NEW TWISTS

IN

LARGE N FIELD TEORIES

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STRINGS 2000

BASED ON J.M. & C. NUÑEZ

• OUTLINE

- LARGE N LIMIT OF BRANES ON CURVED MANIFOLDS
- TWISTED FIELD THEORIES
- D3 BRANE ON $R^2 \times \Sigma_2 \longrightarrow$ (2,2) SUSY
  (4,4) SUSY
- M5 BRANE ON $R^4 \times \Sigma_2 \longrightarrow \mathcal{N}=2 \quad d=4$
  $\mathcal{N}=1 \quad d=4$

  NEW CFT_4 IF genus >1.

- A NO GO THEOREM FOR R-S & de SITTER COMPACTIFICATIONS OF 10 & 11 D SUPERGRAVITIES.

- WE DON'T HAVE A SYSTEMATIC METHOD FOR OBTAINING THE LARGE N DUAL OF A FIELD THEORY

- THERE ARE WAYS OF OBTAINING NEW DUALITIES BY DEFORMING KNOWN RESULTS

e.g. START WITH $\mathcal{N}=4$ SYM $= AdS_5 \times S_5$

- ORBIFOLD S^5

- RELEVANT PERTURBATIONS

$\downarrow$

MODIFY B.C. FOR A SUGRA FIELD

$\downarrow$

FIND NEW SOLUTION

$\downarrow$

UNDERSTAND THE IR GEOMETRY

- EASIEST IF WE HAVE AN AdS_5 IN THE IR.

- TODAY WE WILL DISCUSS ANOTHER OPTION

$\rightarrow$ STUDY THE FIELD THEORY ON A CURVED MANIFOLD.

e.g. $\mathbb{R}^{D-2} \times \Sigma_2$

INTERESTING THEORIES IN D-2 DIMENSIONS

WITTEN

(SIMPLE EXAMPLE $R^{D-1} \times S^1_\beta \to$ THERMAL THEORY)

- WRAPPING BRANES → ENGINEER FIELD THEORIES
- FIELD THEORIES ON CURVED MANIFOLDS

- $\eta_{\mu\nu} \to h_{\mu\nu}$

- NEW B.C. AT THE BOUNDARY

WITTEN
HENNINGSON SKENDERIS

$$ds^2 \simeq \frac{h_{\mu\nu}\, dx^\mu dx^\nu + dr^2}{r^2} + O(1)$$

$$r \to 0$$

- IN GENERAL → WE BREAK SUSY
  - NO COVARIANTLY CONSTANT SPINOR

$$\nabla_\mu \varepsilon = 0 \qquad [\nabla_\mu, \nabla_\nu] \sim R_{\mu\nu}$$

→ THEORY HAS AN R- SYMMETRY
↓
COUPLE THE THEORY TO AN EXTERNAL GAUGE FIELD

$$(\partial_\mu + \omega_\mu + A_\mu)\epsilon \stackrel{?}{=} 0$$

$\to$ CHOOSE $A_\mu = \omega_\mu$ (EMBEDDING THE SPIN CONNECTION IN THE GAUGE CONNECTION)

$\to$ PRESERVE SOME SUSY $\to \partial_\mu \epsilon = 0$
$\to \epsilon =$ CONST.

THE RESULTING THEORY IS CALLED "TWISTED"

$\downarrow$

IT IS LIKE CHANGING THE SPINS OF VARIOUS FIELDS ACCORDING TO THEIR R-SYMMETRY CHARGES.

- IN OUR CASES THE NON TRIVIAL PART OF ω_μ IS IN TWO OF THE DIMENSIONS $\Rightarrow U(1)$ CONNECTION

$\to$ EMBED THIS $U(1)$ IN THE R-SYMMETRY GROUP.

. WHEN WE WRAP A BRANE ON A CYCLE IN A STRING COMPACTIFICATION → THIS IS PRECISELY HOW IT MANAGES TO PRESERVE SOME SUSY.

BERSHADSKY, SADOV, VAFA

R-SYMMETRY → ROTATIONS IN DIRECTIONS ORTHOGONAL TO THE BRANE → A_μ TAKES INTO ACCOUNT THAT THE NORMAL DIRECTIONS ARE NON-TRIVIALLY FIBERED

. LIMIT IN WHICH WE GET THE TWISTED <u>FIELD THEORY</u>

$\ell_p \sim \ell_s \to 0$, SIZE OF CYCLE FIXED.

$\Rightarrow$ SIZE OF CYCLE → LARGE IN STRING UNITS.

. SO WE SET $\omega_m = A_m$

. FIELD OF SPIN s & CHARGE q

$$D_m \phi = \left(\partial_m + i s \omega_m + i q A_m\right)\Phi$$

. THERE ARE ALSO SOME CURVATURE COUPLINGS COMING FROM SUSY

$$\frac{q}{2} R\, |\phi|^2$$

VAFA WITTEN

. EXAMPLE:

$\mathcal{N}=4$ SYM. ON $\mathbb{R}^2 \times \Sigma_g$

$$\underset{123456}{SO(6)} \supset \underbrace{\underset{12}{SO(2)}}_{\text{TWIST THIS.}} \times \underbrace{\underset{34}{SO(2)} \times \underset{56}{SO(2)}}$$

$$U(1) \times SO(4) = SU(2)_L \times SU(2)_R$$

SUSY IS PRESERVED IF CHIRALITY ON Σ IS THE SAME AS CHIRALITY ON 12 NORMAL DIRECTIONS $\Longrightarrow$ PRESERVES $\frac{1}{2}$ SUSY OF D3

. IN 1+1 DIM. NOTATION → PRESERVES

(4,4) SUSY.

BERSHADSKY JOHANSEN SADOV VAFA

. A D3 BRANE WRAPPED ON A CYCLE $\Sigma_2 \subset K3$ GIVES RISE TO THIS THEORY. IN THE FIELD THEORY LIMIT (IN D=4) → TRANSVERSE SCALARS ARE EFFECTIVELY NON-COMPACT.

. $r \sim \alpha' \phi$

(IF WE HAD KEPT K3 WITH FINITE SIZE IN STRING UNITS → WE WOULD OBTAIN A "T-DUAL" VERSION OF THE D1-D5 SYSTEM)

. ANOTHER EXAMPLE

$$SO(6) \supset SO(2) \times SO(2) \times SO(2)$$

$$\begin{array}{cccc} 123456 & 12 & 34 & 56 \\ & T_1 & T_2 & T_3 \end{array}$$

$$T = \tfrac{1}{2}(T_2 + T_3) \longrightarrow U(1) \subset SU(2)_L \qquad \underbrace{T_2 \quad T_3}_{SU(2)_L \times SU(2)_R}$$

→ PRESERVES 1/4 OF SUSY.

→ (2,2) SUSY IN 1+1 DIM.

≈ BRANE IN A CY MANIFOLD.

. CFT IN THE I.R. , Σ_g $g > 1$

. WE CAN CALCULATE THE CENTRAL CHARGE FROM THE FIELD THEORY POINT OF VIEW

$$c = 3 N^2 (g-1)$$

. GRAVITY SOLUTION.

. GAUGED SUGRA ← TRUNCATION OF 10-D EQNS.

↓

$U(1) \times U(1)$ TRUNCATION.

CVETIC, LÜ, PAGE, POKHA, LIU, LÜ, LU, MARTINEZ-ACOSTA, POPE, SATI, TRAN

↓

SET TO ZERO THE SUSY VARIATIONS

$$\delta \psi_\mu = 0 \qquad \delta \chi = 0$$

ANSATZ:

$$ds_5^2 = e^{2f(r)}\left(-dt^2 + dz^2 + dx^2\right) + e^{2g(r)} \frac{dx^2 + dy^2}{y^2}$$

$\uparrow$
H_2
$\uparrow$
$\Sigma = H_2/\Gamma$

$\psi(r)$
↓
SCALAR DUAL TO

$$\theta_2 = \mathrm{Tr}\left[\tfrac{2}{3}\left(X_1^2 + X_2^2\right) - \tfrac{1}{3}\left(X_3^2 + X_4^2 + X_5^2 + X_6^2\right)\right]$$

1^{st} ORDER DIFF. EQNS. FOR f, g, ψ

$$\dot\psi = F(g, \psi)$$
$$\dot g = G(g, \psi)$$
$$\dot f = H(g, \psi)$$

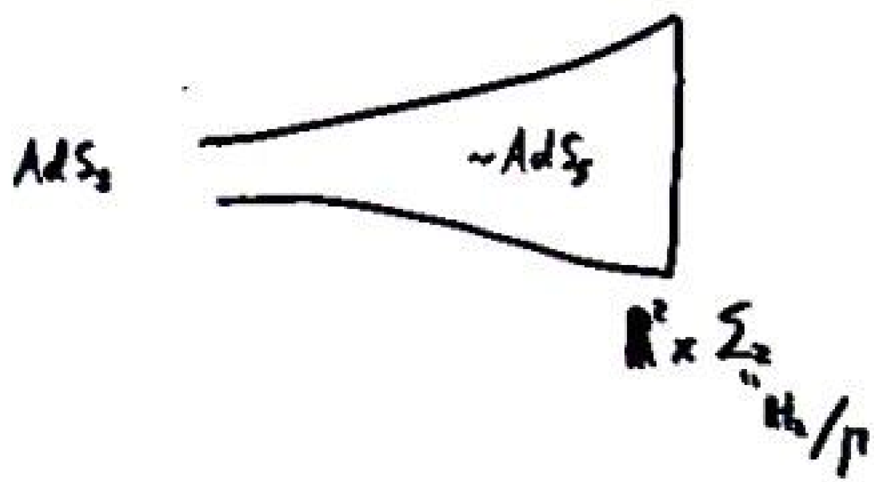
→ GET ONE PARAMETER FAMILY OF SOLUTIONS

↓

ADJUST THE INTEGRATION CONSTANT SO THAT THE IR IS AN AdS_3 REGION WITH

g : CONST

φ : CONST.



WE CAN LIFT THIS SOLUTION TO 10 D.

. CALCULATE c_{SUGRA} → SAME AS $c_{FIELD\ TH.}$

M5 BRANE EXAMPLES

- START FROM THE (0,2) SCFT IN 6 DIMENSIONS

R-SYMMETRY: $SO(5) \to SO(2) \times SO(3)$

$ 12345 \quad\quad \underbrace{12}_{\text{THIS } U(1)} \quad\quad 345$

THIS $U(1) \to \mathcal{N}=2$ IN $D=4$

$$\underset{12345}{SO(5)} \supset \underbrace{SU(2)_L \times SU(2)_R}_{1234}$$

$U(1) \subset SU(2)_L \to \mathcal{N}=1$ IN $D=4$.

IN BOTH OF THESE CASES WE FIND SUGRA SOLUTIONS INTERPOLATING BETWEEN $AdS_7 \longrightarrow AdS_5 \times \Sigma_g$ IF $g>1$.

- CENTRAL CHARGES $C_{SUGRA} \sim N^3(g-1)$

. THE 11-D GEOMETRY IN THE I.R.

→ WARPED AdS_5

SIMILAR:
ALISHAHIHA, OZ
CVETIC, LU, POPE.
VAZQUEZ-PORITZ
FAYYAZUDDIN, SMITH

$$ds^2_{11} = \Omega(y)\, ds^2_{AdS_5} + ds^2_{M_6}$$

NON SINGULAR → BOUNDED ABOVE & BELOW,
NEVER VANISHING.

. IT WOULD BE NICE TO FIND GENERAL CONDITIONS ON M_6, WE HAVE FOUND SOME PARTICULAR EXAMPLES

WORK IN PROGRESS..

- NS-5-BRANE ON S^2 - TWISTED
  $\downarrow$
  PURE $N=1$ SYM IN $D=4$

- FIND GAUGED SUGRA SOLUTION IN
  $d=7 \to d=10$

  TOWNSEND - VAN NIEUWENHUIZEN
  CHAMSEDDINE - SABRA
  CVETIC LU POPE SADRZADEH, TRAN

$\to$ SINGULAR IN IR $\rightsquigarrow$ SOLUTION SHOULD BE CHANGED

- IN THE U.V. $\to$ CORRECT LOGARITHMIC RUNNING OF THE COUPLING $\sim \frac{1}{g_{YM}^2} \sim Vol(S^2)$

BETTER ANALYSIS NECESSARY $\to$ BREAK $U(1)_R$ SYMMETRY.

SEE... KLEBANOV - STRASSLER -

S^2 EXAMPLES

IF $\Sigma_2 = S^2$, BOTH FOR D3 & M5

→ ALL SOLUTIONS ARE SINGULAR.

→ WE EXPECT THAT ONLY ½ OF THE SINGULARITIES GET RESOLVED

→ EITHER BY GOING TO 10-D OR 11-D , OR A DUAL DESCRIPTION

WHY:

$$\mathcal{O}_2 \sim \mathrm{Tr}\left[\tfrac{2}{3}(X_1^2+X_2^2) - \tfrac{1}{3}(X_3^2+\cdots+X_4^2)\right]$$

• TWISTED SCALARS → MASSIVE → CANNOT HAVE VEVS.

$\Rightarrow \langle \mathcal{O}_2 \rangle \geq 0 \quad (\text{or} \leq 0)$

• SINGULARITIES → COULOMB BRANCH

$\langle \mathcal{O}_2 \rangle \sim$ RELATED TO BEHAVIOUR OF φ

$\varphi(r) \sim r^2 \lg r + C_1 r^2$

INSERTION OF OPER. ↳ RELATED TO $\langle \mathcal{O}_2 \rangle$

PURE $\mathcal{N}=1,2$ SYM IN $d=2+1$

D4 ON S^2 < IN K3 — 8 SUPERCH.; IN CY — 4 SUPERCH.

M5 ON S^2 → REDUCE ON A CIRCLE

→ BOTH SEEM TO HAVE COULOMB BRANCH SINGULARITIES

JOHNSON, PEET, POLCHINSKI

– RELATION TO ENHANÇON PICTURE

→ SIMILAR IN THE IR → ANALYSIS SHOULD BE DONE

↓

D6 ON K3 → YM LIMIT → SMALL K3 →

→ "T DUAL" → D4 ON S^1 & SMALL S^2

. A CRITERION FOR ALLOWED SINGULARITIES

- LOOK FOR A SIMPLE NECESSARY CONDITION ON THE SUGRA SOLUTION TO HAVE A FIELD THEORY INTERPRETATION
- GUBSER → POSSIBLE TO EXTEND SOLUTION TO FINITE T → TRANSLATED → V BOUNDED ABOVE ($V_{IR} \leq V_{UV}$)
- WE :

  g_{00} . BOUNDED ABOVE

  . SHOULD NOT INCREASE AS WE APPROACH THE SINGULARITY.

  PHYSICAL REDSHIFT FACTOR → 10-D OR 11-D METRIC. (EINSTEIN)
- SINGULARITY SHOULD NOT BE REPULSIVE (HARD TO FORM BY GRAV. COLLAPSE)
- UV / IR CONNECTION
- <u>NOT</u> IMPLIED BY C-THEOREM.

– NO GO THEOREM FOR R-S & dS COMPACTIFICATIONS OF SUPERGRAVITY

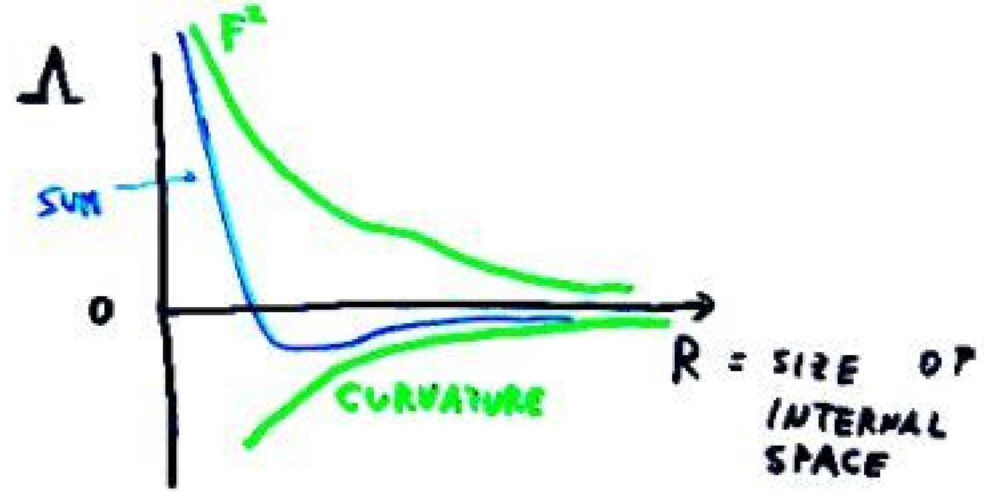
- THERE WERE SOME NO GO THEOREMS FOR SMOOTH SUSY R-S COMPACTIFICATIONS OF 5-d GAUGED SUGRA KALLOSH, LINDE, WIJNHOLT, ZUREV
- WE WILL CONSIDER 10-D OR 11-D SUGRA.

ASSUMPTIONS

- EINSTEIN'S EQUATION HOLDS WITH NO HIGHER DERIVATIVE CORRECTIONS

$$R_{MN} = \tilde{T}_{MN}$$

- $\tilde{T}_{MN}$ → CONTRIBUTIONS FROM m-FORM $F_{M_1 \cdots M_m}$ FIELD STRENGTHS.
- $V \leq 0$ NEGATIVE POTENTIAL
- FINITE G_N IN d DIMENSIONS.



Λ
F^2
SUM
0
R = SIZE OF INTERNAL SPACE
CURVATURE

$$ds^2_D = \Omega^2(y)\Big(dx^2_d + \underbrace{\hat{g}_{mn}\, dy^m dy^n}_{D-d}\Big)$$

$\mathbb{R}^d$ OR dS^d

- ASSUME MAXIMAL SYMMETRY

$\Rightarrow$ ONLY $F_{01\ldots m}$, $m \geq d$ FORMS CAN HAVE EXPECTATION VALUES IN d DIMENSIONS.

EQUATION OF MOTION FOR Ω.

(A) $$\hat{\nabla}^2 \Omega^{D-2} = \Omega^{D-2} R(x) + \Omega^D\left[(n-1)F_e^2 - F_i^2\right] \geq 0$$

AND $= 0$ ONLY IF
- $V = 0$
- $F_{m_1 \ldots m_n} = 0 \quad m > 1$
- $R(x) = 0 \quad (\mathbb{R}^d)$.

- IF THE SPACE IS COMPACT & SMOOTH

$\rightarrow$ INTEGRATE L.H.S. OF (A) $\Rightarrow$ WARP FACTOR IS CONSTANT (FOR $\mathbb{R}^d$) OR INCONSISTENCY FOR dS_d

. SUPPOSE WE HAVE ONLY REGIONS WHERE $\Omega \to 0$ (AS IN ADS) OR SINGULARITIES WHERE Ω IS NON-INCREASING.

$\mathcal{R}$ = EVERYTHING EXCEPT A NEIGHBOURHOOD OF THE SINGULARITIES

INTEGRATING Ⓐ OVER $\mathcal{R}$

$$\int_{\partial\mathcal{R}} \vec{\nabla}\Omega^{D-2}\cdot\vec{n} = \int \text{R.H.S. of Ⓐ} \geq 0$$

$$\leq 0$$

$\Rightarrow = 0$ & AGAIN NO R.S OR dS.

. MASSIVE IIA , $V > 0$,

$\nabla^2 \Omega^{D-2} = \dots$

$\nabla^2 \phi = \dots$

COMBINING THEM $\to$ NO dS_d OR R^d COMPACTIF

- IN STRING THEORY, AS OPPOSSED TO SUGRA, THERE **ARE** HIGHER DERIVATIVE CORRECTIONS

  → ESSENTIAL FOR THESE COMPACTIFICATIONS

  ↓

  ALL KNOWN EXAMPLES RELY ON THEM.

  MANY REFERENCES...

- IN MASSIVE 2A → BOUNDARIES → D8 BRANES..

SUMMARY

- SUGRA SOLUTIONS FOR TWISTED FIELD THEORIES
- SOME HAVE IR CONFORMAL FIXED POINTS
- SOME CHECKS IN 3+1 DIM. $\rightarrow$ 1+1
- M5 BRANE
- $\rightarrow$ D4 BRANE $\rightarrow$ 2+1 DIM THEORIES

FUTURE

- OTHER EXAMPLES
- UNDERSTAND AdS_5 COMPACTIFICATIONS OF M-THEORY & THEIR ASSOCIATED FIELD THEORIES.
- RESOLVING THE SINGULARITY IN THE $\mathcal{N}=1$ $d=4$ CASE FROM NS5 ON S^2.