

Stable Non-BPS States and Their Holographic Duals

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Based on:

S.M. and N.V. Suryanarayana, in progress.

S.M. and N.V. Suryanarayana, hep-th/0003219

S.M., N.V. Suryanarayana, D. Tong, hep-th/0001066

1. Introduction

- Type II string theory has various stable, BPS Dp -branes:

$$IIA: \quad p = 0, 2, 4, 6, 8$$

$$IIB: \quad p = -1, 1, 3, 5, 7, 9$$

and unstable non-BPS Dp -branes:

$$IIA: \quad p = -1, 1, 3, 5, 7, 9$$

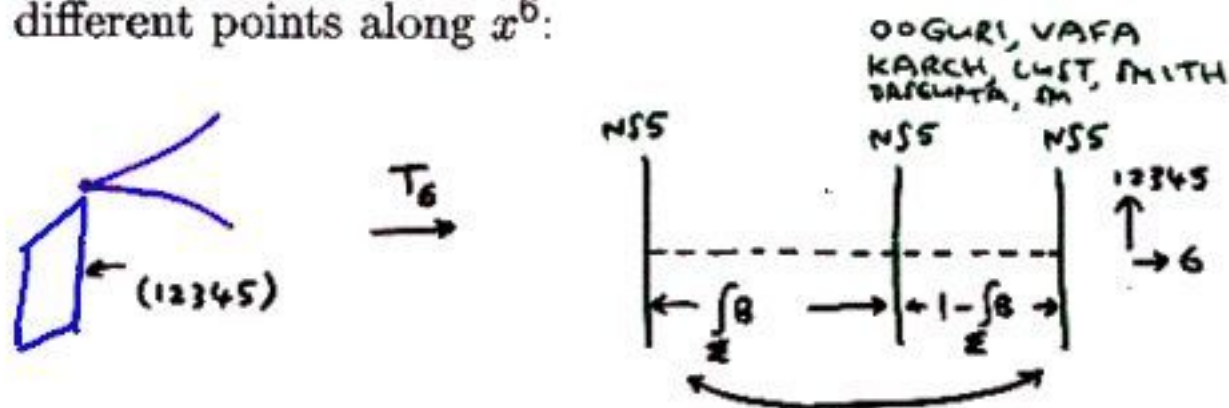
$$IIB: \quad p = 0, 2, 4, 6, 8$$

- The spectrum on the latter branes is the spectrum of a single open string, but without GSO projection. Hence there is a real tachyon.
- The BPS branes are of course stable, while the non-BPS branes can decay, via tachyon condensation, into the vacuum, or into lower (BPS or non-BPS) branes.
- A pair of a BPS brane and its antibrane is also unstable and can decay similarly.

- This is quite a general paradigm. In flat backgrounds, type II branes are either BPS and stable, or non-BPS and unstable.
- It is interesting to look for backgrounds which admit non-BPS but stable branes. In this situation, masses are not protected by BPS formulae. We can hope to disentangle effects of duality from effects of supersymmetry.
- If the backgrounds are themselves non-supersymmetric then things rapidly become difficult. The most accessible situations are those where the backgrounds are supersymmetric, but the states that we study are not.
- Some examples are: orbifolds, orientifolds, Calabi-Yau compactifications. Another class of examples is provided by ^(DIANAMY-WITTEN) *suspended brane constructions*. These all have lower supersymmetry than flat space, which helps to find stable non-BPS states. → SM, SUBVANDANA, TONG
 → SEN BERGMAN, GABRIEL ANTONIADIS, ... LERDA, ..., DIVECKHA ...
- In the following I will make extensive use of the conifold singularity and its brane-construction dual. ALE spaces will also play an auxiliary rôle.

2. Singularities, Brane Duals and Fractional Branes

- Let us start with type IIB on a Z_2 ALE singularity along the (6789) directions.
- Via T-duality along x^6 , the ALE singularity turns into a pair of NS5-branes in type IIA string theory, extending along the (12345) directions and located at different points along x^6 :



- The ALE singularity hides a 2-cycle Σ of zero size, which can be resolved to get an Eguchi-Hanson space. But at the orbifold point, the NS-NS B -field has a flux of $\frac{1}{2}$ through this 2-cycle. In the brane dual, the NS5-branes are symmetrically located along the x^6 circle.
- This duality extends beyond the orbifold point. Varying the B -flux in the ALE corresponds to varying the relative x^6 separations of the NS5-branes.

- If we bring a D3-brane into the plane of an ALE singularity, it can split into a pair of fractional D3-branes $f3, f3'$ of charge and tension α and $1 - \alpha$ where $\alpha \equiv \int_{\Sigma} B$ is the B -flux.

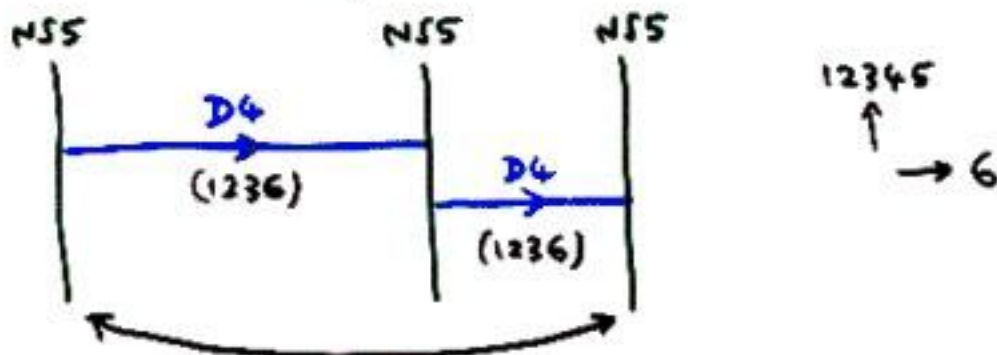
POLCHINSKI
DOUGLAS, MOORE

- The fractional branes are interpreted as:

$f3$: D5 wrapped on Σ

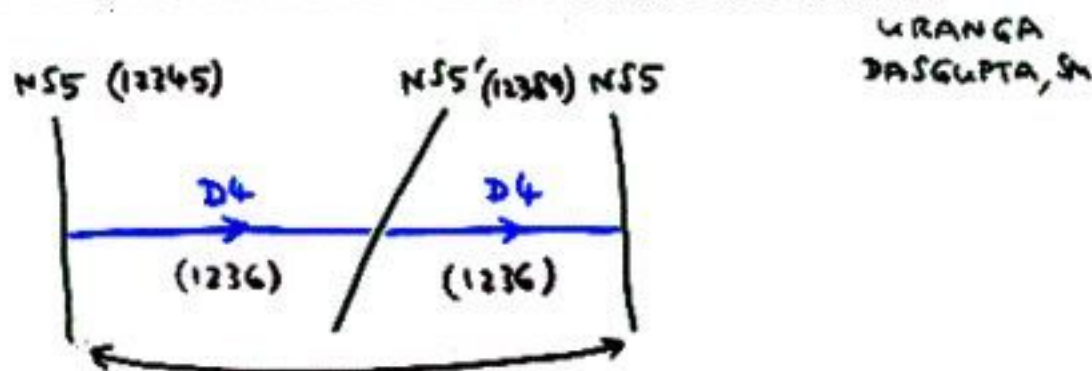
$f3'$: $\bar{D}5$ wrapped on Σ , $\int_{\Sigma} F = 1$

- In the dual brane construction, a D4-brane wrapped on x^6 can be brought in to touch the NS5-branes, where it can break into two pieces:



- The gauge group $U(1) \times U(1)$ and the presence of bi-fundamental matter is also evident from the brane construction.

- An analogous relation holds for the *conifold* singularity along the (456789) directions. It is dual to a similar brane construction but with rotated NS5-branes:



- This model too has bi-fundamental matter, but also a quartic superpotential.

KLEBANOV, WITTEN

- Aspects of fractional branes at a conifold (to be used later in this talk) were discussed in:

DASGUPTA, SM
hep-th/9904131

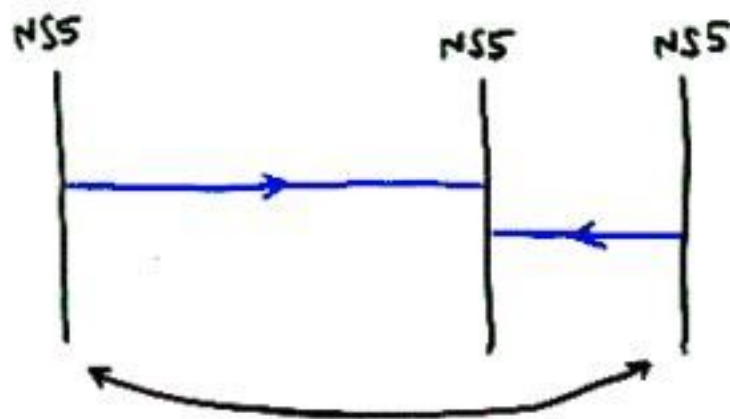
This paper also explored a relation to Witten's MQCD.

3. Fractional Branes and a Stable Non-BPS Configuration

- An interesting class of non-BPS brane configurations is obtained from the system of an *adjacent brane-antibrane pair*. In some cases, this can be analysed using perturbative string theory, via duality to ALE or conifold singularities.

SM, SURYANARAYANA, TONG

- The configuration of interest contains a pair of parallel NS5-branes oriented as was just discussed. In the two intervals between the NS5-branes, we place a D4-brane and a $\bar{D}4$ -brane:



- The NS5-brane configuration is T-dual to an ALE singularity. The D4 and $\bar{D}4$ -brane in the intervals T-dualise into a fractional brane and a fractional antibrane. Let us try to understand this correspondence in more detail.

- A $D3 - \bar{D}3$ pair at a Z_2 ALE singularity splits into 4 distinct types of fractional branes, which we call $f_3, f_3', \bar{f}_3, \bar{f}_3'$.
- These are interpreted as follows:

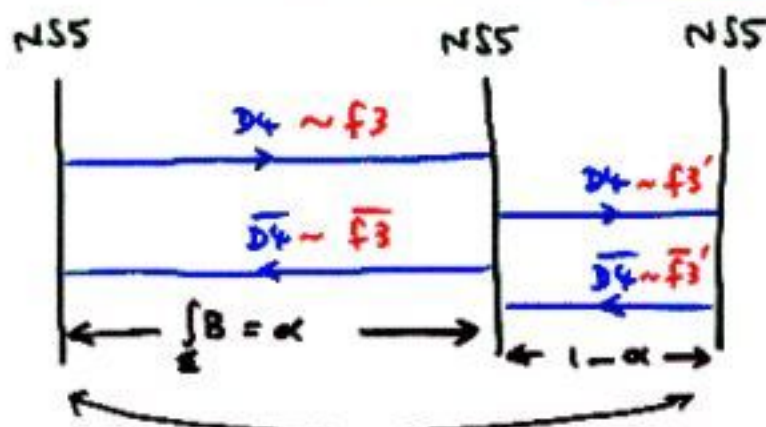
$$f_3 : D5 \text{ wrapped on } \Sigma, \quad \int_{\Sigma} F = 0$$

$$f_3' : \bar{D}5 \text{ wrapped on } \Sigma, \quad \int_{\Sigma} F = 1$$

$$\bar{f}_3 : \bar{D}5 \text{ wrapped on } \Sigma, \quad \int_{\Sigma} F = 0$$

$$\bar{f}_3' : D5 \text{ wrapped on } \Sigma, \quad \int_{\Sigma} F = 1$$

- Introducing a $D4 - \bar{D}4$ pair in the brane construction, we see that it too can break into four distinct pieces:



- This is the Coulomb branch, and we can identify the four fractional branes as in the figure.

- Since we are interested in studying an adjacent D4– $\bar{D}4$ pair, we see that the dual fractional branes are f_3 and \bar{f}_3' .
- This system has a net D5-brane charge of +2, and a net D3-brane charge of $2\alpha - 1$.
- The open strings connecting adjacent branes correspond in the ALE dual to the following Chan-Paton factors:

$$\begin{aligned}
 f_3 - \bar{f}_3' &: \frac{1}{2}(\sigma_3 + i\sigma_2) \otimes (\sigma_1 + i\sigma_2) \\
 \bar{f}_3' - f_3 &: \frac{1}{2}(\sigma_3 - i\sigma_2) \otimes (\sigma_1 - i\sigma_2) \\
 f_3' - \bar{f}_3 &: \frac{1}{2}(\sigma_3 - i\sigma_2) \otimes (\sigma_1 + i\sigma_2) \\
 \bar{f}_3 - f_3' &: \frac{1}{2}(\sigma_3 + i\sigma_2) \otimes (\sigma_1 - i\sigma_2)
 \end{aligned}$$

- These are all odd under the ALE projection. Therefore the strings connecting f_3 to \bar{f}_3' have no tachyonic or massless bosonic states. In fact, these strings only give massless fermions.

- Next we construct the boundary states corresponding to the fractional D3-branes, and use them to compute the force between the adjacent pair of interest.
- There are four independent consistent boundary states for D3, $\bar{D}3$, which can be identified with the four fractional branes $f_3, f_3', \bar{f}_3', \bar{f}_3$.

$$|D3, +\rangle = \frac{1}{2}(|U\rangle_{NSNS} + |U\rangle_{RR} + |T\rangle_{NSNS} + |T\rangle_{RR})$$

$$|D3, -\rangle = \frac{1}{2}(|U\rangle_{NSNS} + |U\rangle_{RR} - |T\rangle_{NSNS} - |T\rangle_{RR})$$

$$|\bar{D}3, +\rangle = \frac{1}{2}(|U\rangle_{NSNS} - |U\rangle_{RR} - |T\rangle_{NSNS} + |T\rangle_{RR})$$

$$|\bar{D}3, -\rangle = \frac{1}{2}(|U\rangle_{NSNS} - |U\rangle_{RR} + |T\rangle_{NSNS} - |T\rangle_{RR})$$

$|U\rangle = \text{UNTWISTED}$
 $|T\rangle = \text{TWISTED}$

- The amplitude of interest is:

$$\int_0^\infty dt \langle \bar{D}3, + | e^{-lH_c} | D3, + \rangle$$

$$= \int_0^\infty \frac{dt}{2t} \text{tr}_{NS-R} \left(\frac{1 - (-1)^F}{2} \frac{1 - R}{2} e^{-2tH_0} \right)$$

$$= \frac{v^{(4)}}{32(2\pi)^4} \int_0^\infty \frac{dt}{t^3} \left\{ \frac{f_3(\bar{q})^8 + f_4(\bar{q})^8 - f_2(\bar{q})^8}{f_1(\bar{q})^8} \right.$$

$$\left. - 4 \frac{f_4(\bar{q})^4 f_3(\bar{q})^4 + f_4(\bar{q})^4 f_3(\bar{q})^4}{f_1(\bar{q})^4 f_2(\bar{q})^4} \right\}$$

(AT ZERO SEPARATION)

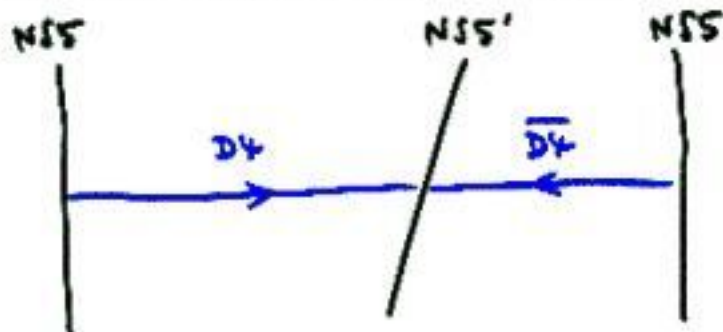
- This simplifies to:

$$\frac{v^{(4)}}{16(2\pi)^4} \int_0^\infty \frac{dt}{t^3} \frac{f_4(\bar{q})^8}{f_1(\bar{q})^8} \left[1 - 4 \frac{f_1(\bar{q})^4 f_3(\bar{q})^4}{f_2(\bar{q})^4 f_4(\bar{q})^4} \right]$$

The integrand is strictly negative, implying that the force between the $f3$ and $\bar{f}3'$ is repulsive.

- Thus we find that the force between an adjacent suspended brane-antibrane pair is repulsive.
- Now consider a "twist" on the configuration of adjacent brane-antibrane pairs that we discussed earlier. We rotate one NS5-brane:

SM, SURYANARAYANA



- Thus we now have an NS5 and an NS5'-brane, making up the brane dual of the conifold. The adjacent brane-antibrane pair is dual to fractional branes at a conifold.

- Physically, we expect a repulsive force between the adjacent brane and antibrane, as was shown earlier in the unrotated model. But there is also a classical attraction since the branes cannot separate without being stretched.
- This leads to a possibility of stable equilibrium at finite displacement.
- In fact we get a more complicated result exhibiting a phase transition as a function of the radius.
- The tension of the stretched D4-brane is

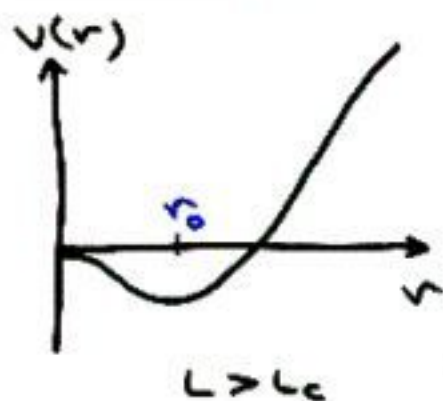
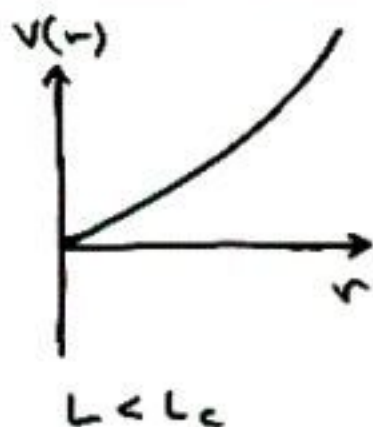
$$\mathcal{V} T_4 \sqrt{L^2 + 2r^2}$$

where \mathcal{V} is an (infinite) volume factor, T_4 is the tension of a BPS D4-brane, and L is the separation between the NS5 and NS5'-branes.

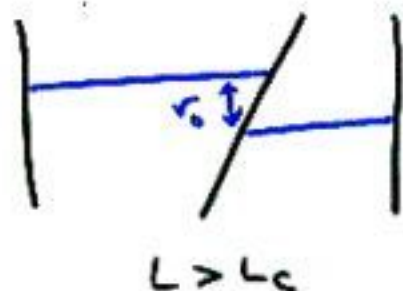
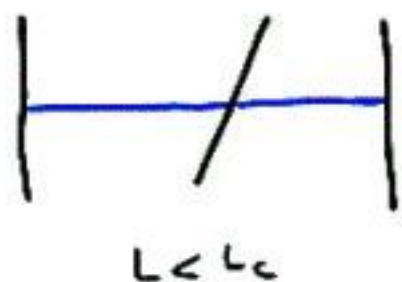
- We assume that the repulsion is as for the ALE (unrotated) case, since it comes from strings connecting the D4 - $\bar{D}4$ pair across each NS5-brane.

	$r \ll 1$	$r \gg 1$
Attraction:	$\sim r^2$	$\sim r$
Repulsion :	$\sim r^2$	$\sim \log r$

- After a calculation, we find that the shape of the potential depends on the separation parameter L .



- Hence the brane and antibrane are aligned for small L but they separate to a finite distance for large L :



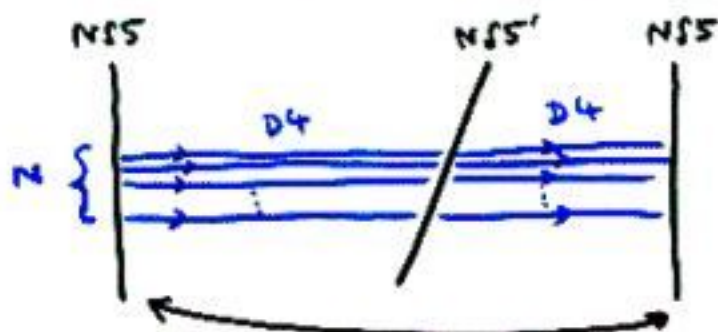
An estimate gives $L_c \sim 0.28 g_s^{-1}$.

r_0 is order 1 in units where $\alpha' = 1$. Numerically,

$L_c :$	$1.02 L_c$	$4 L_c$	$4000 L_c$
$r_0 :$	0.1	1.5	50

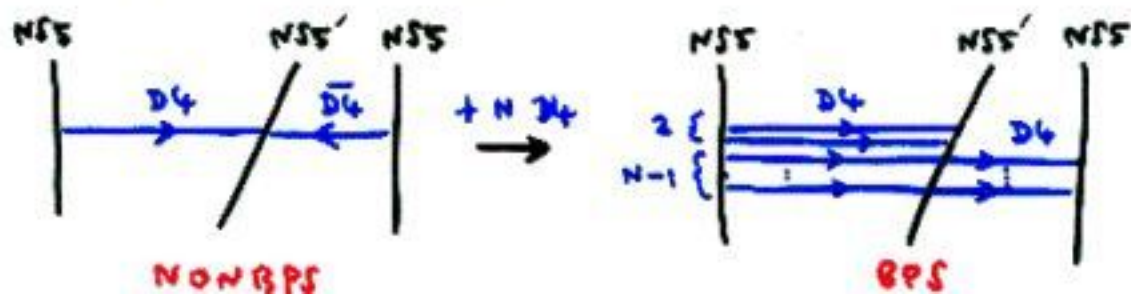
4. Branes at a Conifold and Non-BPS States in AdS_5

- If we bring N D3-branes to a conifold singularity and take the large- N limit, we end up with a $\frac{1}{4}$ -supersymmetric background of type IIB: $AdS_5 \times T_{1,1}$ where $T_{1,1}$ is a particular Einstein 5-manifold.
- If we T-dualise the conifold we get a model of rotated NS5-branes. N D3-branes at the conifold become N D4-branes wrapped round the x^6 circle:

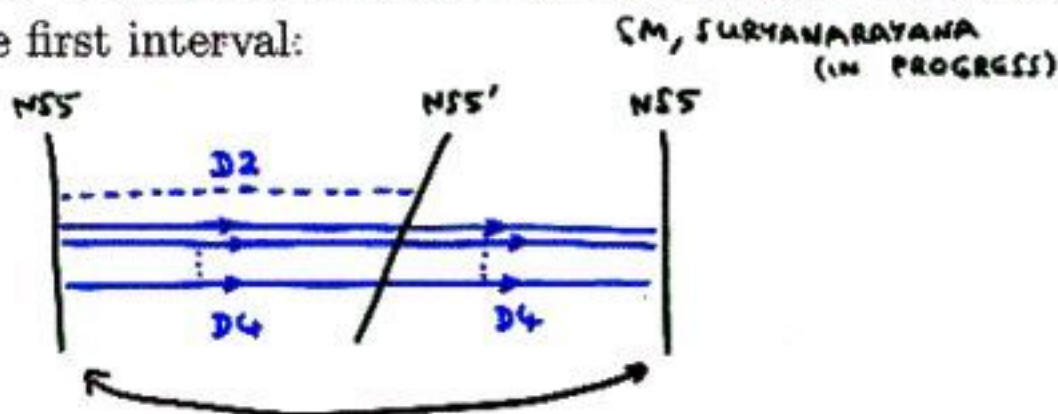


URANGA
DASGUPTA, SM

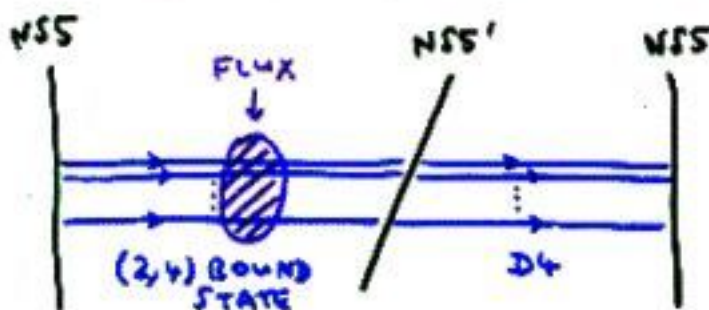
- The adjacent brane-antibrane model that we have described does not have an AdS dual. If we add N D4-branes then the $\bar{D}4$ will annihilate against a fractional D4-brane, leaving $N - 1$ whole D4-branes plus two fractional D4-branes:



- Let us now describe a stable non-BPS brane construction that, instead, does have an AdS dual.
- Take N D4-branes as before and introduce a D2-brane in the first interval:



- In the conifold geometry, this corresponds to the introduction of a fractional D-string in the plane of the singularity.
- This configuration is clearly non-supersymmetric. For example, the strings joining a D2-brane and N D4-branes in the interval will be tachyonic. The stable result should be a bound state of the ^{fractional} D4-branes and the D2-brane. While this is BPS by itself, the neighbouring interval still has only D4-branes:



- The (2, 4) bound state and the D4-branes preserve incompatible supersymmetries. Hence the whole system is non-BPS, much as for an adjacent brane-antibrane pair.
- In the conifold geometry, we have a fractional D-string bound to N $f3$ -branes and coincident with N $f3'$ branes.
- Now we can take the large N limit. What does this state become?
- The conifold geometry is replaced by its 5-manifold base, the Einstein space $T_{1,1}$. Topologically,

$$T_{1,1} \sim S^2 \times S^3$$

- The S^2 is the same 2-cycle that was of vanishing size before taking the large- N limit. The fractional D-string was actually a D3-brane wrapped on this S^2 .
- Hence, in the large N limit, the fractional D-string can be identified with a "fat string" obtained by wrapping a D3-brane on S^2 .

- Before going further, let us list all the unwrapped and wrapped branes of this model:

Dim.	--	S^2	S^3	$S^2 \times S^3$
-1	$D(-1)$	$D1$	$UD2$	$UD4$
0	$UD0$	$UD2$	$D3$	$D5$
1	$D1$	$D3$	$UD4$	$UD6$
2	$UD2$	$UD4$	$D5$	$D7$
3	$D3$	$D5$	$UD6$	$UD8$
4	$UD4$	$UD6$	$D7$	$D9$

(U = UNSTABLE)

- The $D5$ wrapped on S^2 is known to be a domain wall that augments the gauge group:

GUBSER, KLEBANOV

T-DUAL: DASGUPTA, JN

$$SU(N) \times SU(N) \rightarrow SU(N+1) \times SU(N)$$

- The $D3$ wrapped on S^2 is our fat string. We would like to understand its holographic dual description.
- The Euclidean D-string wrapped on S^2 gives rise to a new instanton, while the (unstable) $UD2$ on S^2 is a new unstable D0-brane. We will examine their holographic duals too.

5. Some Properties of the Fat String

- The nature of the fat string depends on the B-flux through S^2 . In general we have

$$\int_{S^2} B_{NS,NS} = \alpha, \quad \int_{S^2} B_{RR} = \beta$$

- The $SU(N) \times SU(N)$ gauge theory on the 3-branes has couplings and θ -angles given by

$$\tau_1 = \beta + \alpha\tau_s$$

$$\tau_2 = -\beta + (1 - \alpha)\tau_s$$

where $\tau_s = \frac{\chi_{RR}}{2\pi} + \frac{i}{g_s}$.

- The fat string carries D-string charge α and F-string charge β , by virtue of the Chern-Simons coupling

$$\int B_{NS,NS} \wedge B_{RR} \rightarrow \alpha \int B_{RR} + \beta \int B_{NS,NS}$$

on a D3-brane.

- It is convenient to choose $\beta = 0$.

- The tension of the fat string can be estimated from integrating the DBI action of a D3-brane over S^2 :

$$T_{\text{fat}} \sim T_3 \int_{S^2} \sqrt{\det g + (B_{NS,NS})^2}$$

In the flat space limit, the S^2 is of zero size and this becomes

$$T_{\text{fat}} \sim T_3 |\alpha|$$

which shows that it is BPS. On the other hand at large N the dominant contribution comes from

$$T_{\text{fat}} \sim T_3 \int_{S^2} \sqrt{g} \sim \frac{N}{(g_s N)^{\frac{1}{2}} \alpha'}$$

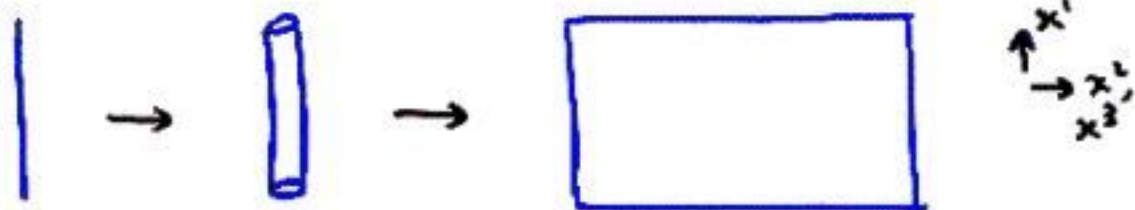
- As with fractional branes, there are really two complementary fat strings, the second one being an anti D3-brane wrapped over S^2 and having a magnetic flux $\int F = 1$ over the cycle. We call this a fat' string. It has a D-string charge $(1 - \alpha)$.
- The non-BPS nature of fat strings, and their charges, imply the reaction

$$\text{fat string} + \text{fat}' \text{ string} \rightarrow \text{D-string}$$

with loss of energy.

- Recall how a D-string is understood in holography. In $AdS_5 \times S^5$, a D-string parallel to the boundary corresponds to a magnetic flux tube. As the string falls towards the horizon, the flux tube fattens and in the limit becomes a constant flux:

BALASUBRAMANIAN,
KRAMS, LAWRENCE, TRVEDI;
BANKS, DOUGLAS,
KOROWITZ, MARTINEZ



- The same result holds for a D-string in $AdS_5 \times T^{1,1}$, but the flux is in the diagonal of the $SU(N) \times SU(N)$ gauge group.
- The fat string is similarly a flux tube in the boundary theory, but this time the flux is only in one $SU(N)$ factor.
- This is consistent with its non-BPS nature. On a 3-brane we have nonlinearly realised supersymmetry that acts on the gauginos as:

$$\delta^* \lambda_{\alpha}^{(1)} = \frac{1}{4\pi\alpha'} \eta_{\alpha}^*, \quad \delta^* \lambda_{\alpha}^{(2)} = \frac{1}{4\pi\alpha'} \eta_{\alpha}^*$$

and linearly realised supersymmetry:

$$\delta \lambda_{\alpha}^{(1)} = F_{23}^{(1)} \sigma_{\alpha}^{23\beta} \eta_{\beta}, \quad \delta \lambda_{\alpha}^{(2)} = F_{23}^{(2)} \sigma_{\alpha}^{23\beta} \eta_{\beta}$$

We see that, if and only if the fluxes are diagonal: $F^{(1)} = F^{(2)} = F$, there is a surviving set of linearly realised supersymmetries, described by choosing

$$\eta_{\alpha}^* = -4\pi\alpha' F_{23} \sigma_{\alpha}^{23\beta} \eta_{\beta}$$

- With this non-BPS fat string, one can now study Wilson/'t Hooft loops in the AdS context and compare predictions at weak and strong 't Hooft coupling (in progress).
- A brief comment on some other wrapped branes:

D1 wrapped on S^2 is a new "D-instanton". It is expected to be dual to a Yang-Mills instanton in the first factor of $SU(N) \times SU(N)$. (*fractional instanton*)

It has its own associated sphaleron, the D2-brane of type IIB wrapped on S^2 , dual to a meron in $SU(N)$. (*fractional meron*)

- The relation between the two is parallel to the one between unwrapped D-instantons and D0-branes, studied recently.

DRUKKER, GROSS, ITZHIKI

6. Conclusions

- The stable brane-antibrane construction could describe an interesting non-SUSY model field theory. Microscopically it has a pair of branes separated by a finite calculable distance (brane-world model?).
- BPS brane constructions are most useful when we can use S-duality or M-theory. What do we learn from these about non BPS brane constructions?
- Is there a physical reason why "fat" objects are associated to one $SU(N)$ factor while "thin" objects are diagonal in $SU(N) \times SU(N)$?
(wrapped on S^2)
- A lot of interesting physical results should emerge from a closer inspection of the AdS/CFT correspondence for non-BPS states.

→ would be the first test of AdS/CFT on stable, non-BPS states.

- ⊙ Can use Myers effect / Polchinski-Strominger picture to study non-Susy situations?