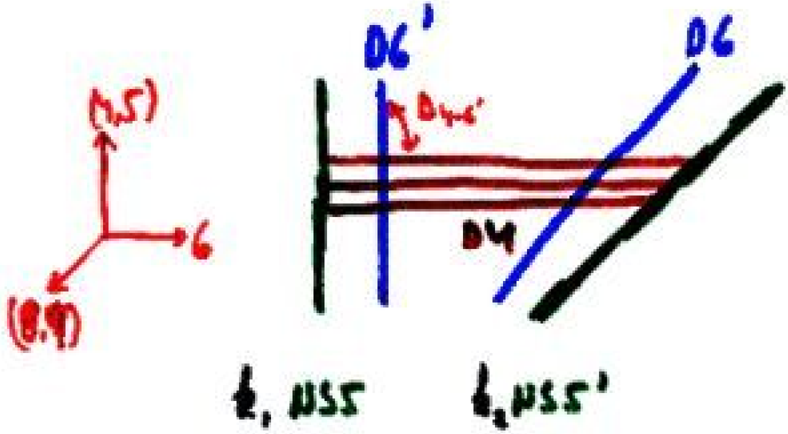
HANANY-WITTEN

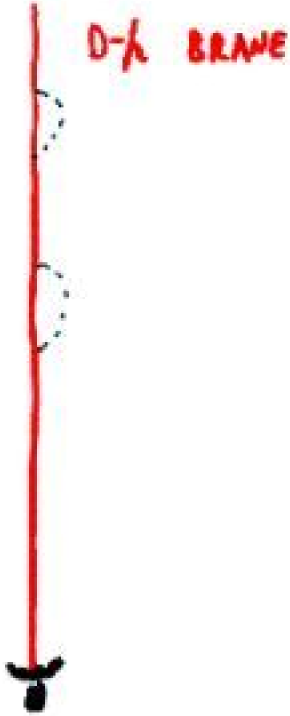
$\mathcal{N}=1$

ELITZUR, GIVEON, KUTASOV

EGK + E.R, SCHWIMMER



$\Delta_{4-6'}(x_4, x_5)$



POLCHINSKI

OPEN STRINGS ARE FREE TO ROAM

ON $p+1$ WORLD VOLUME

16 SUPERCHARGES, U(N) SUSY YM

PERTURBATIVE ANALYSIS VALID.

IN THIS WORK WE DEAL WITH

MORE COMPLICATED SETTINGS

COMIC STRIPS VS. WORLD SHEET METHODS

B.D

PHYSICALLY INTERESTING

strings 2000
Ann-Arbor

hep-th/0005052

D-BRANES IN THE BACKGROUND OF NS5S

S. ELITZUR

A. GIVEON

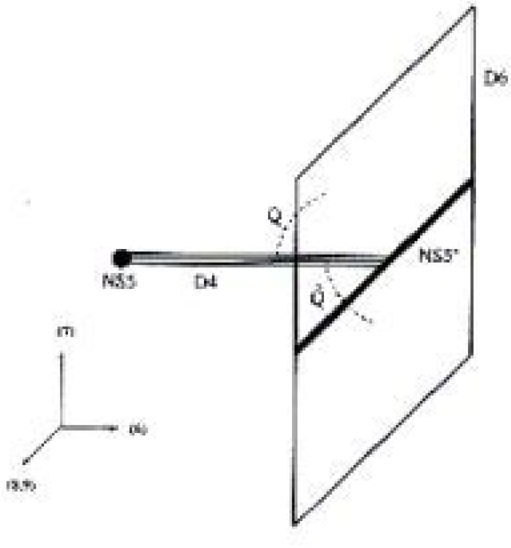
D. KUTASOV

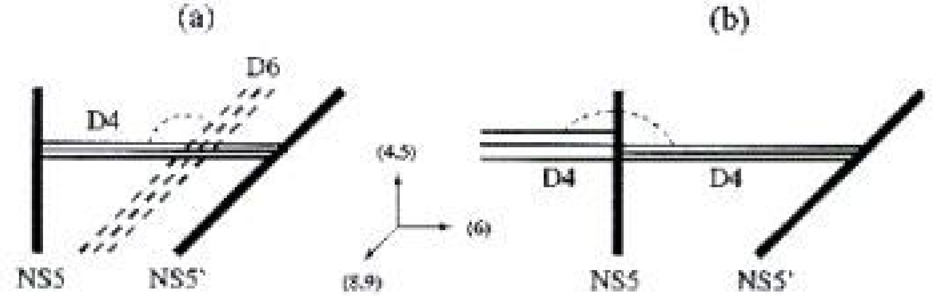
E. R.

G. SARKISSIAN

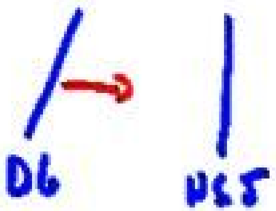
CHIRALITY

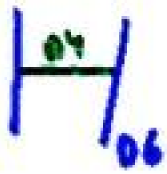
BRODIE – HANANY




(a)
D6
D4
NS5
NS5'
(4,5)
(6)
(8,9)
(b)
D4
D4
NS5
NS5'

HW TRANSITION



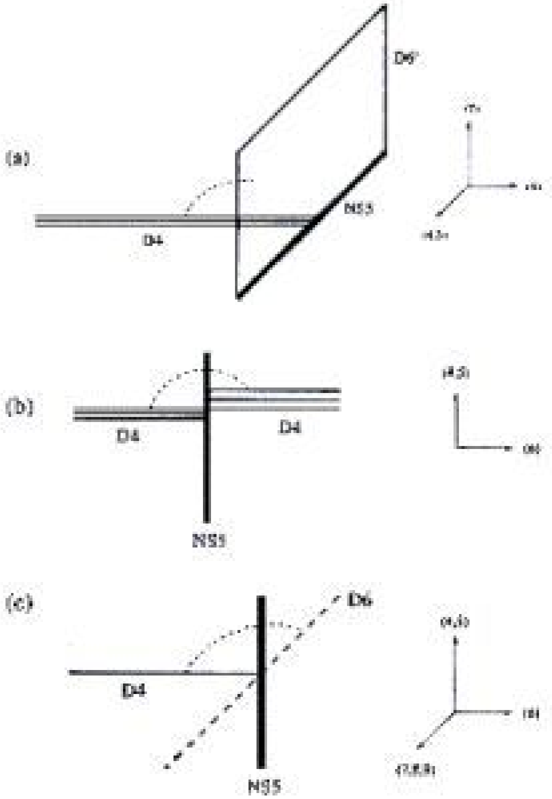


CONDENSATION

CHIRALITY

HIGGS

CONDENSE



STATES "NEAR" THE NS5 BRANE

D4, D6 INTERSECT THERE.

SIMILAR METHODS TO CY WITH SMALL CYCLES

SINGULARITIES..

NS 5 BRANES AND THEIR NHL

CONSIDER k PARALLEL NS-5 BRANES

$$e^{2(\Phi-\Phi_0)} = 1 + \sum_{j=1}^{k} \frac{\ell_s^2}{|\vec{x}-\vec{x}_j|^2} \qquad \left(\begin{matrix} g_s \to \infty \\ \vec{x} \to \vec{x}_j \end{matrix} \right)$$

$$g_s \to 0 \quad |\vec{x}| \to \infty$$

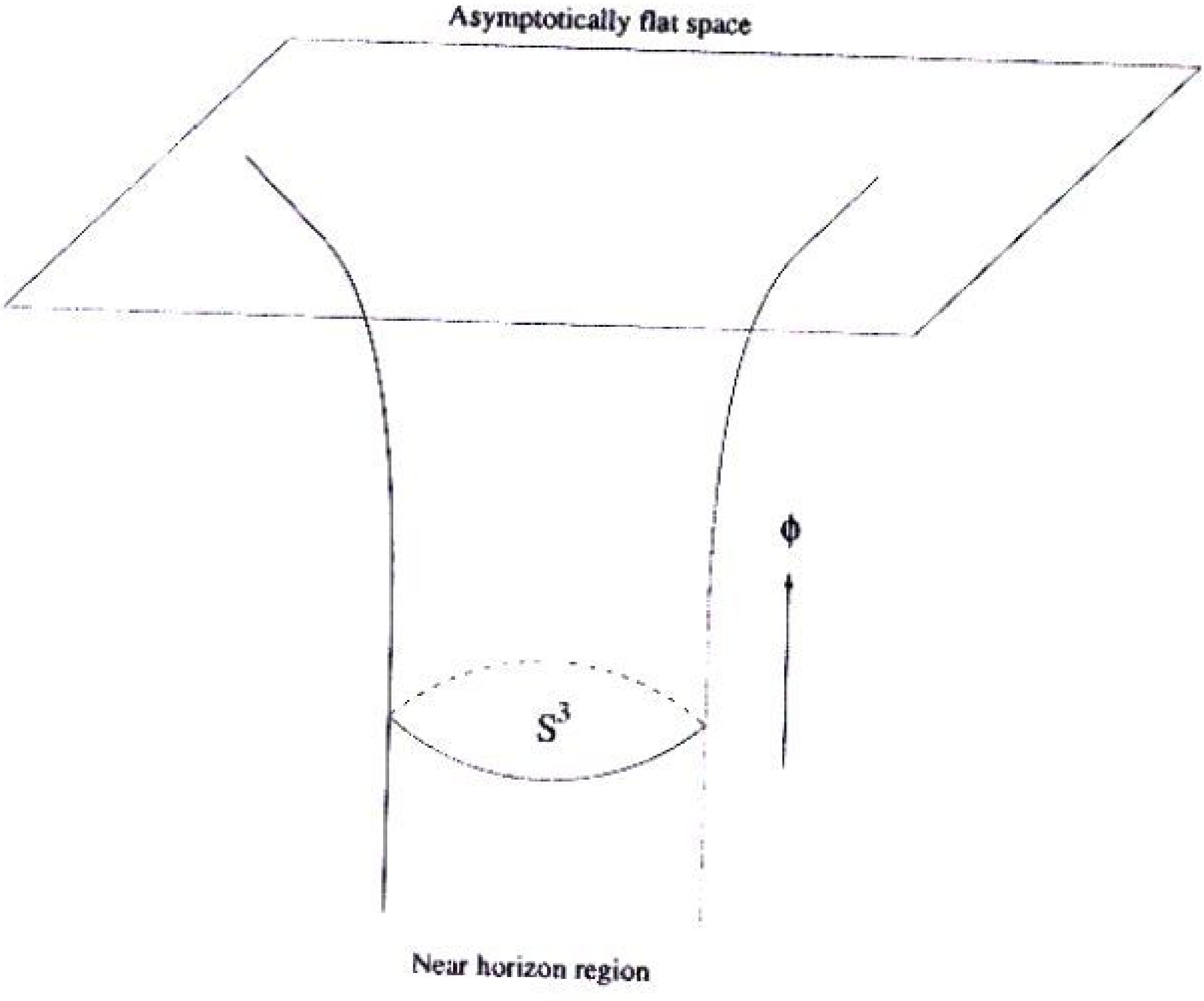
$G_{IJ} = \exp\left(2(\phi-\phi_0)\right) \delta_{IJ}$; $I, J = 6, 7, 8, 9$

$G_{\mu\nu} = \eta_{\mu\nu}$; $\mu, \nu = 0, 1, \ldots, 5$

$H_{IJK} = - \epsilon_{IJKL} \partial^L \Phi$; $\vec{x}_i = (x^6, x^7, x^8, x^9)$

INTERPOLATES BETWEEN FLAT SPACE $G_{IJ} = \delta_{IJ}$

AND NHL (NEAR HORIZON LIMIT)


Asymptotically flat space
ϕ
S^3
Near horizon region

NHL

$$e^{2(\Phi-\Phi_0)} = \frac{k\ell_s^2}{|\vec{x}|^2}$$ (ALL BRANES AT $k=0$)

$$G_{IJ} = \exp\left(2(\Phi-\Phi_0)\right)\delta_{IJ}$$

$$G_{\mu\nu} = \eta_{\mu\nu}$$

$$H_{IJK} = -\varepsilon_{IJKn}\,\partial^n \Phi$$

THE TARGET SPACE IS:

$$\boxed{R^{5,1} \times R_\phi \times SU_k(2)}$$ CALLAN, HARVEY, STROMINGER

ϕ ALONG THE $r = |\vec{x}|$ DIRECTION

$$\phi = \frac{1}{Q}\log\frac{|\vec{x}|^2}{k\ell_s^2} \; ; \quad \Phi = -\frac{Q}{2}\phi \; ; \; Q = \sqrt{\frac{2}{k}}$$

$$\Phi_0 = 0$$

$$g(x) = \frac{1}{|\vec{x}|}\left[-x^6 \mathbb{1} + i\left(x^8\sigma_1 + x^9\sigma_2 + x^7\sigma_3\right)\right]$$

$R^4 = (x^6, x^7, x^8, x^9)$ $\quad SO(4) \sim SU(2)_L \times SU(2)_R \quad S^3$

ROTATIONS $h_{L(R)}$ IN $SO(4)$ ACT $\quad g \to h_L\, g\, h_R$

$J^3 - \bar{J}^3$ ROTATION IN (x^6, x^7) PLANE

$J^3 + \bar{J}^3$ " " (x^8, x^9) PLANE

BOSONIC COORDINATES x^μ, ϕ, g

FERMIONIC " ψ^μ, χ^r, χ^a $a = 1, 2, 3$

FREE

THE WORLD SHEET $N=1$ SUPERCONFORMAL GENERATORS

$$T(z) = -\frac{1}{2}(\partial x^\mu)^2 - \frac{1}{2}\psi^\mu \partial \psi_\mu - \frac{1}{2}(\partial\phi)^2 - \frac{Q\partial^2\phi}{2} - \frac{1}{2}\chi^r \partial \chi^r$$

$$- \frac{1}{k} J^a J^a - \frac{1}{2}\chi^a \partial \chi^a$$

(SIMILAR $\bar{T}(\bar{z})$)

$$G(z) = i\psi_\mu \partial x^\mu + i\chi_r \partial\phi + iQ\left(\chi_a J^a - i\chi_1\chi_2\chi_3 + \partial\chi_r\right)$$

J^a GENERATE $SU(2)_{k_B = k-2}$ (BOSON) $\Longrightarrow$ $\boxed{k \geq 2}$

$$J^a_{total} = J^a + J^a_F \quad ; \quad J^a_F = -\frac{i}{2}\varepsilon^{abc}\chi_b\chi_c$$

ALSO GSO $(-1)^{F_L} = (-1)^{F_R} = 1$ (16 SUPERCHARGES)

REPLACE

$$R^{5,1} \times R_\phi \times SU(2)_k$$

BY

$$\boxed{R^{5,1} \times \frac{SL(2,R)}{U(1)} \times \frac{SU(2)_k}{U(1)}}$$

k NS5 ON TOP OF EACH OTHER AND SINGULAR

BY

k NS5 ⁘ RESOLVED

CAPPING

2-d BLACK-HOLE IN (x_6, x_7)

WEAK COUPLING

$$R_\phi \times SU(2)_k \to R_\phi \times U(1) \times \frac{SU(2)_k}{U(1)} \to \frac{SL(2,R)_k}{U(1)} \times \frac{SU(2)_k}{U(1)}$$

LST
HOLOGRAPHY
LINEAR DILATON

OOGURI - VAFA
GIVEON - KUTASOV
AHARONY, BERKOOZ, KUTASOV, SEIBERG
SFETSOS

OBSERVABLES ARE PRIMARIES OF THE

$\mathcal{N}=1$ SUPERCONFORMAL GROUP WITH

SCALING DIMENSIONS $\left(\frac{1}{2}, \frac{1}{2}\right)$

$$R^{5,1} \times \frac{SU(2)_k}{U(1)} \times \frac{SL(2,R)_k}{U(1)}$$

FREE

KNOWN BUT BRANES FAILS ON

FAILS ON

$$\frac{SL(2,R)_k}{U(1)}$$

(*) $\mathcal{N}=2$ SUPERCONFORMAL

$c = 3 + \frac{6}{k}$ (NS,NS) SECTOR

$\mathcal{N}=2$ PRIMARIES $V_{j,m,\bar{m}}$ HAVE SCALING

$$(h, \bar{h}) = \frac{1}{k}\left(m^2 - j(j+1),\ \bar{m}^2 - j(j+1)\right)$$

(*) $(m, \bar{m}) = \frac{1}{2}(p + wk,\ p - wk)$

$p, w \in \mathbb{Z}$ BOF WITHOUT GSO

$\uparrow \quad \uparrow$

MOMENTUM WINDING

CIGAR

(*) $p \in \mathbb{Z}$ WITH GSO

$w \in \frac{\mathbb{Z}}{k}$

UNITARITY AND NON-NORMALIZABILITY REQUIRE

(*) $j \in \mathbb{R}$ $-\frac{1}{2} < j < \frac{k-1}{2}$

($j = -\frac{1}{2} + is,\ s \in \mathbb{R}$ DELTA FUNCTION NORMALIZABLE
IN OFF SHELL OBSERVABLES
CONTINUUM ABOVE GAP IN LST...)

(*) OTHER STATES OBTAINED BY $N=2$ ACTION
OBSERVABLES

(*) $\langle V V \rangle$ HAS POLES FOR

$$|m| = j + n,\ |\bar{m}| = j + \bar{n};\ \ n, \bar{n} = 1, 2, 3$$

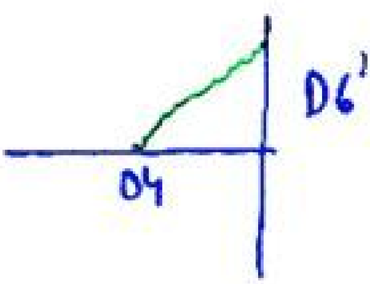
PARTICLES (HAGEDORN IN LST...)   PARTICLES IN BULK

FACTS ON D-BRANES (TO BE PUT ON WZW $\frac{SU(2)}{U(1)} \times \frac{SL(2,R)}{U(1)} \times R^{5,1}$)

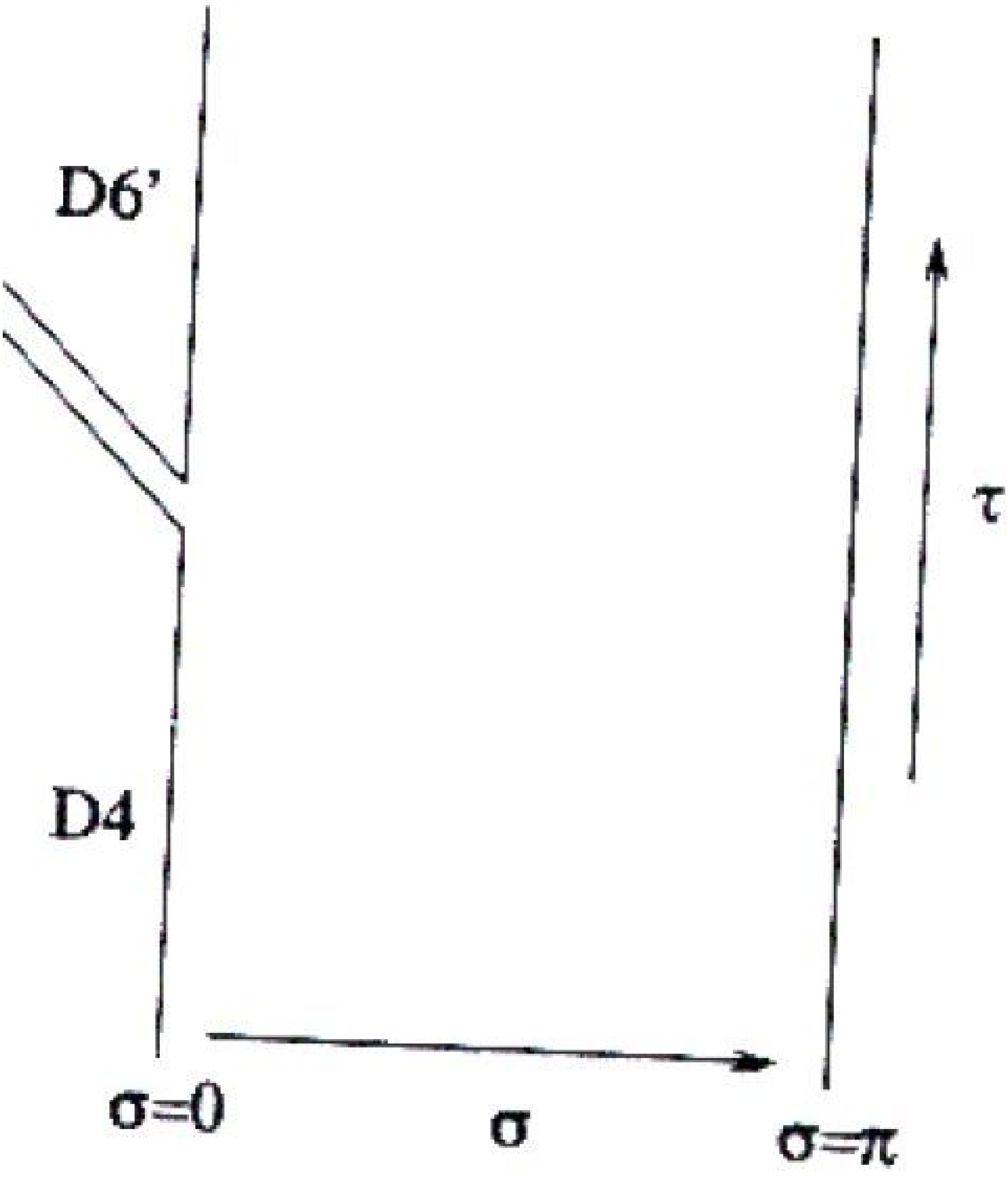
D-BRANES IN FLAT SPACE AND S^3 (WZW)

D-BRANES IN FLAT SPACE - A WORLD SHEET PICTURE

CALCULATE VERTEX OPERATORS FOR STRINGS BETWEEN D4 AND D6 BRANES



CONSIDER AN OPEN STRING ENDING ON D4, ONE SIDE OF WHICH HOPS TO A D6'


D6'
D4
σ=0
σ
σ=π
τ

FACTS ON D-BRANES (TO BE PUT ON WZW $\frac{SU(2)}{U(1)} \times \frac{SL(2,R)}{U(1)} \times R^{5,1}$)

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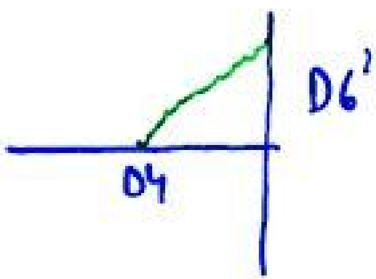


CONSIDER AN OPEN STRING ENDING ON D4, ONE SIDE OF WHICH HOPS TO A D6'

	0	1	2	3	4	5	6	7	8	9
D4	N	N	N	N	D	D	N	D	D	D
D6'	N	N	N	N	N	N	D	N	D	D
						ND				

CONSIDER A ND COORDINATE FOR EXAMPLE X_4

FOR $\quad 0 < z' < z \qquad \partial X^4(z') + \bar{\partial} X^4(z') = 0$ ● D

FOR $\quad z < z' \qquad \partial X^4(z') - \bar{\partial} X^4(z') = 0$ ● N

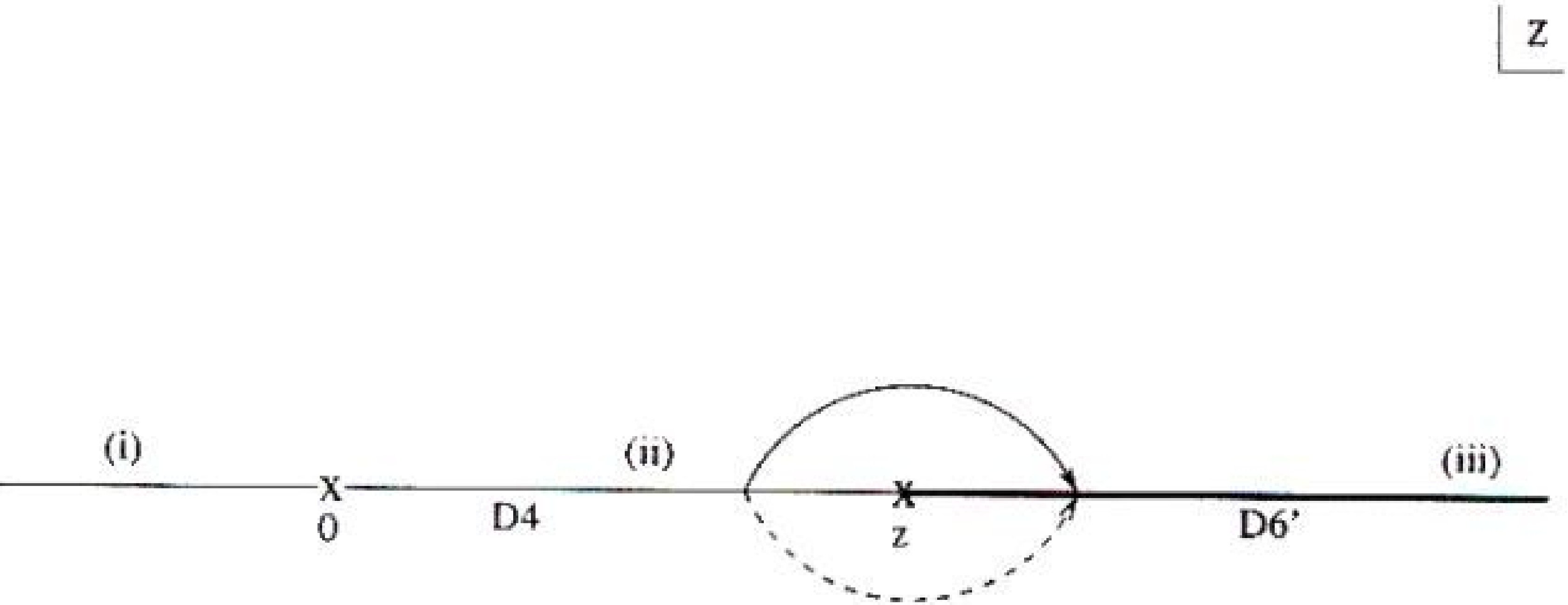
$V(z)$ THE STRING EMITTING OPERATOR INSERTED AT z SHOULD DO THE JOB OF:

POLCHINSKI, CHAUDHURI,
JOHNSON, HASHIMOTO

VERTEX OPERATOR FOR 4-6′ STRING

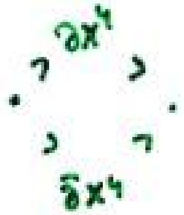
IN FLAT SPACE

$$V = V_{NN} \, V_{ND} \, V_{DD} \, e^{-\phi}$$


z
(i)
(ii)
(iii)
X
0
D4
X
z
D6'

THE RELATIVE SIGN OF ∂X^4 AND $\bar{\partial} X^4$ SHOULD CHANGE.

THAT MEANS:



THE OPE:

$$V(z)\,\partial X^4(z')$$

SHOULD CONTAIN A BRANCH CUT IN $z-z'$.

NAME: TWIST OPERATOR
GAME: ORBIFOLDS
WEIGHT: $1/16$ (THE LEAST)

SAME ARGUMENT FOR ALSO X^5, X^6, X^7

SO THE TWIST OPERATOR, σ_{4567},

WILL HAVE WEIGHT $4 \cdot \frac{1}{16} = \frac{1}{4}$

FOR SPACE-TIME BOSONS V BELONGS TO NS SECTOR ON THE WORLD-SHEET. IT IS LOCAL WITH RESPECT TO $G = i \sum \psi_a \partial X^a$.

V HAS A SQRT CUT WITH ∂X^4 SO IT <u>NEEDS</u> ONE WITH $\psi_{4,5,6,7}$ AS WELL.

THUS V WILL ALSO CONTAIN A SPIN-FIELD $S_{4,5,6,7}$ FOR THE WORLD SHEET FERMIONS.

IT WILL ALSO HAVE DIMENSION $\frac{1}{4}$. V_{ND}

(X^0, X^1, X^2, X^3) ARE NN AND THUS STANDARD. V_{NN}

(X^8, X^9) ARE DD.

IF D4 IS AT $(0,0)$ AND D6' AT $(a,0)$

SIMILAR METHODS GIVE RESULTS.

4-6' STRINGS TRANSFORM AS $\bar{N}$ OF $U(N)$.

SO 2 COMPLEX SCALARS Q IN N

$\tilde{Q}$ IN $\bar{N}$

THE D4-D6 SYSTEM KEEPS 8 (OF 32) SUPER-CHARGES ($N=2$ SUSY IN $D=4$ AND IS THUS AS EXPECTED A HYPER-MULTIPLET)

FROM FLAT SPACE TO A WZW BACKGROUND.

PRADISI, SAGNOTTI, STANEV
KLINCIK, SEVERA
KATO, OKADA
BIANCHI, STANEV
ALEKSEEV, SCHOMERUS
STANCIU
KAWEDZKI
BIRKS, FUCHS, SCHWEIGERT
GARCIA-COMPEAN, PLEBANSKI
BEHREND, PEARCE, PETKOVA, ZUBER
ALEKSEEV, RECKNAGEL, SCHOMERUS x2
FELDER, FROHLICH, FUCHS, SCHWEIGERT
FIGUEROA-O'FARRILL, STANCIU
BACHAS, DOUGLAS, SCHWEIGERT

FACTS ON D BRANES ON SU(2) MANIFOLD
GROUP

WHERE TO PLACE D ON G ?

(*) NO BOUNDARY – k (WZ COEFF.) QUANTIZED.

SYMMETRY – GIVEN $g(z,\bar{z})$

$$g \to h_L(z)\, g(z,\bar{z})\, h_R(\bar{z}) \quad \text{SYMMETRY}$$

$$G_L \times G_R$$

(*) IF THE WS HAS A BOUNDARY

$G_L \times G_R$ IS BROKEN

ONE CAN ATTEMPT TO PRESERVE

$$g \to h\, g\, h'^{-1}$$

WITH THE RELATION BETWEEN h AND h'

FIXED.

EXAMPLE! $h' = h$

THIS SYMMETRY LEADS TO CONSTRAINTS.

IF SYMMETRY PRESERVED IS G

AND IF $g(\text{BOUNDARY}) = f \in G$

THEN ALSO hfh^{-1} SHOULD BE ALLOWED FOR ANY $h \in G$

ALL THE CONJUGACY CLASS OF f IS ALLOWED.

FOR $G = SU(2)$

C_{class} DENOTED BY $0 \leq \theta \leq \pi$

$$f = \exp(i\theta\sigma_3)$$

ALLOWED BOUNDARY STATES g

$$g \in C_\theta \qquad \left(J^a = \bar{J}^a \quad a = 1,2,3 \right.$$
ON THE BOUNDARY)

(*) NOT ALL θ ALLOWED!

IN FACT $\theta = 2\pi \frac{j}{k}$ $\quad 0 \leq j \leq \frac{k}{2}$ $\quad j \in \mathbb{Z}, \mathbb{Z}+\frac{1}{2}$

THUS $k+1$ VALUES ALLOWED (LIKE # OF PRIMARIES)

(e) REASON

HINT - Σ W.S. HAS BOUNDARIES THUS CAN'T BE ITSELF A BOUNDARY. TO DEFINE THE WZ TERM

$$\int_{d=3} d^3x \, \frac{k}{4\pi} \, \omega^{(3)}$$

$$\omega^{(3)} = \frac{1}{3} \mathrm{Tr}\left((g^{-1} dg)^3\right)$$

ONE NEEDS TO FILL DISC ..... RESTRICTIONS...

(f) CAN BE HANDELED FOR

$$g(\text{boundary}) \in C_0 f$$

$$C_0 f = h \, (C_0 f) \, f^{-1} h^{-1} f$$

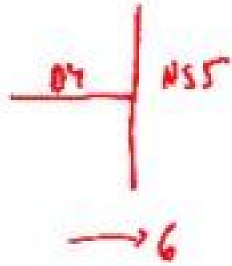
ON TO THE PROBLEM AT HAND!

PLACE THE BRANES ON S^3

$$g(\vec{x}) = \frac{1}{|\vec{x}|}\left[-x^6\, \mathbb{1} + i\left(x^8 \sigma_1 + x^9 \sigma_2 + x^7 \sigma_3\right)\right]$$

D4 ENDS ON & NS5 S

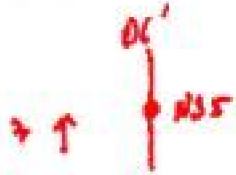
$x_6 \to 0_-$ $\qquad x^7 = x^8 = x^9 = 0$



$g = 1$ $\qquad ;\ f = 1$ $\qquad \theta = 0$

ALLOWED

& NS5 S PARTITION THE D6'

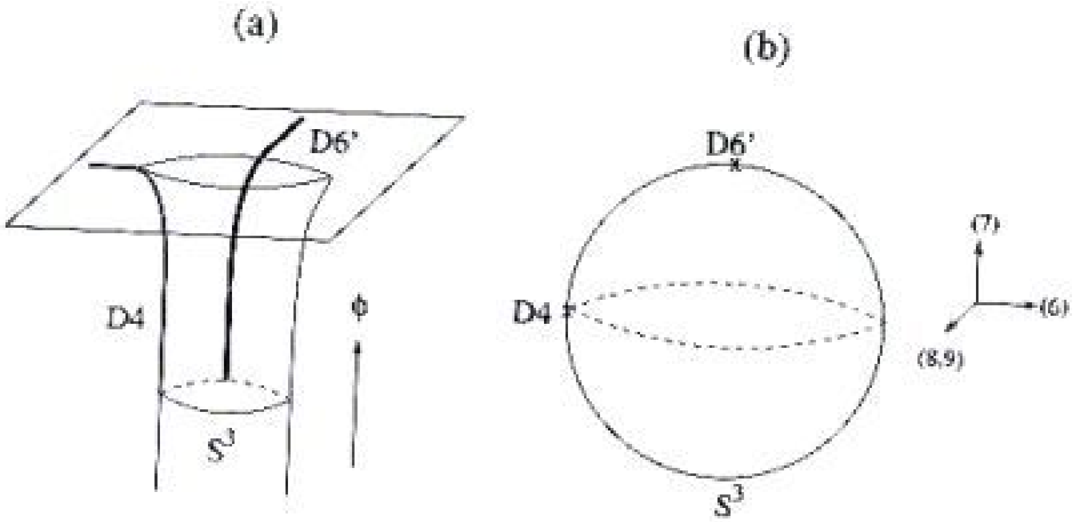


$x^6 = x^8 = x^9 = 0$ $\qquad x_7 = 0_+$

$g = i\sigma_3$

$f = \exp(i\pi\sigma_3/2)$ $\qquad \theta = \frac{\pi}{2}$

ALLOWED


(a)
D6'
D4
ϕ
S^3
(b)
D6'
D4
S^3
(7)
(6)
(8,9)

BUILD $V_{4-6'}$ IN NS5 BACKGROUND

$$R^{5,1} \times R_\phi \times R_Y \times \frac{SU(2)_k}{U(1)}$$

	0	1	2	3	4	5	6	7	8	9
D4	N	N	N	N	D	D		N		
D6'	N	N	N	N	N	N		N		

$\underbrace{N\ N\ N\ N}_{NN}\ \underbrace{N\ N}_{ND}$

$$V = \exp(-\varphi)\,\underbrace{\sigma_{45}\; S_{45}\; \exp(ik_\mu X^\mu)}_{}\exp(\beta\phi)\,V_{SU(2)}$$

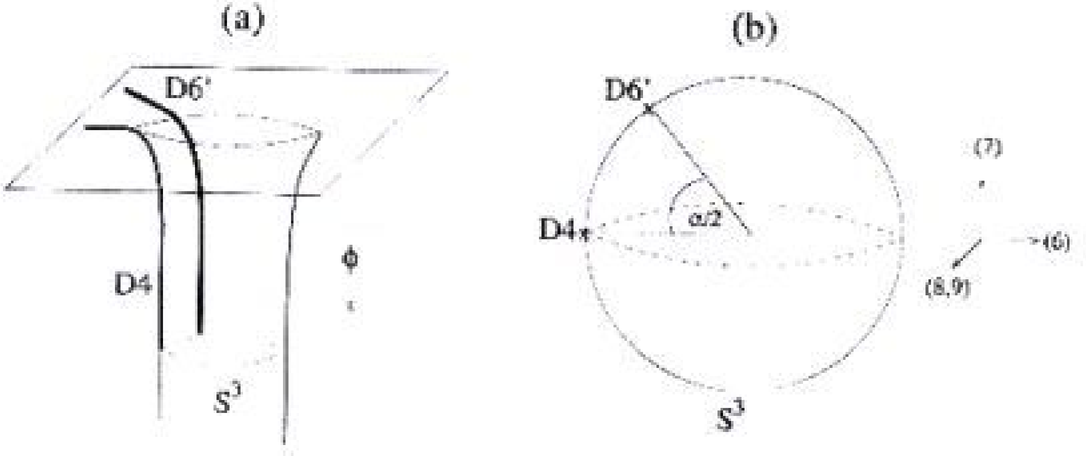
$\longleftarrow$ LIKE FLAT SPACE $\longrightarrow$

CONSTRUCT $\mathbf{V}_2$ - (FOR GENERAL α)

(A) $g = 1$ $\qquad J^a_4 = \bar{J}^a_4\, I_B \quad a = 1,2,3 \qquad \chi^a = \bar{\chi}^a\, I_B$

(B) $g = f_\alpha$ $\qquad J^3 = \bar{J}^3 \qquad J^\pm = \exp(\mp i\alpha)\bar{J}^\pm$

$\qquad \chi^3 = \bar{\chi}^3 \qquad \chi^\pm = \exp(\mp i\alpha)\bar{\chi}^\pm$


(a)
D6'
D4
ϕ
S^3
(b)
D6'
D4
α/2
(7)
(6)
(8,9)
S^3

THIS PRESERVES $J^3 + \bar{J}^3$ ($\partial\varphi$ ROTATIONS)

$V_{SU(2)}$ WILL CONTAIN A TWIST FIELD σ_α

SUCH THAT

$$\sigma_\alpha(z)\, J^3(z') \sim (z-z')^{n_1}\, O_1(z') \qquad \text{REGULAR}$$

$$\sigma_\alpha(z)\, \chi^3(z') \sim (z-z')^{n_2}\, O_2(z') \qquad \text{"}$$

$$\sigma_\alpha(z)\, J^\pm(z') \sim (z-z')^{\mp\frac{\alpha}{2\pi}+n_1}\, O_3(z') \qquad \text{CUT}$$

$$\sigma_\alpha(z)\, \chi^\pm(z') \sim (z-z')^{\mp\frac{\alpha}{2\pi}+n_2}\, O_4(z') \qquad \text{"}$$

LOWEST

σ_α CAN BE FOUND (TWISTED AFFINE SO(2)
SPECTRAL FLOW

$$h(\sigma_\alpha)_{n=0} = \frac{k_B\, \alpha^2}{4\, 4\pi^2} \qquad k_B = k-2$$

$$m(\sigma_\alpha) = -\frac{k_B}{2}\,\frac{\alpha}{2\pi}$$

THUS $V_{4-6'}$ FOR A STRING STRETCHED BETWEEN D4 at $(x_8, x_9) = (0,0)$ AND D6' AT $(x_8, x_9) = (a,b)$ IS:

$$V_{4-6'} = \underbrace{e^{-\phi}}_{(-1)\text{ PICTURE}} \underbrace{\sigma_{4567}}_{\substack{ND \\ B}} \underbrace{S_{4567}}_{\substack{ND \\ F}} \underbrace{\exp\left(\frac{i}{\pi}\left(a(x_L^8 - x_R^8) + b(x_L^9 - x_R^9)\right)\right)}_{DD} \underbrace{\exp(ik \cdot X)}_{NN}$$

N=1 SUPCONF. PRIMARY, BRST $h = \frac{1}{2}$

$$1 \underset{\text{BRST}}{=} \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{a^2+b^2}{2\pi^2} + \frac{k_\mu^2}{2}$$

$$\Longrightarrow \quad -k_\mu^2 = \frac{a^2+b^2}{\pi^2}$$

$$M_{4-6}^2 = \frac{a^2+b^2}{\pi^2}$$

FOR $(a,b) = (0,0)$

$$\boxed{M^2 = 0 \;\; !}$$

A SCALAR OF 3+1 LORENTZ. 2 REAL SCALARS GSO 4→2 OF S_{4567}; FOR N D4's N OF U(N).

$$V^{+}_{4-6'} = \exp(-\varphi)\,\sigma_{45}\,S_{45}\,\exp(ik_\mu x^\mu)\,V_{jm}$$

$$\frac{k_\mu^2}{2} + \frac{m^2 - j(j+1)}{k} = \frac{1}{4}$$

FINALLY LOWEST LYING STATE $e^{-\varphi}\sigma_{45}S_{45}V_{\frac{k}{2}-1,-\frac{k}{4}}e^{ik_\mu x^\mu}$

$$\boxed{M^2(\alpha) = \frac{1}{2}\left(\frac{\alpha}{\pi} - 1\right)}$$ no k DEP.

(*) FOR $\alpha = \pi$ $\boxed{M^2(\alpha) = 0 \ !!!}$ CHIRAL FERMION

S_{45} $2 \xrightarrow[GSO]{} 1$ $(4\text{-}6')_+$, ALSO $(6'\text{-}4)$

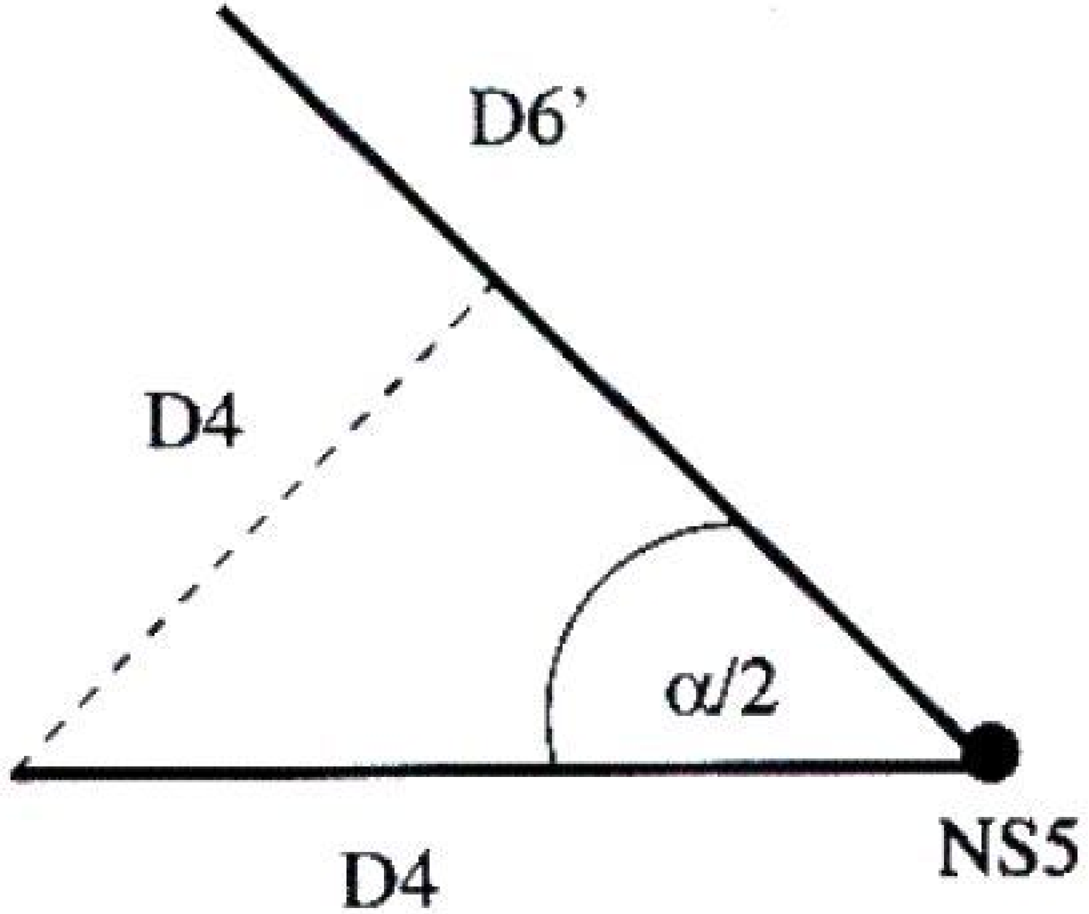
Q Q^+

$4\text{-}6'_-$ $6'_-\text{-}4$

$\tilde{Q}^+$ $\tilde{Q}$

* VALID FOR $\alpha > \alpha_c = \frac{2\pi}{k}$ $(k \geq 2)$

(**) TACHYONS FOR $\alpha < \pi$


D6'
D4
α/2
D4
NS5

(*) $4 - 6'_+$ Q (μ_c, μ_R)

$6'_+ - 4$ Q^+

$6'_- - 4$ $\hat{Q}$ $(\bar{\mu}_c, \bar{\mu}_R)$

$4 - 6'_-$ $\hat{Q}^+$

(*) m QUANTIZED ?

A. NO WILSON LINES

B.

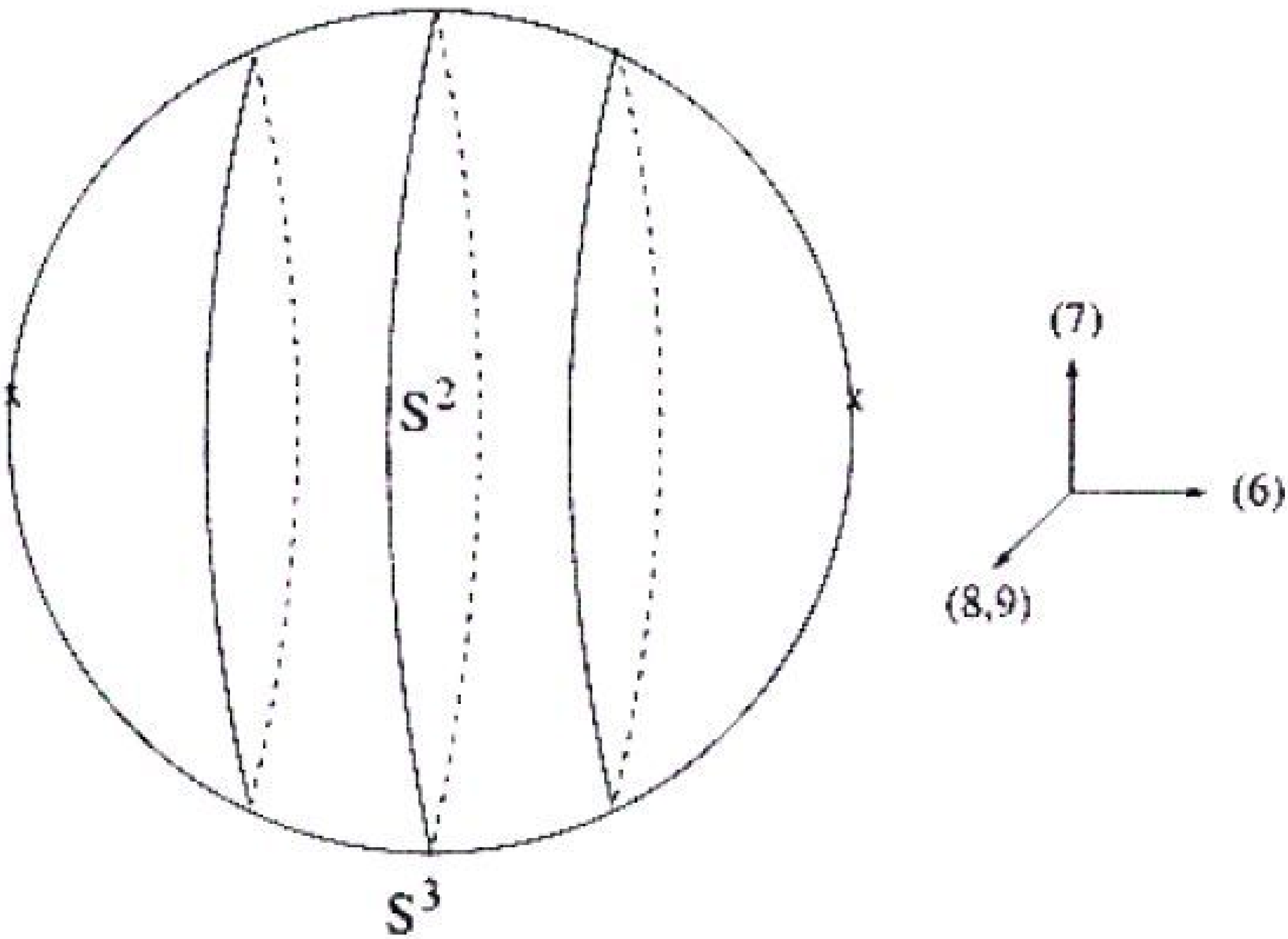
$\theta = n \frac{2\pi}{k}$

(*) AGREES WITH FI $\sim (Q^+ Q - \hat{Q}^+ \hat{Q} - r)^2$

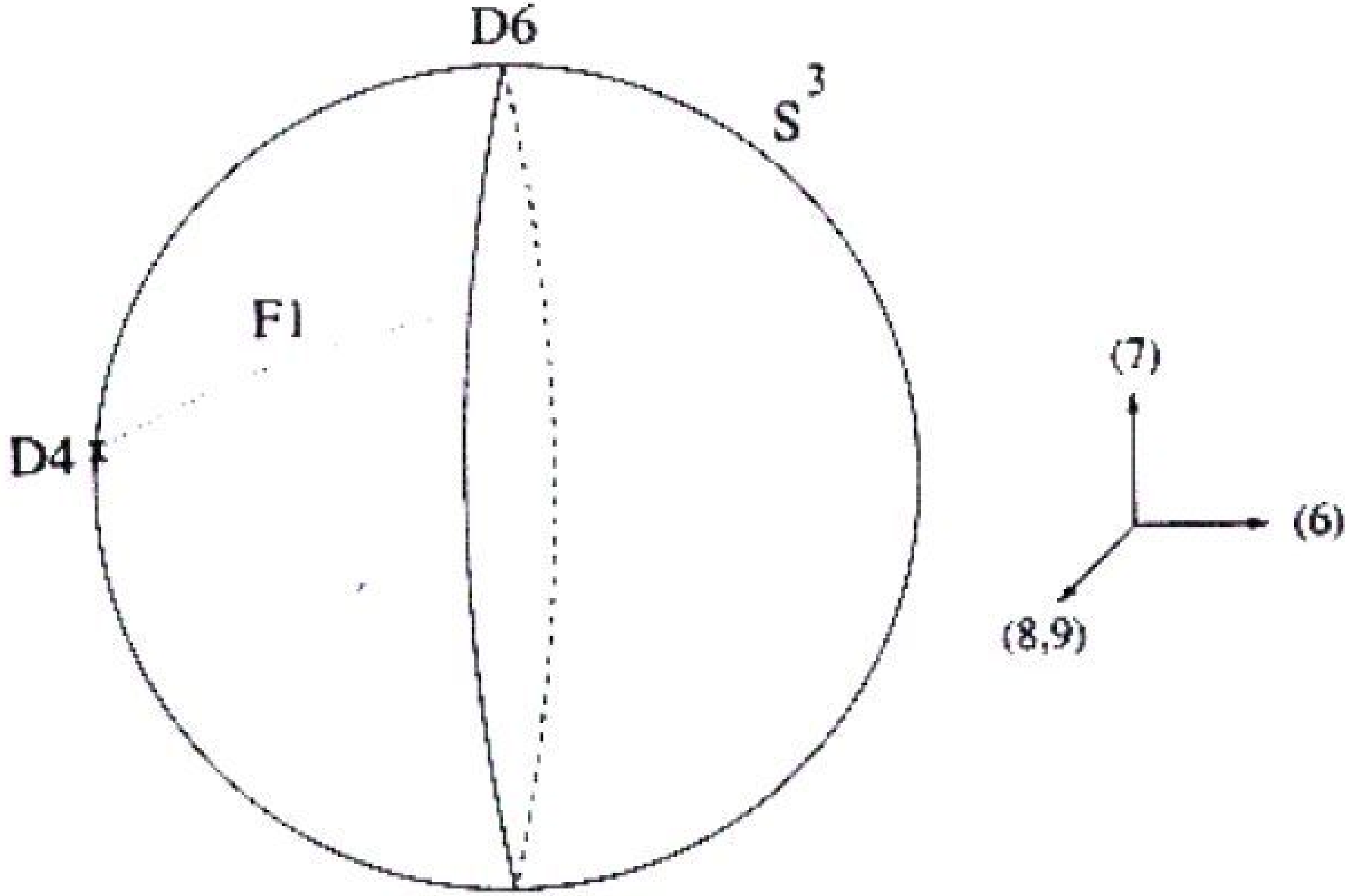
$r \propto \Delta x^7$

(*) MASSIVE STATES FORM HYPERMULTIPLETS

(*) $j = -\frac{1}{2}$ [illegible] mass gap $\sim \frac{1}{\ell_s}$



S²
S³
(7)
(6)
(8,9)


D6
S^3
F1
D4
(7)
(6)
(8,9)

THE D6 WRAPS $S^2_n \in S^3$

IT IS TO THE LEFT OF n OUT OF k

NS5-BRANES. MOVING IT TO THE RIGHT

PAST n NS5 CREATES n D4-s

WHICH IN NHL SEEM LIKE n

POINTS ON S^2.

(n D0 $\longleftrightarrow$ D2 WRAPPED ON S^2 WITH RADIUS $\sim n$)

THIS IS HW TRANSITION.