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Superstrings in $AdS_5 \times S^5$

- Review: GS σ-model on supercoset
- Semiclassical expansion near "long" string
- Light cone gauge

Metsaev, A.T. hep-th/9805028
Kallosh, A.T. hep-th/9808088
Drukker, Gross, A.T. hep-th/0001204
Metsaev, A.T. hep-th/0007036

Strings in AdS + R-R backgrounds

→ understanding (non) supersymmetric large N gauge theories

Type IIB "vacua":
- Flat 10-d space
- $AdS_5 \times S^5$ + F_5-backgr.

Solvable string theory?

Need GS description (X, θ)

(one loop in GS → non-perturbative in NSR description)

$$\bar{\theta}\gamma^{\cdots}\theta\, \partial X\, \partial X\, \mathcal{F}_{\cdots}$$

"quantum" interaction:

classical string does not feel R-R backgr.

Why GS Action?

(X^m, θ_I^α)

collective coordinates of
string-like soliton in susy field theory
fundamental string in supergravity

Follows from field theory
manifest symmetries.

Nambu action $\longmapsto$ Green-Schwarz action

Natural action for quantizing
"long" fundamental superstring

$$AdS_5 = \frac{SO(2,4)}{SO(1,4)}$$

$$S^5 = \frac{SO(6)}{SO(5)}$$

Bosonic string on G/H :

$$\mathcal{L} = Tr\left[\left(g^{-1}\partial g\right)_{G/H}\right]^2$$

Non-conformal theory $\qquad R_{MN} \neq 0$

Superstring : 2-d CFT

AdS_5 Bosons $\longleftrightarrow$ Fermions θ^α $\longleftrightarrow$ S^5 Bosons

$$R_{MN} - (F_5 F_5)_{MN} = 0$$

$\mapsto$ Action ?

Quantization ?

Spectrum ?

GS Superstring Action

Flat superspace $= \dfrac{\text{super Poincaré}}{SO(1,9)} = \dfrac{G}{H}$

$(P_m, J_{mn}, Q^I_\alpha)$ $\quad I = 1,2$

$$\{Q, Q\} \sim P$$
$$[P, P] = 0$$
$$[J, J] \sim J$$

$$(g)_{G/H} = \exp\left(x \cdot P + \theta \cdot Q \right)$$

$$g^{-1} dg = L^m P_m + L^I Q_I + L^{mn} J_{mn}$$

$$L^m = dx^m - i\, \bar{\theta}^I \Gamma^m d\theta^I$$

$$L^I_\alpha = d\theta^I_\alpha$$

$$S = \int_{\partial M_3} L^m L^m + \int_{M_3} H$$

Henneaux Mezincescu

$$H = L^m \wedge \left(\bar{L}^I \Gamma_m L^J \right) s_{IJ}$$

$s_{IJ} = \mathrm{diag}(1, -1)$ $\qquad dH = 0$

No $\bar{L}^I L^I \sim \partial\bar{\theta}\, \partial\theta$

- global 10-d susy
- κ- symmetry

$$S = \int d^2\sigma \Big[\sqrt{g} \left(\partial_\mu X^m - i \bar\theta \Gamma^m \partial_\mu \theta \right)^2$$
$$- 2i \, \epsilon^{\mu\nu} S_{IJ} \, \bar\theta^I \Gamma_m \partial_\mu \theta^J \left(\partial_\nu X^m - \tfrac{i}{2} \bar\theta \Gamma^m \partial_\nu \theta \right) \Big]$$

Gauge fixing:

- Covariant: e.g. $\theta^1 = \theta^2$ + $\sqrt{g} g^{\mu\nu} = \eta^{\mu\nu}$

$$\mathcal{L} \sim (\partial X)^2 + \partial X \, \bar\theta \Gamma \partial \theta + \theta^4$$

well-defined expansion near "long" strings
$(X^0 = \tau, \; X^1 = \sigma)$
but degenerate for "short" strings

- Light-cone:

(i) fermionic : $\Gamma^+ \theta^I = 0$

$$\mathcal{L} \sim (\partial X)^2 + \partial X^+ \bar\theta \Gamma^- \partial \theta$$

(ii) bosonic : $\sqrt{g} g^{\mu\nu} = \eta^{\mu\nu}, \quad X^+ = \tau$

$$\mathcal{L} \sim (\partial X)^2 + \bar\theta \partial \theta \longrightarrow \text{spectrum}$$

Superstring in $AdS_5 \times S^5 = \frac{SO(2,4)}{SO(1,4)} \times \frac{SO(6)}{SO(5)}$

SuperPoincare $\longrightarrow$ $SU(2,2|4)$

$SO(1,9)$ $\longrightarrow$ $SO(1,4) \times SO(5)$

Supercoset Space $= \dfrac{SU(2,2|4)}{SO(1,4) \times SO(5)}$

$SU(2,2|4)$: $\left(P_A , J_{AB}\right) + \left(P_{A'} , J_{A'B'}\right) + \left(Q^{\alpha\alpha' I}\right)$

5+5 split SO(2,4) SO(6) Super

$(A = 0, \dots, 4)$ $(A' = 1, \dots, 5)$

$$[P,P] \sim J \ , \quad [Q,P] \sim Q \ , \quad [Q,J] \sim Q$$

$$\{Q^I, Q^J\} \sim \delta^{IJ}\left(\Gamma^A P_A + \Gamma^{A'} P_{A'}\right) + \epsilon^{IJ}\left(\Gamma^{AB} J_{AB} - \Gamma^{A'B'} J_{A'B'}\right)$$

$$g^{-1} dg = L^m P_m + L^{mn} J_{mn} + L^I Q_I$$

$(m = A, A')$ $(mn = (AB, A'B'))$

GS Action:

Metsaev, A.T.

bosonic part = σ-model on $AdS_5 \times S^5$

global $SU(2,2|4)$; κ-invariance ; flat limit

exists and unique

$$\boxed{S = \int_{\partial M_3} L^m L^m + \int_{M_3} H}$$

Same as in flat case

$$H = L^m \wedge (\bar{L}^I \Gamma^m L^J)\, s_{IJ}\,, \qquad dH = 0$$

$$(g)_{coset} = g(\underset{10}{x},\underset{32}{\theta}) = g(x)\, e^{\theta\cdot Q} \rightarrow$$

$$\left[\begin{array}{l} L^m = e^m - i\,\bar{\theta}^I \Gamma^m \underline{D}\theta^I + \cancel{O(\theta^4)} \\ L^I = \underline{D}\theta^I + O(\theta^3) \end{array}\right.$$

$\underline{D}$ = generalized covariant derivative

$D^2 = 0 \leftrightarrow \hat{R} \equiv d\omega + \omega\wedge\omega \pm e\wedge e = 0$

$DE = 0 \rightarrow$ Killing spinors, global susy

$$D_\mu = \partial_\mu + \tfrac{1}{4}\omega_\mu^{mn}\Gamma^{mn} - \Gamma^{m_1\ldots m_5}\Gamma_\mu F_{m_1\ldots m_5}$$

$$\boxed{\begin{array}{l} \mathcal{L} = (e^m_\kappa\, \partial x^\kappa - \bar{\theta}\Gamma^m \underline{D}\theta)^2 + \ldots \\ \qquad -2i\,\varepsilon^{\mu\nu} e^m_\kappa\, \partial_\mu x^\kappa\, \bar{\theta}\Gamma^m \underline{D}_\nu\theta + \ldots \end{array}}$$

direct covariantization of flat action

+ RR coupling $\quad \bar{\theta}\,\Gamma^{\ldots}\Gamma_m\theta\, \partial x^m \partial x^n F_{\ldots}$

All orders in θ form . Kallosh, Rahmfeld, Rajaraman

Exact conformal invariance :

- nonrenormalization of WZ term
- global symmetry
- κ-symmetry (relating $L^m L^m$ and WZ coeffs)

κ - symmetry gauge fixing ("covariant")

e.g. $\theta^1 = \theta^2$ or $\theta^1 = \Gamma_{0123}\theta^2$

Kallosh
Rahmfeld
Pesando

$$ds^2 = y^2 \underbrace{dx^p dx^p}_{4} + \frac{1}{y^2}\underbrace{dy^s dy^s}_{6}$$

$$\mathcal{L} = \sqrt{g}\Big[y^2(\partial x^p - \bar\theta\Gamma^p\partial\theta)^2 + \frac{1}{y^2}\partial y^s \partial y^s$$
$$+ 4i\,\varepsilon^{\mu\nu}\partial_\mu y^s\, \bar\theta\Gamma^s\partial_\nu\theta\Big]$$

θ^4 only

Quadratic in θ after T-duality

$$\partial_\mu x^p \to \varepsilon_{\mu\nu}\partial_\nu \tilde{x}^p$$

Kallosh
A.T.

$$y^2\partial x\ \bar\theta\ \partial\theta + \partial y\ \bar\theta\ \partial\theta$$

degenerate for "small" strings

As in flat case

But well-defined expansion near "long" string configurations

Semiclassical (one-loop) Approximation

$\theta^1 = \theta^2$ gauge

$$S_F = \int d^2\sigma \left(\sqrt{g}\, g^{\mu\nu}\, \bar{\theta} \rho_\mu \mathcal{D}_\nu \theta - \underbrace{\tfrac{i}{2} \epsilon^{\mu\nu} \bar{\theta} \rho_\mu \tilde{\rho}_\nu \theta}_{\text{"mass term"}} \right)$$

"mass term"
$m = \frac{1}{R_{AdS}} = 1$

$\mathcal{D} = \partial + \frac{1}{4} \omega \cdot \Gamma$

$$\rho_\mu \equiv \left(\Gamma_A e^A_m + \Gamma_{A'} e^{A'}_m \right) \partial_\mu x^m$$ projected Γ's

$$\tilde{\rho}_\mu \equiv \left(\underbrace{\Gamma_A e^A_m}_{AdS_5} + i \underbrace{\Gamma_{A'} e^{A'}_m}_{S_5} \right) \partial_\mu x^m$$

Direct check of conformal invariance

Transformation to "2-d Dirac spinor" form

Action for 2-d scalars θ^α in curved $G_{mn}(\bar{x})$ $\longrightarrow$ Action for 2-d Dirac Spinors in 2-d induced geometry

$$\bar{g}_{\mu\nu} = G_{mn}(\bar{x})\, \partial_\mu \bar{x}^m \partial_\nu \bar{x}^n$$

Local $SO(1,9)$ Rotation:

$$\theta \to \Lambda(\sigma)\, \theta$$

$$\rho^{\text{tang}}_\mu(\sigma) \to \Lambda^{-1}(\sigma)\, \Gamma_\mu\, \Lambda(\sigma)$$

$\Lambda = \Lambda(x(\sigma))$

Polyakov
Wiegmann
....

2-d representation $\quad \Gamma_\mu \to \tau_\mu \times I_8$ $\quad (\mu = 0,1)$

$$(G_{mn}, \bar{x}^n) \to (\bar{g}_{\mu\nu}, \bar{A}_\mu)$$

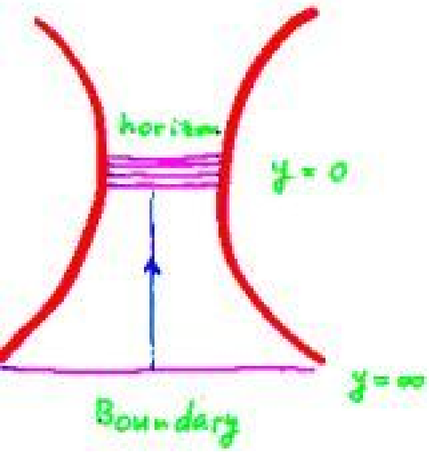
Simple example: $\quad$ World sheet $\subset AdS_3 \subset AdS_5$

3-d space, one normal direction, $\bar{A}_\mu = 0$

θ- determinant $\longrightarrow$ 2-d Dirac determinant in induced 2-d metric

Straight string configuration

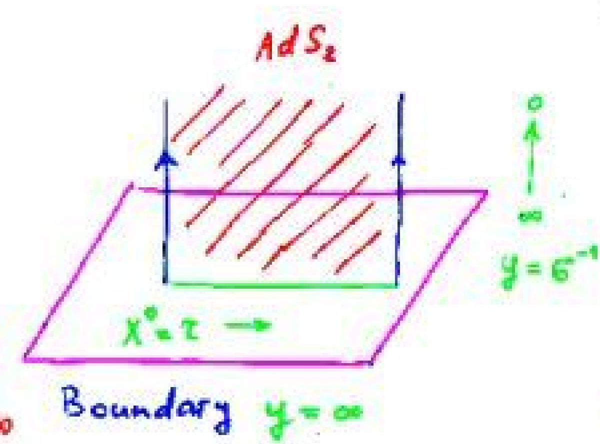
Drukker
Gross
A.T.



$$ds^2 = y^2\, dx_a dx_a + \frac{dy^2}{y^2}$$

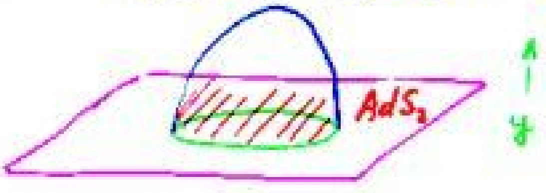
$$x^0 = \tau \,, \quad x^4 \equiv y = \sigma$$

$$ds^2_{induced} = \sigma^2 d\tau^2 + \frac{d\sigma^2}{\sigma^2} = AdS_2$$



Relevant example:

minimal surfaces $\xrightarrow{y\to\infty} AdS_2$



BPS configuration : $Z \overset{?}{=} 1$

One-loop partition function

$$S_F = \int d^2\sigma \sqrt{g} \left(i \bar{\theta} \tau^\mu \hat{\nabla}_\mu \theta - \bar{\theta} \tau_3 \theta \right)$$

2-d Dirac operator in AdS_2

$$Z = \frac{\left[\det \left(-\hat{\nabla}^2 + \frac{1}{4} R^{(2)} + 1 \right) \right]^{8/2}}{\left[\det \left(-\nabla^2 + 2 \right) \right]^{3/2} \left[\det \left(-\nabla^2 \right) \right]^{5/2}}$$

Fermions

Bosons

$\uparrow$ AdS_5 $\quad\quad$ $\uparrow$ S^5

Susy field theory in AdS_2

$R^{(2)} = -2$
$(R_{AdS} = 1)$

$\mathcal{N}=1$ scalar multiplet: $\quad m_B^2 = \mu^2 - \mu$
$m_F = \mu$

$\left[\begin{array}{l} 3 \text{ "massive" multiplets} \\ m_B^2 = 2,\ |m_F| = 1,\ \mu = -1 \end{array} \right] \oplus \left[\begin{array}{l} 5 \text{ "massless" multipl.} \\ m_B^2 = 0,\ |m_F| = 1,\ \mu = 1 \end{array} \right]$

($\mathcal{N}=8$ in AdS_2 ?)

$Z = 1$? $\quad$ Subtle in AdS

Vacuum energy: $\quad$ susy b.c.'s, discrete spectrum in spatial direction

$E = 0$?

$$E = \frac{1}{2} \sum_{n=0}^{\infty} \left[3 \left(\omega_n^B - \omega_n^F \right)_{\mu=-1} + 5 \left(\omega_n^B - \omega_n^F \right)_{\mu=+1} \right]$$

$$\omega_n^B = n + |\mu| + \tfrac{1}{2}$$
$$\omega_n^F = n + |\mu - \tfrac{1}{2}| + \tfrac{1}{2}$$

ζ- function regularization:

$$E = -\frac{1}{4} \left[3 \times \left(2 + \tfrac{1}{6}\right) + 5 \times \tfrac{1}{6} - 8 \times \left(1 - \tfrac{1}{12}\right) \right] = 0$$

Jacobian of $g_{\alpha\beta} \to 1$ is 1 $\to$ $Z = 1$

Similar partition function $\neq 1$ for rectangular Wilson loop configuration

determines string 1-loop correction coefficient in strong-coupling expansion of $q\bar{q}$ potential

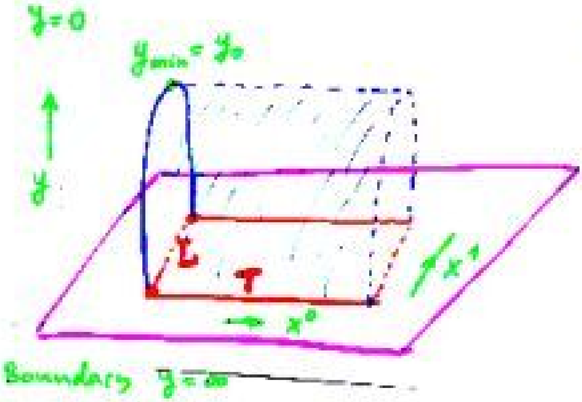
$$V = -\frac{d_0 \sqrt{\lambda}}{L} - \frac{d_1}{L} + O\left(\frac{1}{\sqrt{\lambda}}\right)$$

Conclusions :

GS action in $AdS_5 \times S^5$:

- well-defined perturbation theory in "long string" sector
- described by effective 2-d theory in induced 2-d geometry (cf. NSR)
- practical formalism : d_1 explicitly computable. sensitive to quantum string coupling to RR fields (computable in GS, not in NSR)

Wilson loop: one loop correction



$$ds^2_{AdS_5} = R^2\left(y^2 dX_m dX_m + \frac{dy^2}{y^2}\right)$$

$$x^0 = T\,\tau$$

$$x^1 = L\,\sigma$$

$$y = y(\sigma)$$

$$y(0) = y(1) = \infty$$

Maldacena
Rey, Lee

$$I_{Nambu} = \frac{R^2}{2\pi\alpha'} T \int d\sigma \sqrt{y'^2 + y^4}$$

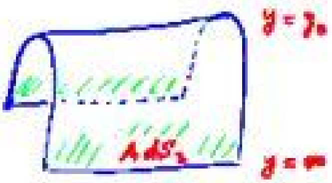
$$y'^2 = \frac{y^8}{y_0^4} - y^4 \qquad y_0 = \frac{k_0}{L}, \quad k_0 = \frac{(2\pi)^{3/2}}{[\Gamma(\frac{1}{4})]^2}$$

Induced geometry:

$$ds^2_2 = y^2(\sigma)\, d\tau^2 + y^6(\sigma)\, d\sigma^2 = y^2\left(d\tau^2 + \frac{dy^2}{y^4 - y_0^4}\right)$$

Non-compact 2-d surface asymptotic ($y \to \infty$) to AdS_2

$y_0 = 0$ limit ($L = \infty$) $\mapsto$ straight string case



Correct Normalization:

$$\bar{Z} = \frac{Z(M_2)}{Z(AdS_2)} = \frac{Z(y_0)}{Z(0)}$$

$$\bar{I} = I(y) - \underbrace{I(y_0=0)}_{\text{boundary term}} = \text{finite}$$

$$= -\frac{R^2 T}{2\pi\alpha'} \cdot y_0^2 \cdot L = -\, d_0 \cdot \sqrt{\lambda} \cdot \frac{T}{L} \qquad R^4 = \lambda\alpha'^2$$

$$d_0 = \frac{(2\pi)^2}{[\Gamma(\frac{1}{4})]^4}$$

$$V_{q\bar{q}} = -\underbrace{\frac{d_0\sqrt{\lambda}}{L}}_{\substack{\text{classical} \\ \text{Nambu string}}} - \underbrace{\frac{d_1}{L}}_{\substack{\text{1-loop string} \\ \text{correction}}} + O\left(\frac{1}{\sqrt{\lambda}}\right)$$

$\frac{R^2}{\alpha'} \sim \sqrt{\lambda}$ $\qquad$ $O(1)$ $\qquad$ $\leftarrow$ quantum superstring fluctuations

Bosonic fluctuations:

$\xi^0 = 0,\ \xi^1 = 0$ static gauge

$$\xi^s \quad s=2,3 \quad ; \quad \zeta = \delta y \quad ; \quad \eta^p \quad (p=1,\dots,5)$$

"parallel" $\qquad$ "radial" $\qquad$ S^5

$$\delta I = \int \xi^s \underbrace{(-\nabla^2 + 2)}_{\text{parallel}} \xi^s + \zeta \underbrace{(-\nabla^2 + R^{(2)} + 4)}_{\text{radial}} \zeta + \eta^p \underbrace{(-\nabla^2)}_{S^5} \eta^p$$

Gliozzi Forste Theisen

$$\nabla^2 = \frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\, g^{ij}\partial_j), \quad R^{(2)} = -2\left(1 + \frac{1}{y^4}\right)$$

induced world sheet geometry

Fermionic action:

$$\rho_0 = y\,\gamma_0, \quad \rho_1 = y\,\gamma_1 + \frac{y'}{y}\gamma_4, \quad \mathcal{D}_i = \partial_i + \frac{1}{2} y\,\gamma_i \gamma_4$$

Rotation: $\rho_1 \to y^3 \gamma_1$ $\qquad$ $e_i^a = (y, y^3)$

$\theta^I \to \Psi^I = S^{-1}\theta^I$

$S = \exp(-\tfrac{1}{2}\beta\gamma_1\gamma_4)$ $\qquad \cos\beta = \frac{1}{y^2}, \ \beta' = 2y$

$\rho_1 = y^3 \ S\gamma_1 S^{-1}$ $\qquad \beta = \beta(\sigma)$

$\theta^1 = \theta^2 \qquad \Psi^1 = \Psi^2 = \Psi$

$$\boxed{\mathcal{L}_{2F} = i\,\bar\Psi\tau^i\hat\nabla_i\Psi - \bar\Psi\tau_3\Psi}$$

$\tau_i = e_i^{\ k}\gamma_k = (y\gamma_0, y^3\gamma_1)$

$\tau_3 = \frac{1}{2}\frac{\epsilon^{ij}}{\sqrt{g}}\tau_i\tau_j = \gamma_0\gamma_1$

2d Dirac matrices

$\gamma_0 = i\sigma_2 \times I_8$

$\gamma_1 = \sigma_1 \times I_8$

$\gamma_0\gamma_1 = \sigma_3 \times I_8$

massive Dirac fermions on curved 2-d surface

$$\boxed{\Delta_F = -\hat\nabla^2 + \frac{R^{(2)}}{4} + 1}$$

same result in $\theta^1 = \gamma_{0123}\theta^2$ gauge: $\qquad$ Kallosh, A.T.

$\int d^2\sigma\left(\sqrt{g}g^{ij}y^2\bar\theta\gamma_i\partial_j\theta - \epsilon^{ij}\partial_i y\,\bar\theta\gamma_4\partial_j\theta\right)$ $\qquad \theta' = y^{-1/2}\theta$

$= \int d^2\sigma\ \bar\theta'\left[\gamma^1\partial_1 + (y^4\gamma^0 + y'\gamma^4)\partial_0\right]\theta'$

Related by rotation: $\mathcal{S} = \exp(-\tfrac{1}{2}\varphi\gamma^0\gamma^4)$

$\cosh\varphi = y^2$

1-loop partition function:

$$Z = c_1 \frac{\left[\det\left(-\hat{\nabla}^2 + \frac{1}{4}R^{(2)} + 1\right)\right]^{8/2}}{\left[\det\left(-\nabla^2+2\right)\right]^{3/2}\left[\det\left(-\nabla^2 + R^{(2)} + 4\right)\right]^{1/2}\left[\det\left(-\nabla^2\right)\right]^{5/2}}$$

$$\bar{Z} = \frac{Z(M_2)}{Z(AdS_2)} = \frac{Z(y_0)}{Z(0)}$$

(can ignore all boundary contributions)

$$= \exp\Big(-\underset{\substack{\parallel \\ \text{finite const}}}{d_1}\ \frac{T}{L}\Big)$$

Comments:

- Z is finite
  with proper definition of path integral measure
  ($\int R^{(2)} \log\Lambda$ cancel)
- $Z \neq 1$: susy broken by $y(\sigma)$
- $\bar{Z}$: standard definition of det Laplacians on non-compact spaces: normalize on fiducial metric of same topology and asymptotic behaviour

$$d_1 \sim \sum \log\det\left(-\nabla^2 + X\right)$$

in static 2-d geometry:

determined by spectra of $\Delta = -\frac{d^2}{d\sigma^2} + U[y(\sigma)]$ one-dim. Schrodinger operators

computable

10/4

Light Cone Gauge Approach

Why:

- quantize string without assuming expansion near non-trivial $\bar{x}^m(\sigma,\tau)$ $\rightarrow$ spectrum of "short" closed strings

Analogy with flat case:

$$\partial x\, \bar{\theta}\Gamma\partial\theta \xrightarrow[\Gamma^+\theta=0]{\mathrm{I}} \partial x^+ \bar{\theta}\Gamma^-\partial\theta \xrightarrow[x^+=\tau]{\mathrm{II}} \bar{\theta}\partial\theta$$

Also possible in $AdS_5 \times S^5$?

- establish relation to boundary theory

$\mathcal{N}=4$ SYM : no maximally susy covariant description, but simple in l.c. gauge superspace

$A^+=0$

Brink
Lindgren
Nilsson
Mandelstam

$$\Phi(x,\theta) = \underset{\underset{A_1+iA_2}{\shortparallel}}{A(x)} + \theta^i \underset{\underset{SU(4)}{\uparrow}}{\psi_i(x)} + \ldots$$

$$S[\Phi] = \int d^4x\, d^8\theta\, \mathcal{L}(\Phi)$$

$1/2$ linear l.c. susy
$\theta \rightarrow \theta + \epsilon$

non-linear $1/2$ Poincaré + conformal susy

Formulate bulk string theory to "match" l.c. form of boundary SYM

(same symmetries - linear and non-linear)

Aims:

- choose "superconformal" parametrization

$$\theta^I_\alpha \to (\theta^i, \eta^i)$$

Q S manifest $SU(4)$

("linear") ("non-linear")

- fix fermionic (κ-symmetry) l.c. gauge where

$y \equiv e^\phi$

$$\mathcal{L}_B = (\partial\phi)^2 + e^{2\phi}(\partial x^+ \partial x^- + \partial x \partial \bar{x}) + G_{MN}(y)\, \partial y^M \partial y^N$$

$$\mathcal{L}_F = e^{2\phi} \partial x^+ (\bar{\theta} D \theta + \bar{\eta} D \eta) + (e^{2\phi} \partial x^+)^2 \, \eta\eta\eta\eta$$

- fix bosonic (2-d diff's) gauge

$x^+ = \tau$ + cond. on $g_{\mu\nu}$

e.g. $e^{2\phi} \sqrt{g}\, g^{00} = -1$

- integrate out x^-, "2-d fermion" form, etc.

find more adequate variables

(phase space approach ?)

Possible ? Useful ?

Bosonic l.c. gauge

Flat space:

BDHP formulation: $\sqrt{g}\, g^{\mu\nu} = \eta^{\mu\nu} \rightarrow \partial^2 x^+ = 0$

$$x^+ = \tau$$

Phase space GGRT formulation:

$$x^+ = \tau \qquad P^+ = p^+$$

Curved space ($\neq R^{1,1} \times M^8$, null Killing)

GGRT approach straightforward to apply — Thorn

BDHP: ($x^+ = \tau$ - solution?)

- if insist on conformal gauge need covariantly constant null Killing — Horowitz Steif
- abandon conformal gauge:

  e.g. $x^+ = \tau, \quad \sqrt{g}\, g^{00} = -1$ — Rudd

  (consistent if $G_{+-} = 1$, $G_{-m} = 0$)

AdS space: $G_{+-} = e^{2\phi} \neq 1$

consistent choice: $e^{2\phi} \sqrt{g}\, g^{00} = -1$

$$x^+ = \tau$$

Equivalent to (after integrating out x^-) $\sqrt{g}\, g^{\mu\nu} = (-e^{-2\phi}, e^{2\phi})$ — Polyakov

No global null Killing ?! (degenerate at horizon $e^{2\phi} \to 0$)

- Boundary theory has well-defined l.c. description on $R^{1,3}$
- formal approach : $e^{2\phi} \to 0$ singularity will be reflected in the l.c. action

Fermionic light-cone Gauge

$$\frac{SU(2,2|4)}{SO(1,5) \times SU(5)} \qquad S = \int_{\partial M_3} L^m L^m + \int_{M_3} L^m \wedge (\bar{L} \Gamma^m L)$$

- "4-d conformal" basis of $SU(2,2|4)$

$$SO(2,4) \times SU(4)$$

$$(\underbrace{P^a, K^a, D, J^{ab}}_{15}) + (\underbrace{J^i_j}_{15}) + (\underbrace{Q^i, S^i}_{32})$$

- light-cone decomposition:

$$x^a \in R^{1,3}, \quad y^A \in S^5, \quad -\infty < \phi < \infty$$

$$\hookrightarrow (x^+, x^-, x, \bar{x}) \qquad x^{\pm} = \frac{1}{\sqrt{2}}(x^3 \pm x^0)$$

$$x, \bar{x} = \frac{1}{\sqrt{2}}(x^1 \pm i x^2)$$

- light-cone κ-symmetry gauge

$$\theta^+_i = 0 \ , \quad \eta^+_i = 0$$

5+5 basis $\longleftrightarrow$ 4+6 basis $\qquad PSU(2,2|4)$

$P_A, J_{AB} \quad \xleftrightarrow{SO(2,4)} \quad P_a, K_a, J_{AB}, D$

$P_{A'}, J_{A'B'} \quad \xleftrightarrow{SO(6)} \quad J^i{}_j \; (= \gamma_{A'} P_{A'} + \gamma_{A'B'} J_{A'B'})$

$Q_I^{\alpha\alpha'} \quad \xleftrightarrow{SUSY} \quad Q^i_\alpha, S^i_\alpha$

$d=4, \mathcal{N}=4$ superconformal algebra

Light-cone decomposition:

Supercharges diagonal w.r.t. $\quad J_{+-}, J_{x\bar{x}}, D$

$SO_{1,1} \times SO_2 \times R \subset SO(2,4)$

$Q^\pm_i, Q^{\pm i} \quad 16$

$S^\pm_i, S^\pm_i \quad 16$

$[J^{+-}, Q^\pm] = \pm\frac{1}{2} Q^\pm$ etc.

Supercoset representative:

$$g = g(x,\theta)\, g(\eta)\, g(y)\, g(\phi)$$

$$g(x,\theta) = \exp\left(x^a P_a + \theta^+_i Q^{-i} + \theta^-_i Q^{+i} + h.c.\right)$$

$$g(\eta) = \exp\left(\eta^+_i S^{-i} + \eta^-_i S^{+i} + h.c.\right)$$

$$g(y) = \exp\left(y^i_j J^j_i\right) \qquad y^i_j = (\gamma^{A'})^i{}_j\, y^{A'}$$

$$g(\phi) = \exp(\phi D)$$

SO(5) $\qquad S^5$

Gauge fixing:

$$\theta_i^+ = 0\,, \quad \eta_i^+ = 0$$

$$\theta_i^- = \theta_i\,, \quad \eta_i^- = \eta_i$$

Action:

$$S\,(x^+, x^-, x, \bar{x}\ \phi\,;\, y^A\,;\, \theta_i, \eta_i)$$

$$= S_B + S_F^{(2)} + S_F^{(4)}$$

$$\mathcal{L}_B = \sqrt{g}\, g^{\mu\nu} \Big[e^{2\phi} \big(\underline{\partial_\mu x^+} \partial_\nu x^- + \partial_\mu x\, \partial_\nu \bar{x} \big) + \partial_\mu \phi\, \partial_\nu \phi + G_{AB}(y)\, \partial_\mu y^A \partial_\nu y^B \Big] \,,$$

S^5

$$\mathcal{L}_F^{(2)} = i \sqrt{g}\, g^{\mu\nu}\, \underline{e^{2\phi} \partial_\mu x^+} \big(\theta^i \partial_\nu \theta_i + \eta^i \partial_\nu \eta_i + M^{j}_{A\,i}(y)\, \eta^i \eta_j\, \partial_\nu y^A \big)$$

$$- \varepsilon^{\mu\nu}\, \underline{e^{2\phi} \partial_\mu x^+}\, \Lambda_{ij}(y)\, \eta^i \big(\partial_\nu \theta^j + i\, e^{\phi} \eta^j \partial_\nu x \big) + \text{h.c.}$$

$$\mathcal{L}_F^{(4)} = \sqrt{g}\, g^{\mu\nu}\, \underline{e^{4\phi} \partial_\mu x^+ \partial_\nu x^+}\, \Sigma^{ij}_{k\ell}(y)\, \eta^k \eta^\ell \eta_i \eta_j$$

G, M, Λ, Σ : explicit functions of $y^A \in S^5$ in terms of $U \in SU(4)$

$$U = \cos\frac{|y|}{2} + i \gamma^A n^A \sin\frac{|y|}{2}\,, \qquad n^A = \frac{y^A}{|y|}$$

Properties of action:

- linear in ∂x^-, depends on $e^{2\phi}\partial x^+$:
  (no $\partial x, \partial y$ in fermion kinetic terms)

- manifest $SU(4)$ in fermion kinetic terms
- simple dependence on θ^i
  ($\theta \to \theta + \epsilon$ $\mapsto$ linear susy as in bndry theory)
  complicated dependence on η^i
  (non-linear conformal susy)
- $\Sigma \cdot \eta^4$-term : curvature of background
  $M \cdot \eta^2$, $\Lambda \cdot \eta^2$: R-R couplings

- Flat space ($R_{AdS} \to \infty$) limit:
  GS action in $\Gamma^+ \theta^I = 0$ gauge

$$\theta^I_\alpha \to \left(\underset{\substack{\downarrow \\ 0}}{\eta^+_i}, \eta^-_i, \underset{\substack{\downarrow \\ 0}}{\theta^+_i}, \theta^-_i \right)$$

$$\mathcal{L}_F = \sqrt{g}\, \partial^\mu x^+ \left(\theta^i \partial_\mu \theta_i + \eta^i \partial_\mu \eta_i \right) - \epsilon^{\mu\nu} \partial_\mu x^+ \, \eta^T_i \partial_\nu \theta^i + \text{h.c.}$$

Fixing bosonic (2-d diff) gauge:

$$x^+ = \tau \,, \qquad \sqrt{g}\, g^{\mu\nu} \overset{?}{=} \eta^{\mu\nu}$$

- phase space GGRT type approach

$$x^+ = \tau \,, \quad P^+ = p^+$$ Thorn

- equivalent Polyakov-type approach

$$x^+ = \tau \,, \quad \sqrt{g}\, g^{00} e^{2\phi} = -1$$

Integrate over x^-:

$$\delta\left[\, \partial_\mu \left(\sqrt{g}\, g^{\mu\nu} e^{2\phi} \partial_\nu x^+\right)\right]$$

Integrate over g^{01}, etc.

Effectively equivalent to:

$$x^+ = \tau \,, \qquad \sqrt{g}\, g^{\mu\nu} = \mathrm{diag}\left(-e^{-2\phi}, e^{2\phi}\right)$$ Polyakov

Action:

$$S = S_B + S_F^{(2)} + S_F^{(4)}$$

$$\mathcal{L}_B = -\sqrt{g}\, g^{\mu\nu}\Big[\, e^{2\phi}\, \partial_\mu x\, \partial_\nu \bar{x} + \partial_\mu \phi\, \partial_\nu \phi + G_{AB}(y)\, \partial_\mu y^A \partial_\nu y^B \Big]$$

$$= |\partial_0 x|^2 - \underline{e}^{4\phi} |\partial_1 x|^2 + \underline{e}^{-2\phi} (\partial_0 \phi)^2 - \underline{e}^{2\phi} (\partial_1 \phi)^2 + G_{AB}(y) \left(\underline{e}^{-2\phi}\, \partial_0 y^A \partial_0 y^B - \underline{e}^{2\phi}\, \partial_1 y^A \partial_1 y^B\right)$$

Price of light-cone (non-conformal) gauge:

S^5 part is <u>coupled</u> to ϕ

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"2-d spinor" form of $\mathcal{L}_F$:

$$e^{-1}\mathcal{L}_F^{(2)} = i e^\mu_m \bar{\Psi}^i \tau^m \mathcal{D}_\mu \Psi_i - \bar{\Psi}^i \Psi_i \underbrace{\partial_1 \phi}$$

$$+ \mathcal{M}^i_{Aj}(y)\, \bar{\Psi}_i \tau^- \Psi^j \underbrace{\partial_0 y^A}$$

$$+ e^{\phi}\, \tilde{\Psi}_i \Psi^i \underbrace{\partial_1 x} + h.c.$$

$$e^{-1}\mathcal{L}_F^{(4)} = \frac{1}{4} \underset{\substack{\shortparallel \\ \text{const}}}{\tilde{\Sigma}^{ij}_{kl}}\, \bar{\Psi}_i \Psi^k \bar{\Psi}_j \Psi_l$$

$$e^m_\mu = (e^{2\phi}, 1) \quad \cancel{\text{const}} \qquad g_{\mu\nu} = e^m_\mu e^n_\nu \eta_{mn}$$

τ^m – 2d Dirac matrices

$$\Psi^i = \begin{pmatrix} \Psi^i_1 \\ \Psi^i_2 \end{pmatrix}, \qquad \Psi^i_{1,2} \sim \theta^i \pm \eta^i$$

$\mathcal{D}_\mu$: generalized covariant derivative on S^5

- G/H bosonic σ-model coupled to Thirring-type 2-d fermionic model in curved 2-d geometry (ϕ-dependent)
- standard fermionic kinetic term degenerate at AdS horizon: $e^{2\phi} \to 0$, $|\det e^m_\mu| \to 0$

Reflection of l.c. gauge singularity

Summary:

- String in R-R background $\rightarrow$
  GS description: useful
- <u>Well-defined</u> perturbation theory
  near <u>"long string"</u> background
- "Short string" sector:
  choose fermionic <u>l.c. gauge</u> (solve ~~$\partial x \bar{\theta} \partial \theta$~~ problem)
- Curved AdS background:
  $x^+ = \tau$ + non conformal gauge
- <u>Well-defined string action</u> ($\phi \neq -\infty$)
  but complicated...

Can be put in simpler form?

Special limits? (e.g. $\alpha' \to \infty$?)

Phase space: new variables?