# STRINGS
# AS
# NON-COMMUTATIVE SOLITONS

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**Talk based on *hep-th/0005031***

**w. Jeff Harvey, Per Kraus, Emil Martinec**

# OUTLINE

- Framework: bosonic open-string field theory.
- Goal: identify the closed fundamental string as a classical solution in this field theory.
- Large non-commutativity is introduced as a tool that facilitates an explicit construction.
- The result: a soliton with tension *exactly* equal to the fundamental string tension; and the same fluctuation spectrum too.
- Some open questions.

**See also talk by J. Harvey (Friday 9 am)**

# INTRODUCTION

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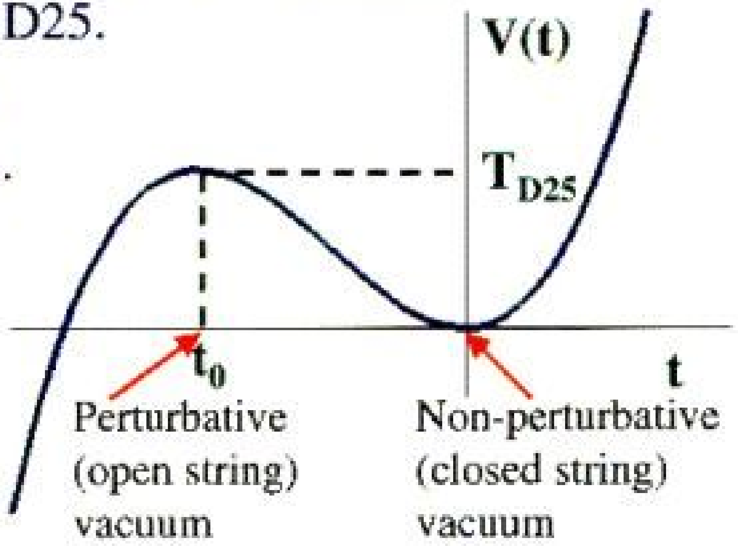
- Consider open bosonic string theory, *i.e.* bosonic string theory with a space-filling D25-brane.
- The open-string tachyon on the D25 signals an instability.
- The true vacuum after condensation of the tachyon is the original closed string theory *without* the D25.

**Sen**

- The energy liberated by the tachyon condensation exactly cancels the tension of the D25.
- This picture was tested in Open String Field Theory.

**Kostelecky+Samuel**
**Sen+Zwiebach**
**Taylor**
**...**



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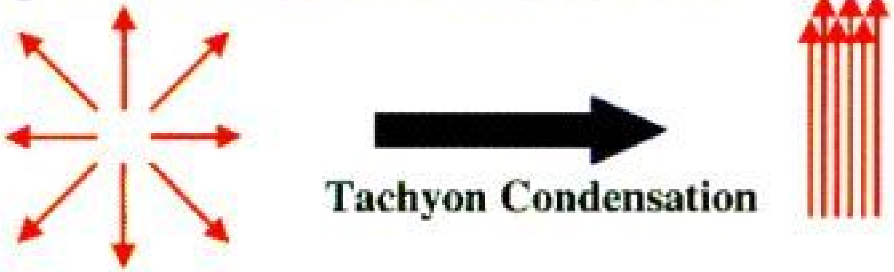
# THE QUALITATIVE PICTURE

- It is interesting to explore *excitations* of the nonperturbative vacuum, *e.g.* fundamental strings. **For D-branes: see Harvey's talk**
- Strings are described *before* tachyon condensation by electric flux tubes on the D25 world-volume.
- Tachyon condensation removes the open-string gauge field.
- Whatever remains after tachyon condensation of a D25 with electric flux is the fundamental (closed) string.
- The process is similar to confinement.

**Sen;Yi;Bergman+**
**Hori+Yi;KRAUS+HARVEY**
**Kogut+**
**Susskind '74**



**Perturbative vacuum:**
**flux-lines spread**

**Non-Perturbative vacuum:**
**flux-lines align (confinement)**

# FIELD THEORY DESCRIPTION

- The tachyon potential V(t) is only known approximately (its qualitative form is sufficient for us).
- General arguments determine the dependence on constant field strengths:

**Sen**

$$S_t = -\int d^{26}x\, V(t)\sqrt{-\det[g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}]}$$

- The kinetic terms and numerous higher derivative terms are determined *in principle* by string field theory.
- In practice they are difficult to compute accurately; *e.g.* all massive fields must be taken into account.
- This is a problem because the unknown terms are of order string scale, as are the solutions we seek.

# NON-COMMUTATIVITY

- B-fields are incorporated by replacing the metric and the coupling with

$$G_{\mu\nu} = g_{\mu\nu} - (2\pi\alpha')^2 (Bg^{-1}B)_{\mu\nu} \qquad G_s = g_s \left( \frac{\det G}{\det(g + 2\pi\alpha' B)} \right)^{1/2}$$

**Seiberg +Witten**

- and multiplying fields using the non-commutative product

$$A * B = \exp\left( \frac{i}{2} \theta^{\mu\nu} \partial_\mu \partial_{\nu'} \right) A(x) B(x')|_{x=x'} \; ; \;\; \theta^{\mu\nu} = -(2\pi\alpha')^2 \left( \frac{1}{g + 2\pi\alpha' B} B \frac{1}{g - 2\pi\alpha' B} \right)^{\mu\nu}$$

- The non-commutativity parameter $\theta$ provides a new scale for classical solutions; solitons simplify for large $\theta/_{\alpha'}$.

**Gopakumar+Minwalla+Strominger**

- Complications of open string field theory are string scale w.r.t. the *open string metric*. They are much smaller than the soliton and therefore negligible.

**See also Dasgupta+ Mukhi+Rajesh; Witten**

- The soliton is string scale w.r.t. the *closed string metric*,

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# THE NC DECOUPLING LIMIT

- String theory with D-branes and B-fields reduce to NCYM in the low energy decoupling limit

  CONNES + DOUGLAS + SCHWARZ

  $$\alpha' \sim \varepsilon^{1/2} \to 0 \quad g_{ij} \sim \varepsilon \to 0 \quad \text{(in NC directions)}$$

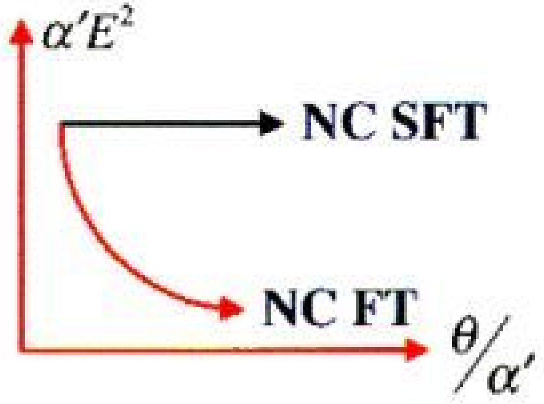
  **Seiberg +Witten**

- This implies large non-commutativity

  $$\theta/\alpha' \sim \varepsilon^{-1/2} \to \infty$$

- We take $\theta/\alpha' \to \infty$ *without* taking a low energy limit; string excitations are therefore kept.

- Our limit is non-commutative String Field Theory (NCSFT).



# EQUATIONS OF MOTION

- Take all directions transverse to string non-commutative. Introduce *-product, open string metric $G^{ij}$, and open string coupling $G_s$ in the BI-type Lagrangean.
- In a given flux sector the corresponding Hamiltonian is:

Flux quantum number

$$H = \int d^{25}x \left[ \sqrt{V(t)^2 + E^2 / (2\pi\alpha')^2} + \lambda \partial_1 E \right] - \lambda p$$

Lagrange multiplier

- The equations of motion become:

$$\frac{V(t)V'(t)}{\sqrt{V(t)^2 + E^2 / (2\pi\alpha')^2}} = 0 \qquad \frac{E}{\sqrt{V(t)^2 + E^2 / (2\pi\alpha')^2}} + \lambda(2\pi\alpha')^2 = 0$$

- In general it is difficult to find *localized* solutions to these nonlinear equations.

# NON-COMMUTATIVITY TO THE RESCUE

**Gopakumar**
**+Minwalla**
**+Strominger**

- Non-commutative multiplication allows nontrivial localized solutions to:

$$\phi * \phi = \phi \qquad (e.g. \quad \phi_0 = 2^{12} e^{-r^2/\theta})$$

- Functionals act in a simple way on such functions:

$$f(\lambda\phi) = \sum_{n=0} a_n \lambda^n \phi^n = f(0) + [f(\lambda) - f(0)]\phi$$

- The equations of motion therefore become algebraic for *ansätze* built on solutions to $\phi * \phi = \phi$.
- It is therefore straightforward to find solutions to the equations of motion.

# STRING SOLUTIONS

- A simple string solution is

$$t = t_0 \phi_0 \qquad E = p\phi_0$$

- The tension

$$T = \frac{1}{2\pi\alpha'}\sqrt{\frac{1}{g_s^2} + p^2}$$

- identifies this solitonic string as a (p,1) string.
- Different solutions to $\phi * \phi = \phi$ give (p,q) strings (q>1) .
- A solution with the tension of p fundamental strings is:

$$t = 0 \qquad E = p\phi_0$$

- In each case the tension formula is *exact* even though the theory is not supersymmetric.

# STRONG COUPLING

- We started with the action:

$$S_t = -\int d^{26}x\, V(t)\sqrt{-\det[\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}]}$$

- A puzzle: how is is possible that we find a soliton tension independent of $g_s$ from a Lagrangean L~V(t)~1/$g_s$ ?
- A hint: V(t)=0 in the solution so a simple scaling argument for the energy is misleading.
- Resolution: the problem is analogous to a massless particle with V(t) playing the role of mass. The Lagrangean degenerates, but the Hamiltonian presents no subtleties.

# THE QUANTUM PROBLEM

- The classical e.o.m. allow numerous solutions potentially interpretable as a fundamental string; generally

LAWRENCE+ SHENKER

$$E = \sum_{n=0} c_n \phi_n \qquad \phi_n * \phi_n = \phi_n \qquad c_n \text{ real}$$

- General quantum properties of the underlying gauge theory require that $c_n$ is integral; countably infinite candidate fundamental strings remain.
- Quantum dynamics mix these. Standard D-brane dynamics would provide a unique quantum ground state but the situation is more involved here.
- The quantum problem deserves a (much) better understanding; we consider the classical problem only.

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# FLUCTUATIONS

- The Born-Infeld type action also describes fluctuations depending on the [redacted] commutative directions $x^0, x^1$:

$$H = \int d^{25}x \left[ \sqrt{E^\alpha M_{\alpha\beta} E^\beta + V(t)^2 \det(\mathcal{I} + \mathcal{F})} + A_0 \nabla_\alpha E^\alpha \right]$$

$$= \int d^{25}x \sqrt{(E^1)^2 (1 + \vec{A}'^2) + \vec{E}^2 + (\vec{E} \cdot \vec{A}')^2} \qquad \text{where} \quad M_{\alpha\beta} = \delta_{\alpha\beta} - F_{\alpha\gamma} F^\gamma{}_\beta$$

Equality uses:

$t = 0 \Rightarrow V(t) = 0$ (closed string vacuum)

$A_0 = 0$ (gauge condition)

$F_{ij} = 0,\ F_{1i} = A_i'$ (derivatives negligible in NC directions)

- The *Ansatz* for the fluctuating string is a common profile for all fields:

$$\phi = \phi_0(x^i - f^i(x^0, x^1))$$

$$E_1 = p\phi_0, \quad \vec{E} = \vec{e}\,\phi_0, \quad \vec{A}' = \vec{a}'\phi_0$$

p=1

# THE REDUCED ACTION

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- Hamiltonian reduction gives an effective description of the fluctuations in 1+1 dimensions with Hamiltonian and corresponding Lagrangean in static gauge $X^\mu=(x^0,x^1,f^i)$ :

$$H=\int dx^1\sqrt{1+\vec{\pi}^2+(\vec{f}')^2+(\vec{\pi}\cdot\vec{f}')^2} \quad \Rightarrow \quad L=-\int d^2x\sqrt{(\dot{\vec{X}})^2(\vec{X}')^2-(\dot{\vec{X}}\cdot\vec{X}')^2}$$

- *The effective action of long wave length fluctuations is the Nambu-Goto action with the correct tension*!
- The spectrum of the soliton is therefore precisely the same as for a fundamental string.
- The graviton and the closed string tachyon seem to appear as collective excitations in open string field theory.
- Unfortunately we can only justify the effective action at long wave length. $\sqrt{\alpha'}F'_{\mu\nu} << F_{\mu\nu}$

# CONCLUSION

- Excitations of the closed string vacuum resulting from tachyon condensation are interesting.
- Non-commutativity facilitates their study in open string field theory.
- An explicit construction of (p,q)-strings follow.
- Solitons with the exact tension and spectrum of fundamental strings were constructed.
- Major problems remain: quantum properties of the gauge dynamics are important; the computation of the spectrum is justified only at large wave length.