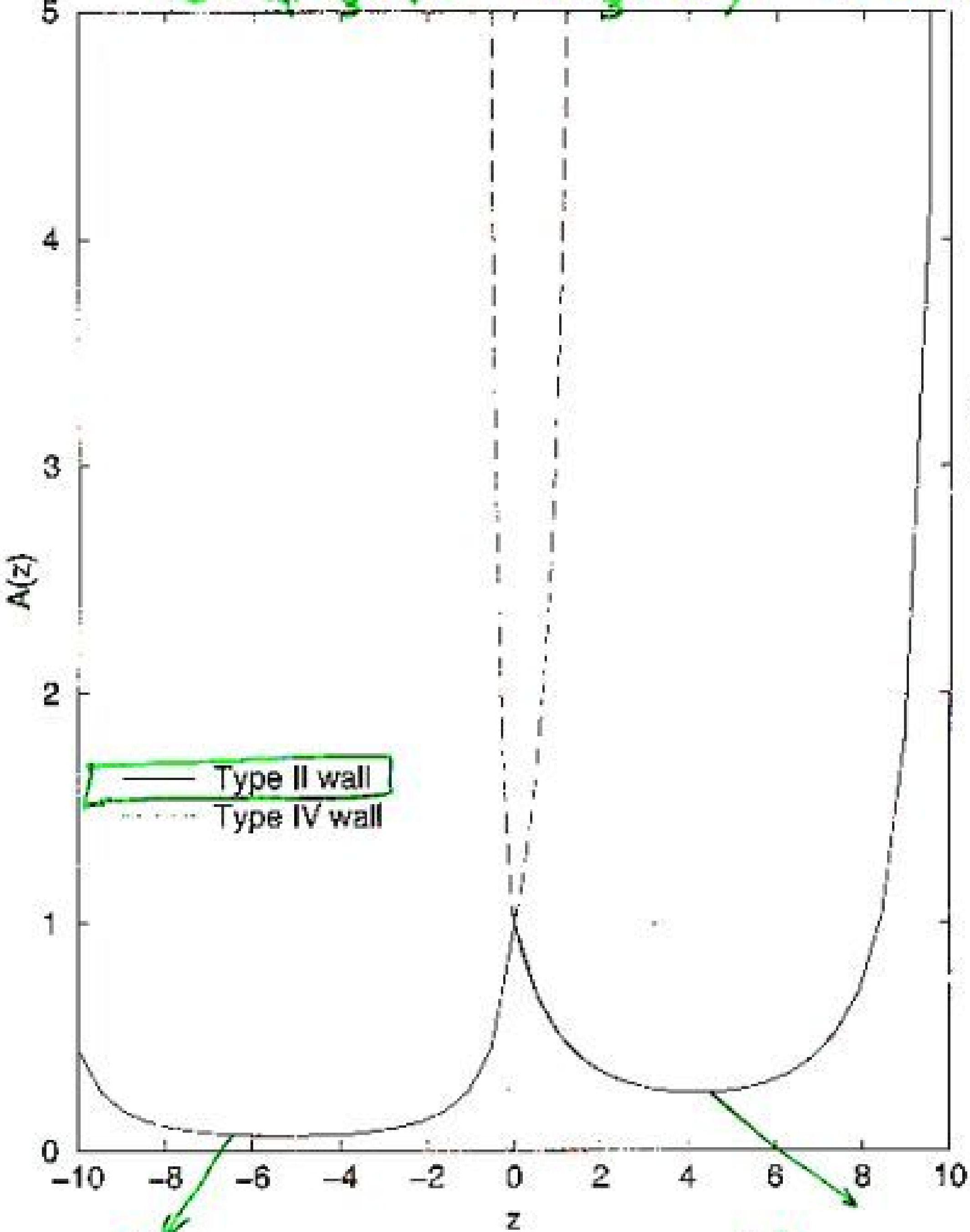



$z \to \pm\infty$ "Cosmological" horizons

w/ Wang
hep-th/9912187

# Bent (anti de Sitter) walls

$$\sigma = 2\left(\sqrt{-\frac{\Lambda_+}{3}-\beta^2} + \sqrt{-\frac{\Lambda_-}{3}-\beta^2}\right) < \sigma_{BPS}$$




c.f.
L. Randall's
talk

$$\frac{\beta^2}{\left[\sqrt{-\frac{\Lambda_-}{3}}\cos(\beta(z+z))\right]^2} \qquad \frac{\beta^2}{\left[\sqrt{-\frac{\Lambda_+}{3}}\cos(\beta(z+z_+))\right]^2}$$

$z \to \pm\infty$ Boundary

# Walls in $\underset{\overset{\shortparallel}{5}}{D}$-dimensions

1 - transverse direction

$\underset{\overset{\shortparallel}{3}}{(D-2)}$ - spatial directions internal to the wall

gravitational effects universal

(thin wall w/ Wang)

$$\underset{D=4}{\sqrt{\frac{-\Lambda_\pm}{3}}} \longrightarrow \sqrt{-\frac{(D-2)\,\Lambda_\pm}{2(D-1)}} \underset{\underset{D=5}{\uparrow}}{=} \sqrt{-\frac{3\Lambda_\pm}{8}}$$

BPS - first order formalism in D-dim: c.f.:

[Freedman, Gubser, Pilch & Warner;
Skenderis & Townsend;
Chamblin & Gibbons;
De Wolf, Freedman, Gubser & Karch ]

⋮

H. Verlinde

Domain walls in fundamental theory?:
as solutions of an effective theory that arises
as compactification of M-/string theories

$\downarrow$ on Einstein-Sasaki spaces
(sphere-most symmetric example)

effective supergravity with $V(\phi_i)$ - gauged supergravity

$\phi_i$ - parameterize internal space deformations

$\downarrow$

Domain walls of $D=5$ $N=2$ gauged supergravity
(with abelian $U(1)_R$ gauging) &
massless vector supermultiplets, only

w/ Behrndt
hep-th/9909058

# D=5 N=2 Supergravity w/ abelian $U(1)_R$ gauging

Günaydin, Sierra & Townsend '84

w/ vector supermultiplets

**Bosonic sector:**

$X^I$ - scalar components of vector supermultiplets (real, neutral)

Subject to the constraint:

$$F = \sum_{IJK} C_{IJK} X^I X^J X^K = 1 \longrightarrow \text{solved for physical scalars } \phi^A$$

Constrained superpotential:

$$W = \sum_I h_I X^I$$

$\{C_{IJK}, h_I\}$ constants

Lagrangian:

$$\mathcal{L} = g_{AB}\, \partial_\mu \phi^A \partial^\mu \phi^B - V + R$$

$$V = \frac{9}{2}\left( g^{AB} \partial_A W \partial_B W - \frac{4}{3} W^2 \right) \qquad g_{AB} \doteq \frac{1}{2}\{\partial_I \partial_J \log F\} \partial_A X^I \partial_B X^J$$

Supersymmetric extrema:

$$\partial_A W_{ext} = 0 \Longrightarrow V_{ext} \leq 0$$

# BPS-domain walls:

between isolated supersymmetric extrema ($\partial_A W_\pm = 0$)

Killing spinor eqs.: $\delta\lambda^\alpha = 0$, $\delta\Psi^\alpha_\mu = 0$

$$\circledast\ \partial_z \phi^A = \mp 3 g^{AB} \partial_B W$$

$$*\ \partial_z \ln A = \pm 2W$$

However $W$ & $g_{AB}$ constrained; at $\partial_A W = 0$:

$$\partial_A \partial_B W \overset{}{\underset{\circledast}{=}} \frac{2}{3} g_{AB} W$$

Günaydin, Sierra & Townsend

Kallosh et al.

Expanding solution around susy extrema: $\partial_A W_\pm = 0$

$$\phi^A = \phi^A_\pm + \delta\phi^A$$

$$\circledast\ \partial_z(\delta\phi^A) = \mp 3 g^{AB}(\partial_B\partial_C W_\pm)\delta\phi^C \underset{\circledast}{=} \mp 2 W_\pm \delta\phi^A$$

$$*\ \partial_z(\ln A) = \pm 2 W_\pm$$

Conspiracy of signs!

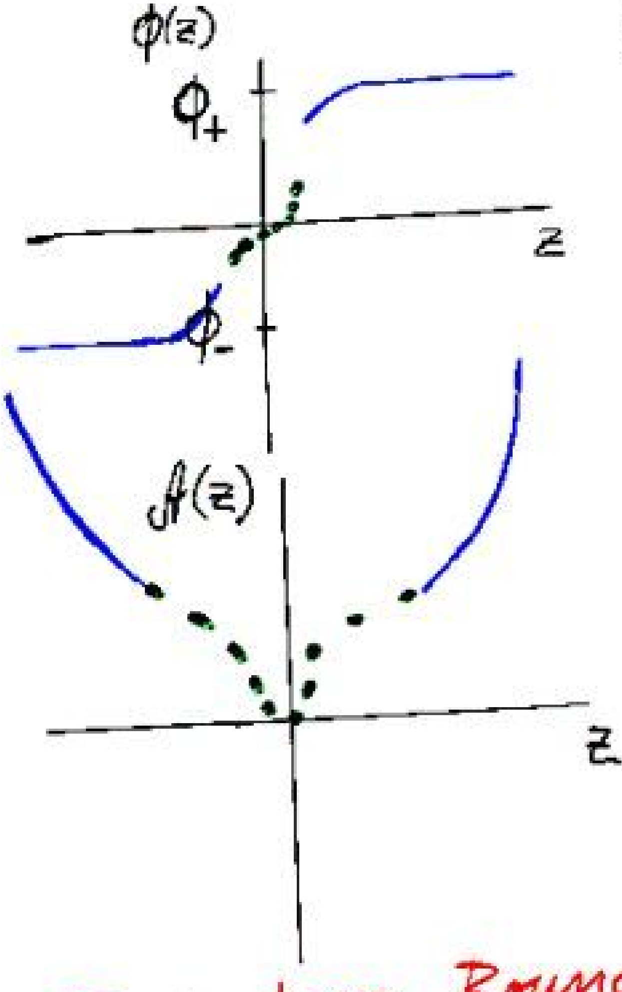
Asymptotic solution: $z \to \pm\infty$

$$\delta\phi^A \sim e^{\mp 2W_\pm z} \xrightarrow{\text{for a kink}} 0 \quad (\text{sign}\, W_+ = -\,\text{sign}\, W_-)$$

$$\Downarrow$$

$$\mathcal{A} \sim e^{\pm 2W_\pm z} \longrightarrow \infty$$ exponential growth!

Kallosh & Linde hep-th/0001071
W/ Behrndt hep-th/0001159



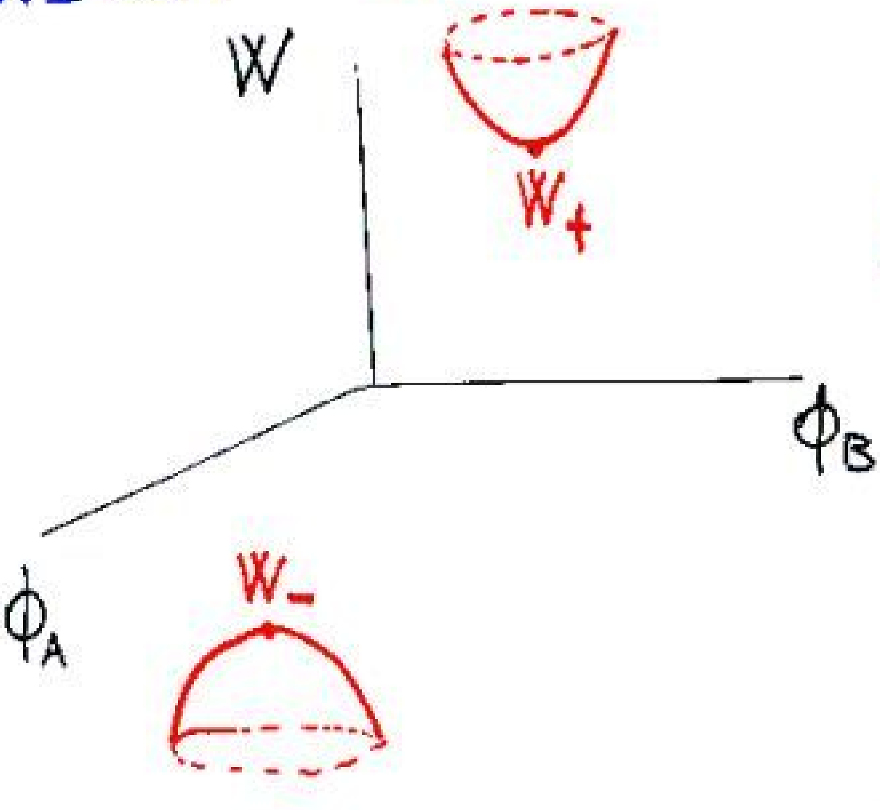

$z \to \pm\infty$ Boundary of AdS
[relevant for AdS/CFT]

Additional constraints:

Assuming : $g_{AB}$ - positive definite
&
$\partial_A \partial_B W_\pm = \frac{2}{3} g_{AB} W_\pm$ (at $\partial_A W_\pm = 0$)

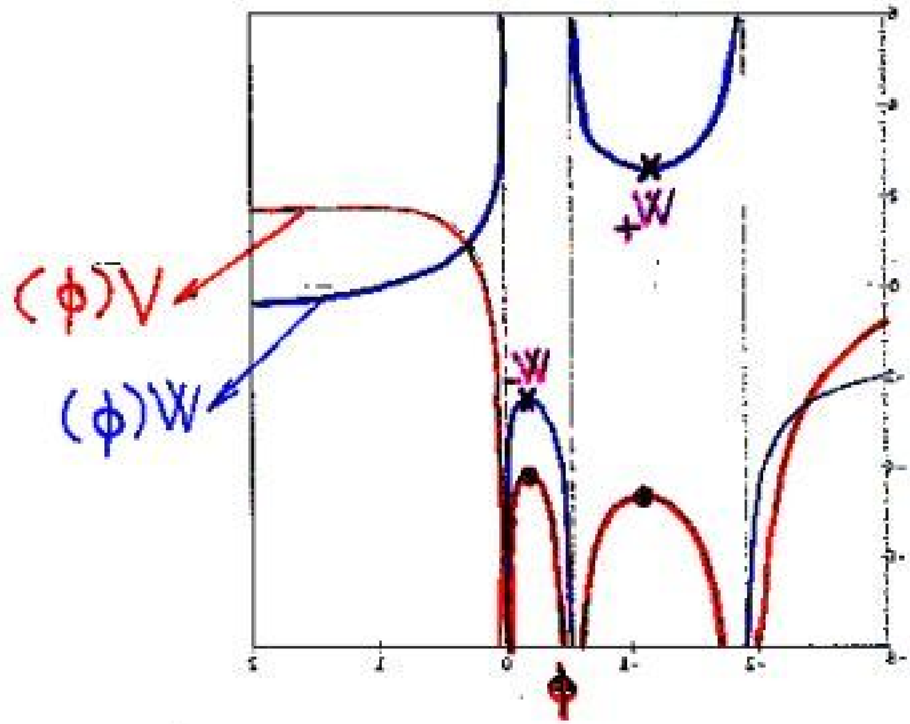
$W_+ > 0$ : $W_+$ - minimum ($\partial_A \partial_B W_+$ - positive definite)

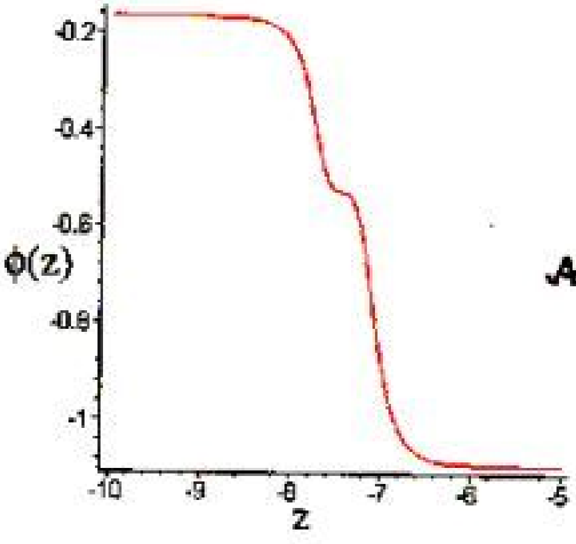
$W_- < 0$ : $W_-$ - maximum ($\partial_A \partial_B W_-$ - negative definite)



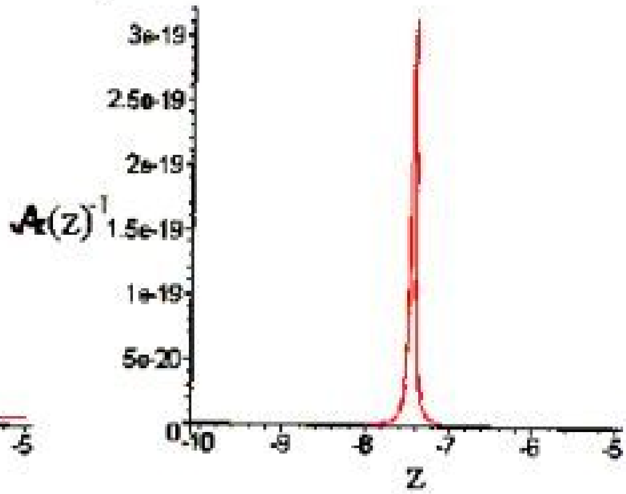

Susy extrema cannot be smoothly connected!

$V_\pm$ - maximum of the potential!


V(ϕ)
W(ϕ)
ϕ



-0.2
-0.4
-0.6
φ(z)
-0.8
-1
-10
-9
-8
-7
-6
-5
z

3e-19
2.5e-19
2e-19
A(z)^-1
1.5e-19
1e-19
5e-20
0
-10
-9
-8
-7
-6
-5
z

# Ways out?

- non-Abelian gauging
  w/ tensor multiplets $\chi_A$

Günaydin & Zagermann
hep-th/9912027

$V \sim m^2 \chi_A^2$ – cannot participate in BPS domain walls

Kallosh & Linde
hep-th/0001071

- w/ hypermultiplets

Ceresole & Dall'Agata
hep-th/0004111

smooth flows may exist?

Behrndt, Louis & Mohaupt
work in progress

↓

Domain walls due to a massive supermultiplet – breathing mode of Type IIB compactified on a sphere!

a potential to trap gravity

W/ Lü & Pope
hep-th/0001002
hep-th/0002054

[related ideas: Chan, Paul & Verlinde hep-th/0003236;
Maldacena & Nunez hep-th/0007018]

# Reduction of Type IIB on $S^5$:

Bremer, Duff, Lü, Pope & Stelle '98

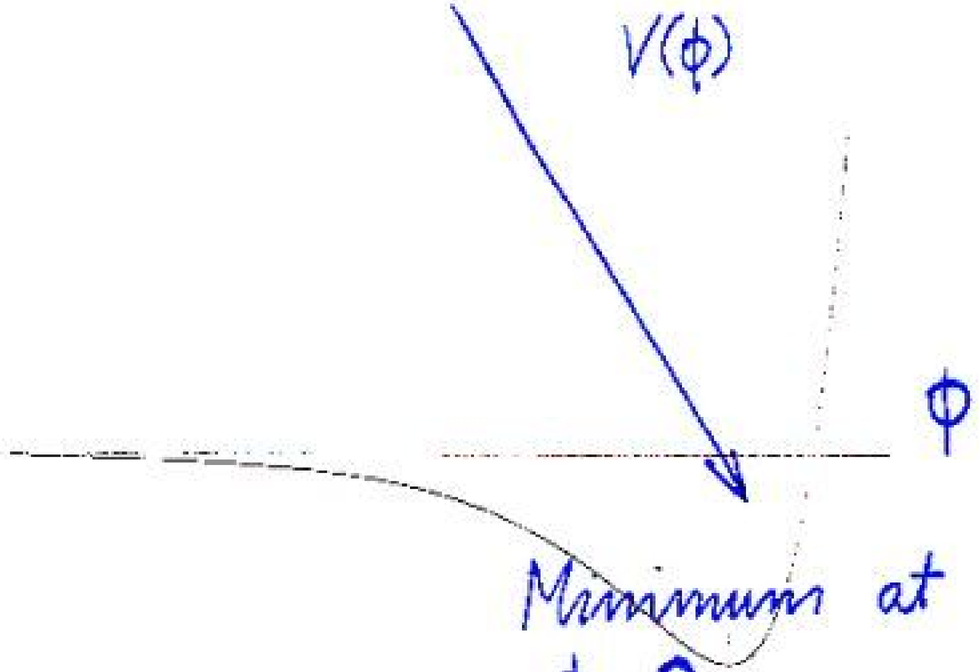
$\phi$ – breathing mode – parameterizes the volume of $S^5$

$$V(\phi) = \frac{g^2}{2}\left(\frac{1}{a_1^2}\, e^{a_1\phi} - \frac{1}{a_1 a_2}\, e^{a_2\phi}\right) \qquad a_1 = 2\sqrt{\tfrac{5}{3}}, \quad a_2 = \tfrac{4}{\sqrt{15}}$$

Einstein curvature

$G_{(5)} - G_{(5)}$ contribution

a "race-track":




Minimum at

$\phi \sim 0$

$R_{adS_5} \simeq R_{S^5}$ (D3-brane horizon) w/ Liu & Pope

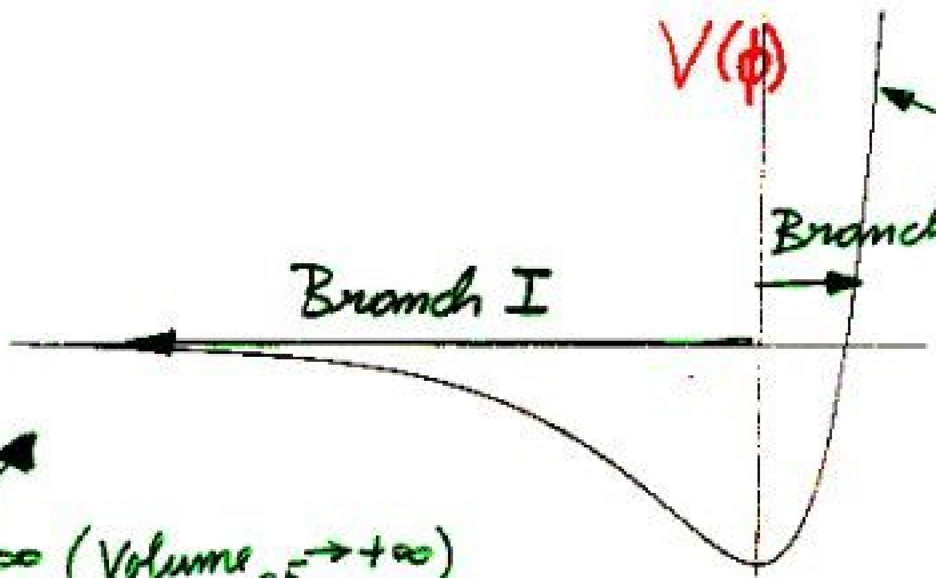
$$V(\phi) = (\partial_\phi W)^2 - \frac{4}{6} W^2 \qquad g_{\phi\phi} = \frac{1}{2}$$

$$W(\phi) = \frac{g}{\sqrt{2}} \left( \frac{1}{a_1} e^{\frac{1}{2} a_1 \phi} - \frac{1}{a_2} e^{\frac{1}{2} a_2 \phi} \right) \qquad a_1 = 2\sqrt{\frac{5}{3}}, \quad a_2 = \frac{4}{\sqrt{15}}$$

BPS-eqs:

$$\partial_z \phi = \sqrt{2} \frac{\partial W}{\partial \phi} \longrightarrow z - z_0 = \frac{4}{a_2 g} e^{-\frac{1}{2} a_2 \phi} {}_2F_1\left( \frac{a_2}{a_2 - a_1}, 1, 1 + \frac{a_2}{a_2 - a_1}; e^{\frac{1}{2}(a_1 - a_2)\phi} \right)$$

$$\partial_z \ln A = -\frac{\sqrt{2}}{3} W$$




$\phi \to +\infty$ (Volume $_{S^5} \to 0$)

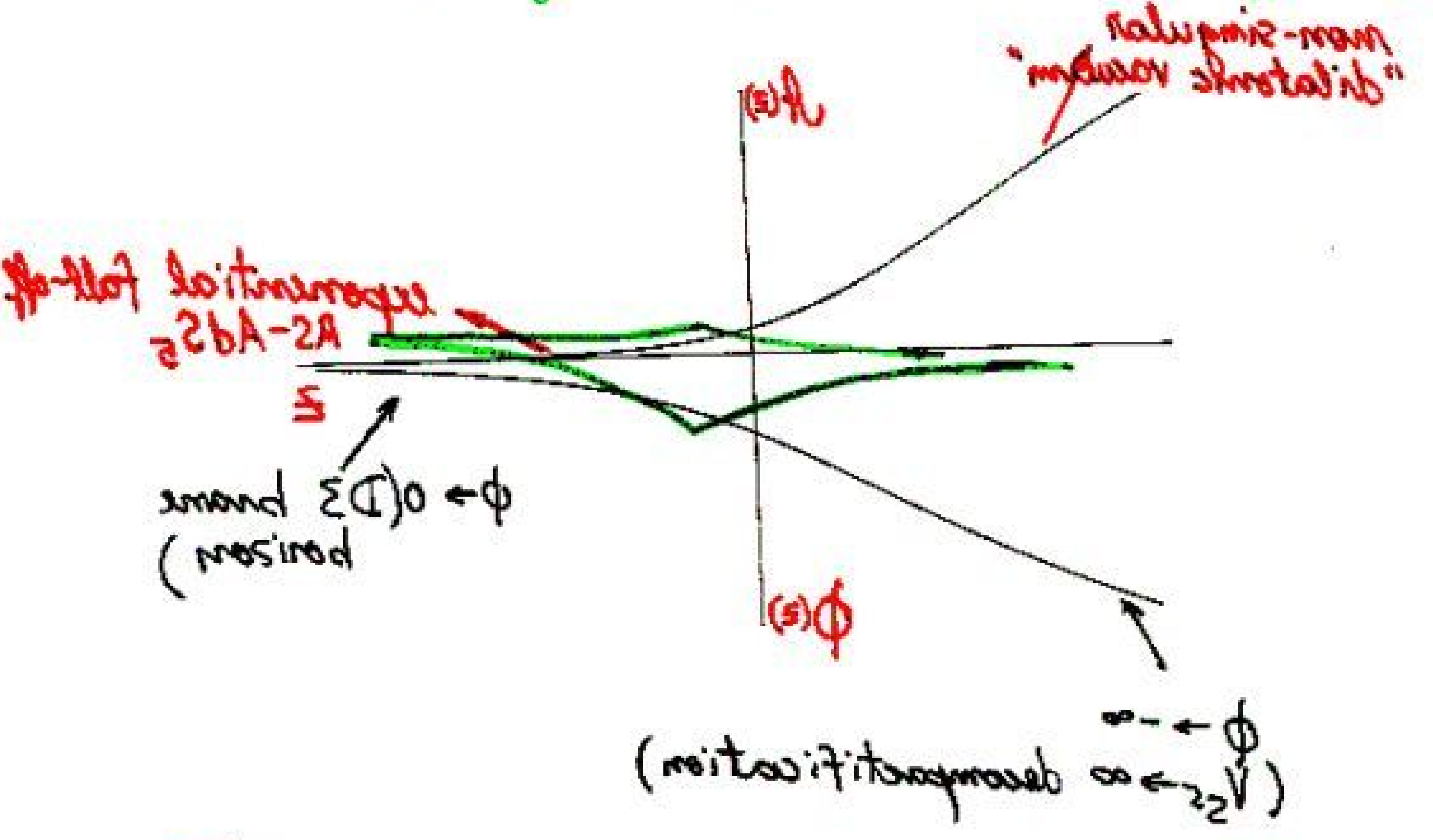
$\phi \to -\infty$ (Volume $_{S^5} \to +\infty$)
decompactification

$\partial_\phi W = 0$
supersymmetric minimum!

hep-th/001002

# Add $\delta(z)$ source : Branch I

[singular D3-brane source in type IIB]




~~smooth wall → no trapping of gravity~~

$Z_2$-symmetric thin wall;

traps gravity

$\phi$ – the role of radion in Goldberger-Wise mechanism

Related work in progress: $S^1/Z_2$-topology

Duff, Liu & Stelle; deAlwis et al.

# Conclusions

I. Overview of supergravity walls

flat $\Longleftrightarrow$ BPS $\sigma = \sigma_{BPS} \sim (\sqrt{-\Lambda_+} + \sqrt{-\Lambda_-})$

bent $\Longleftrightarrow$ non-extreme $\sigma \gtrless \sigma_{BPS}$

gravitational effects

II. Domain walls in $D=5$ gauged supergravity

(a) massless supermultiplets – "no-go" for RS
relevant for AdS/CFT

(b) massive – breathing mode (parameterizes volume of e.g., $S^5$ of sphere reduced Type IIB)

(i) Branch I – no gravity trapping
w/ $\delta(z)$ source – breathing mode radion of Goldberger-Wise mechanism

(ii) Branch II – singular in interior
– a potential to trap gravity
[resolution of singularity ?]