# Nonabelian D-branes
# and
# Noncommutative Geometry

1. Nonabelian D-brane Action

   (RCM; hep-th/9910053)

2. Dielectric Branes

   (RCM; hep-th/9910053)

3. Dielectric F-strings

   (Tafjord + RCM)

4. Bion Core

   (Constable,Tafjord + RCM; hep-th/9911136)

5. D1-D5 Core

   (Constable, Tafjord + RCM)

6. Ghidrah vs. the Giant Gravitons

   (Grisaru, Tafjord + RCM)

## Abelian World-Volume Action:

$$S = S_{BI} + S_{WZ}$$

Born-Infeld action: (Leigh)

$$S_{BI} = -T_p \int d^{p+1}\sigma \, e^{-\phi} \sqrt{-\det P[G_{ab} + B_{ab}] + F_{ab}}$$

Wess-Zumino action: (Polchinski; Douglas; Li; Green, Harvey + Moore)

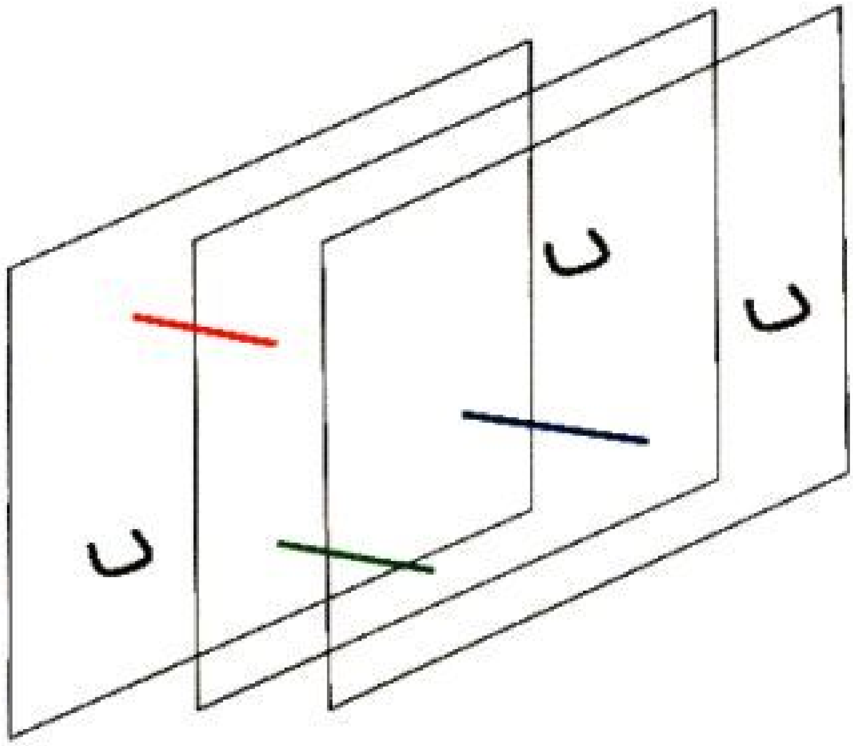
$$S_{WZ} = \int P\left[\sum C^{(n)} e^{B}\right] e^{F}$$

$$= \int P\left[C^{(p+1)} + C^{(p-1)} \wedge (B + F) + \ldots\right]$$

D-brane bound states
(Douglas; Witten)

Coincident D-Branes

→ **Nonabelian** Yang-Mills



$U(1)^N \longrightarrow U(N)$

(Witten; hep-th/9510135)

How do we incorporate nonabelian character in D-brane dynamics?

(Douglas, Ooguri, Kato, Tseytlin, . . . . )

# Nonabelian World-Volume Action:

$$S = S_{BI} + S_{WZ}$$

(Taylor + van Raamsdonk, hep-th/9910052;
RCM, hep-th/9910053)

Born-Infeld action:

$$S_{BI} = -T_p \int d^{p+1}\sigma \; \mathrm{STr}\Big\{ e^{-\phi} \left(\det Q^i{}_j\right)^{1/2}$$
$$\times \left(-\det P\left[E_{ab} + E_{ai}\left(Q^{-1} - \delta\right)^{ij} E_{jb}\right] + F_{ab}\right)^{1/2} \Big\}$$

Wess-Zumino action:

$$S_{WZ} = \mu_p \int \mathrm{STr}\left(P\left[e^{i\, i_\Phi i_\Phi} \sum C^{(n)} e^B\right] e^F\right)$$

⟶ consistent with T-duality

## Nonabelian Wess-Zumino action:

$$S_{WZ} = \mu_p \int \mathrm{STr}\left(P\left[e^{i\, i_\Phi i_\Phi} \sum C^{(n)} e^{B}\right] e^{F}\right)$$

1. nonabelian field strength: $F_{ab}$

2. nonabelian Taylor expansion:

$$C^{(1)}_{\mu}(\sigma,\Phi) = \sum_{n=0}^{\infty} \frac{1}{n!} \Phi^{k_1} \cdots \Phi^{k_n} \partial_{k_1} \cdots \partial_{k_n} C^{(1)}_{\mu}(\sigma, X^i)\Big|_{X^i=0}$$

static gauge: $X^a = \sigma^a$

$$X^k(\sigma^a) \approx \Phi^k(\sigma^a)$$

3. nonabelian pullback:

$$P\left[C^{(1)}_a\right] = C^{(1)}_a + D_a \Phi^k\, C^{(1)}_k$$

with $D_a \Phi^k = \partial_a \Phi^k + i\left[A_a, \Phi^k\right]$

## Nonabelian Wess-Zumino action:

$$S_{WZ} = \mu_p \int STr\left(P\left[e^{i\, i_\Phi i_\Phi} \sum C^{(n)} e^B\right] e^F\right)$$

4. nonabelian interior product:

$$i_v B = v^\mu \; B_{\mu\nu} \, dx^\nu$$

$$i_w i_v B = w^\nu \; v^\mu \; B_{\mu\nu} \quad = -i_v i_w B$$

$$\mathbf{i}_\Phi \mathbf{i}_\Phi B = \tfrac{1}{2}\left[\Phi^j, \Phi^i\right] B_{ij}$$

nonvanishing because of nonabelian character of vector

⟶ Dp-brane couples to all RR potentials!

(a la Matrix model: BFSS; Banks, Seiberg + Sherker)

5. maximally symmetric gauge trace: STr(...)

- symmetric average over all orderings of $F_{ab}$, $D\Phi^i$, $[\Phi^i, \Phi^j]$ and individual $\Phi^k$ in functional dependence of background fields

(a la Matrix model: T + vR, hep-th/9904095, hep-th/9910052)

## Nonabelian Born-Infeld action:

$$S_{BI} = -T_p \int d^{p+1}\sigma \, \mathrm{STr}\Big\{ e^{-\phi} \left(\det Q^i{}_j\right)^{1/2} \times \left(-\det P\left[E_{ab} + E_{ai}\left(Q^{-1} - \delta\right)^{ij} E_{jb}\right] + F_{ab}\right)^{1/2} \Big\}$$

where $E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}$

(e.g., Giveon, Porrati + Rabinovici, hep-th/9401139)

$$Q^i{}_j = \delta^i{}_j + i\left[\Phi^i, \Phi^k\right] E_{kj}$$

→ in flat space:

$$\sqrt{\det Q^i{}_j} = 1 - \tfrac{1}{4}\left[\Phi^i, \Phi^k\right]\left[\Phi^i, \Phi^k\right] + \cdots$$

Warning: Symmetrized Trace needs corrections at order $F^6$!

(Taylor + Hashimoto, hep-th/9703217; Bain, hep-th/9909154)

## Noncommutative Geometry

replace spacetime coordinates by noncommuting operators: $x^\mu \rightarrow \hat{x}^\mu$

typical noncommuting relations:

1. canonical structure

$$\left[\hat{x}^\mu, \hat{x}^\nu\right] = i\,\theta^{\mu\nu} \qquad \theta^{\mu\nu} \in \mathbf{C}$$

$\langle B_{\mu\nu} \rangle \neq 0$ on planar D-brane

(e.g., Connes, Douglas + Schwarz, hep-th/9711162; Seiberg + Witten, hep-th/9908142)

2. quantum space structure

$$\hat{x}^\mu\,\hat{x}^\nu = q^{-1}\,R^{\mu\nu}{}_{\rho\tau}\;\hat{x}^\rho\,\hat{x}^\tau \qquad R^{\mu\nu}{}_{\rho\tau} \in \mathbf{C}$$

(e.g., Zumino, Wess, Madore ....)

3. Lie-algebra structure

$$\left[\hat{x}^\mu, \hat{x}^\nu\right] = i\,R^{\mu\nu}{}_{\rho}\,\hat{x}^\rho \qquad R^{\mu\nu}{}_{\rho} \in \mathbf{C}$$

⟶ fuzzy spheres (e.g., Hoppe, Madore, ....)

D-branes naturally produce noncommutative geometries of latter form since transverse scalars $\Phi^k \in \mathrm{adj}\big(U(N)\big)$

## Dielectric Branes

standard flat space potential:

$$V = -\tfrac{1}{4}\left[\Phi^i,\Phi^k\right]\left[\Phi^i,\Phi^k\right] + \cdots$$

stationary points: $\left[\Phi^i,\Phi^k\right] = 0$

$$\Phi^i = \begin{bmatrix} \lambda_1^i & & \\ & \lambda_2^i & \\ & & \ddots \end{bmatrix}$$

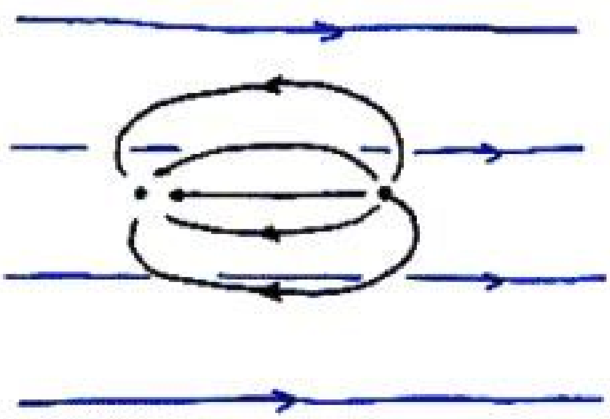
⟶ Parallel Dp-branes can sit in static equilibrium with arbitrary separation in transverse space

However, couplings to background fields can introduce many new terms to scalar potential

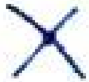
⟶ new stationary points!

## Dielectric Effect for D-branes

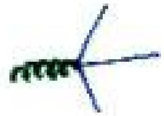
D-branes can be polarized by background supergravity fields (for which they might normally be considered neutral) into extended (noncommutative) geometry which carries multipole moment of background.



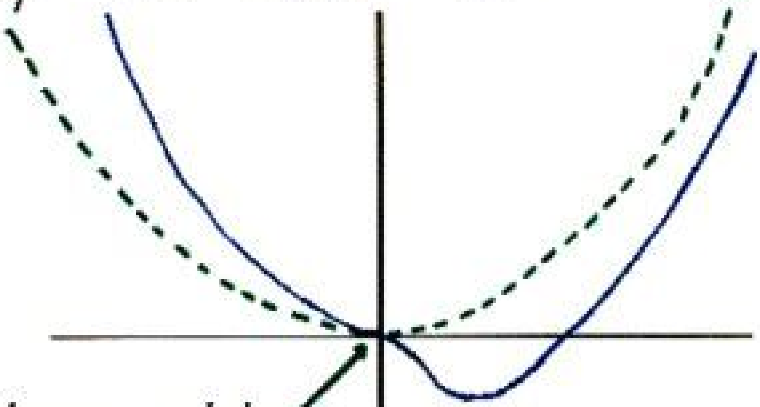
## **Dielectric Cartoon** (no indices)

basic scalar potential 

$$H = \tfrac{1}{4}\Phi^4 - \tfrac{1}{3} C\,\Phi^3$$

interaction with bulk field, C 

if $\langle C \rangle = c$ , $V_{scalar} = \frac{1}{4}\Phi^4 - \frac{1}{3} c\,\Phi^3$




$\langle \Phi \rangle = c \neq 0$ induces new couplings to bulk field C

$$H_{int} = -\tfrac{1}{3} C \langle \Phi^3 \rangle = -\tfrac{1}{3} c^3\, C$$

## Dielectric Brane Example

N D0-branes in constant $F^{(4)}$ background

$$V \cong -\mu_0\left( \tfrac{1}{4}\left[\Phi^i,\Phi^k\right]^2 + \tfrac{i}{3}\mathrm{Tr}\left(\Phi^i\Phi^j\Phi^k\right)F^{(4)}_{tijk} \right)$$

with $F^{(4)}_{tx_1x_2x_3} = -2f$

stationary points: $\dfrac{\delta V}{\delta \Phi^i} = 0$

1. $\left[\Phi^i,\Phi^k\right] = 0$ with $V_0 = 0$

2. $\Phi^i = \dfrac{f}{2}\alpha^i$ with $\left[\alpha^i,\alpha^j\right] = 2i\varepsilon^{ijk}\alpha^k$

and $V_N = -\dfrac{\pi^2\ell_s^3 f^4}{6g} N\left(N^2-1\right) < 0!$

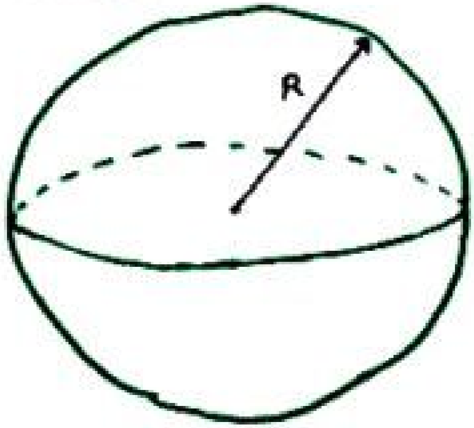
N D0-branes condense into fuzzy $S^2$ with radius

$$R = \pi\,\ell_s^2\,f\,N\sqrt{1-1/N^2}$$

and dipole coupling for spherical D2-brane

$$H_{int} = \frac{\mu_0 R}{3\pi\,\ell_s^2}\sqrt{1-1/N^2}\int dt\, F^{(4)}_{tx_1x_2x_3}$$

ground state in $F^{(4)}$ background is spherical D2-D0-brane bound state!




Dual formulation: examine D2-brane carrying N units of magnetic flux

⟶ potential, radius and dipole moment all match up to $1/N^2$ corrections!

D2-brane calculations valid for large radius

$$R >> \ell_s$$

D0-brane calculations valid for small commutators

$$R << \sqrt{N}\ell_s$$

( ⟶ densely packed D0-branes: $N/R^2 >> 1/\ell_s^2$ )

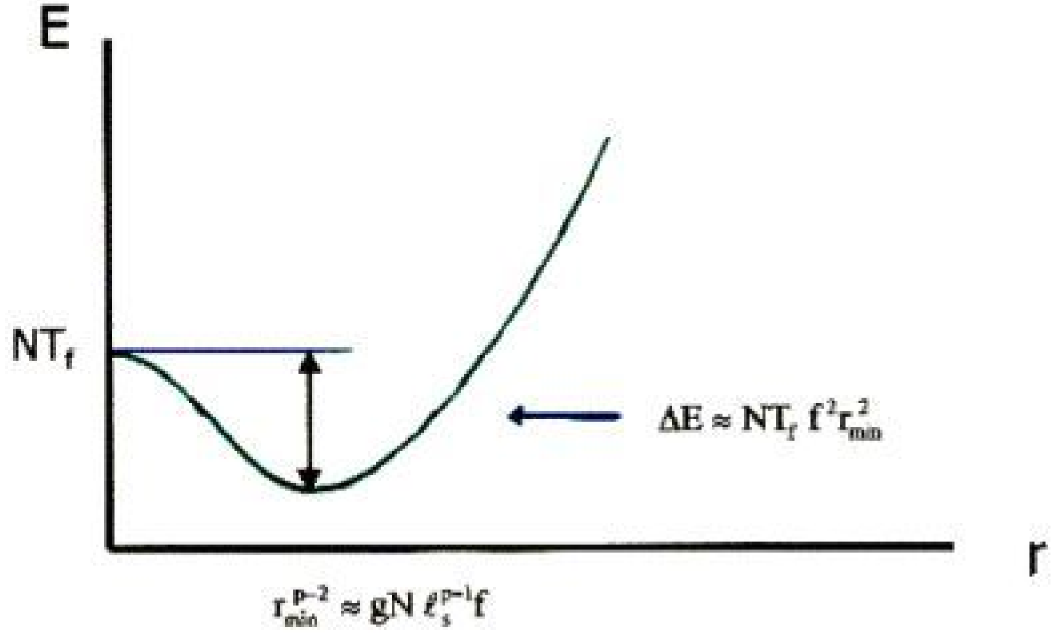
(Emparan, hep-th/9711106;
Tafjord + RCM )

## Dielectric F-strings

under influence of $F^{(p+2)}$, collection of fundamental strings can condense into a Dp-brane (with topology $R \times S^{p-1}$)

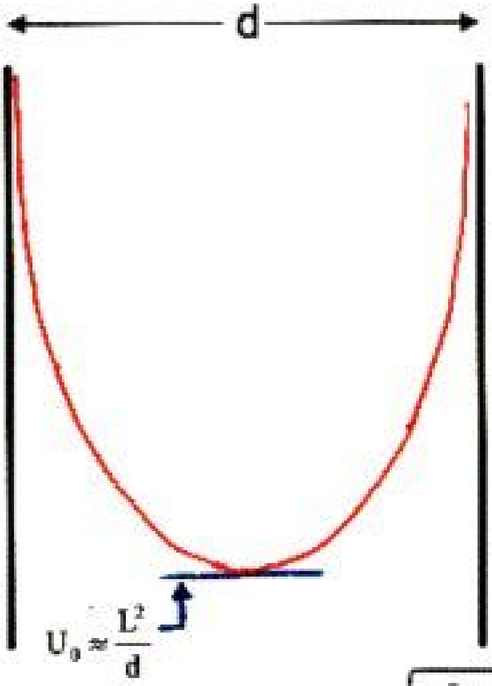
dual Dp-brane formulation with $\langle F_{tz} \rangle \neq 0$:

RR background: $F^{(p+2)}_{tz x_1 \cdots x_p} = f$

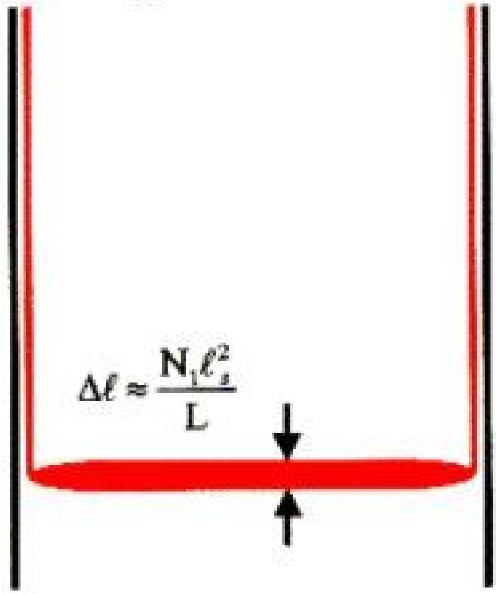



for $\frac{1}{Ng} < f\,\ell_s < \frac{1}{\sqrt{Ng}}$ , expansion is macroscopic but backreaction is small e.g., modify the quark-antiquark potential in N=4 SYM for quarks in higher (large) rep.

AdS/CFT prescription: (Maldacena, hep-th/9803002; Rey + Yee, hep-th/9803001)




Approximation:




$$V \approx -\frac{N_1\sqrt{g_{YM}^2 N_3}}{d}\left(1+\tfrac{1}{3}\, g_{YM}^2 N_3\, \frac{N_1^2}{N_3^2}\right)$$

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Proper formulation of superstring theory in RR backgrounds must show this unusual collective behavior!

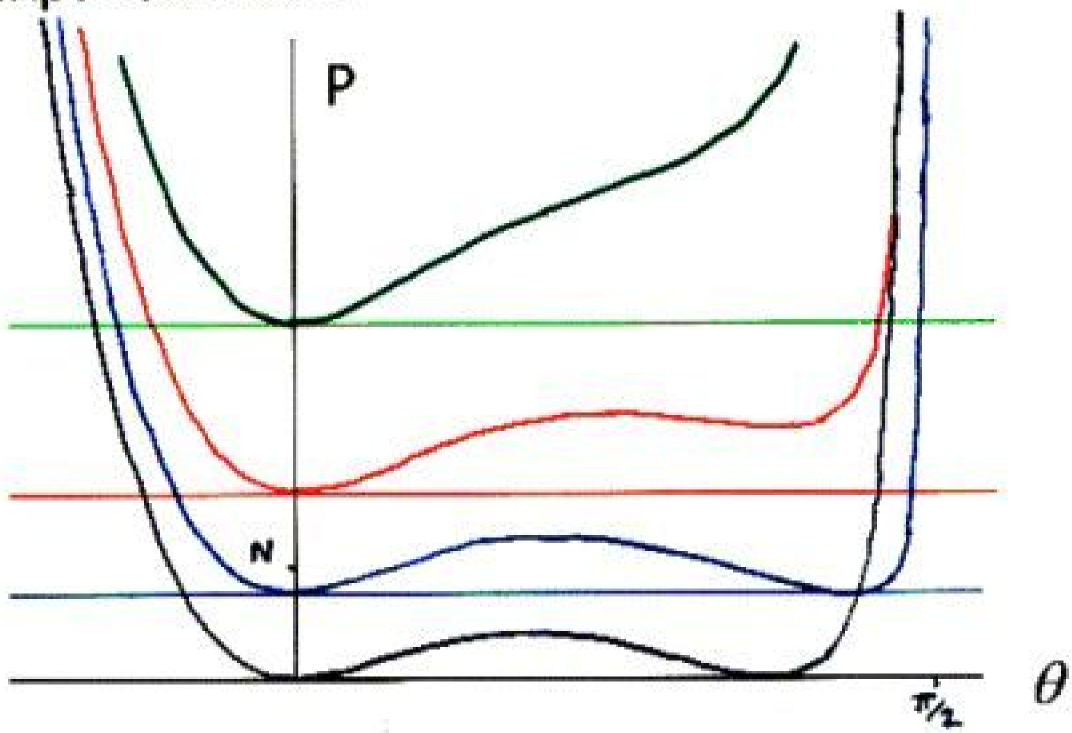
⟶ Matrix strings(?) (Schiappa, hep-th/0005145)

# Ghidrah versus the Giant Gravitons

(McGreevy, Susskind + Toumbas, hep-th/0003075)

## Giant Gravitons:

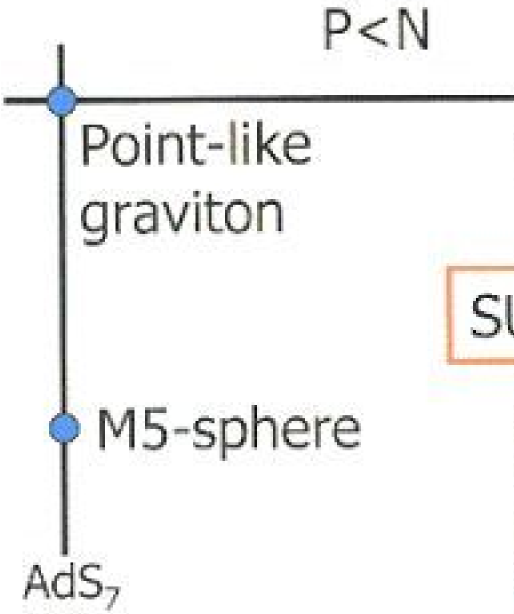
M-theory graviton in $AdS_7 \times S^4$ carrying angular momentum P on the $S^4$ is best described as a spherical M2-brane which expands into $S^4$



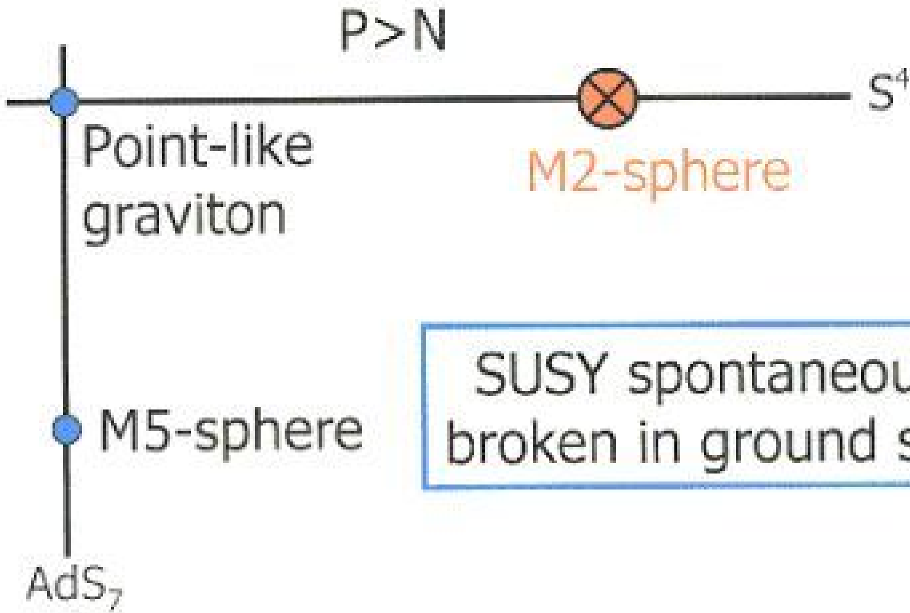

Spherical M2-brane state only exists as a BPS state for P<N (4-form flux on $S^4$)

⟶ stringy exclusion principle

(Maldacena + Strominger; Ho, Ramgoolam + Tatar; Jevicki + Ramgoolam; Gubser, . . . )




SUSY ground state!


SUSY spontaneously broken in ground state

(Grisaru, Tafjord + RCM)