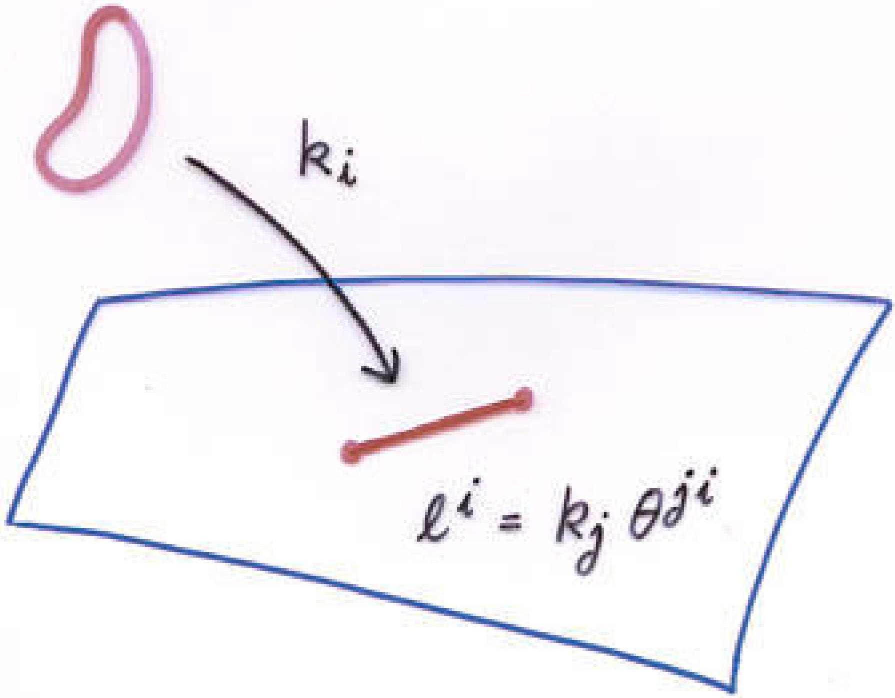
HOW NONCOMMUTATIVE GAUGE THEORIES COUPLE TO GRAVITY

HIROSI OOGURI (CALTECH)

BASED ON A WORK WITH YUJI OKAWA

(HEP-TH /0012218)



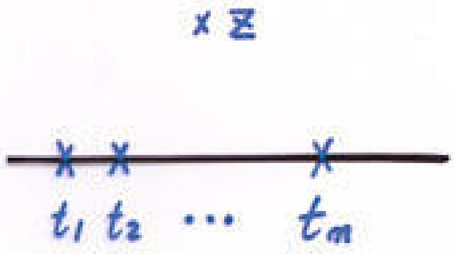
$$[x^i, x^j] = -i\,\theta^{ij}$$

MOTIVATIONS

WE STUDY COUPLING OF NONCOMMUTATIVE GAUGE THEORIES ON BRANES TO CLOSED STRING IN THE BULK IN ORDER TO UNDERSTAND

- GAUGE INVARIANT OBSERVABLES
- DUAL GRAVITATIONAL DESCRIPTION AT LARGE N
- NONCOMMUTATIVE THEORIES IN CURVED SPACES
  ($\longleftrightarrow$ MATRIX THEORY IN CURVED SPACES)

METHOD



COMPUTE DISK AMPLITUDES.

FIND OPERATORS IN GAUGE THEORY
DUAL TO CLOSED STRING STATES.

SURPRISES

- IN THE LIMIT $\theta^{ij} \to 0$, THE OPERATORS WE FOUND IN NONCOMMUTATIVE THEORY <u>DO NOT</u> ALWAYS REDUCE TO THOSE IN COMMUTATIVE THEORY.

$$\mathcal{O}_{\text{NONCOMMUTATIVE}} \not\longrightarrow \mathcal{O}_{\text{COMMUTATIVE}}$$

- THE OPERATORS DERIVED IN BOSONIC STRING AND SUPERSTRING ARE DIFFERENT.

$$\mathcal{O}_{\text{BOSONIC}} \neq \mathcal{O}_{\text{SUPERSTRING}}$$

NOTATIONS

$X^M(z)$

X^i : NONCOMMUTATIVE
X^μ : COMMUTATIVE
} PARALLEL TO THE BRANE

X^α : TRANSVERSE

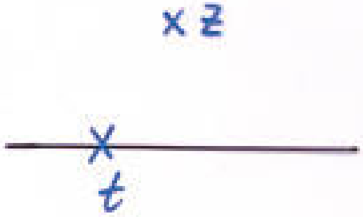
g_{MN} : CLOSED STRING METRIC

G_{MN} : OPEN STRING METRIC

$$G^{ij} = \frac{1}{(2\pi\alpha')^2}\,\theta^{im}\theta^{jn} g_{mn}\,, \quad G^{\mu\nu} = g^{\mu\nu}$$

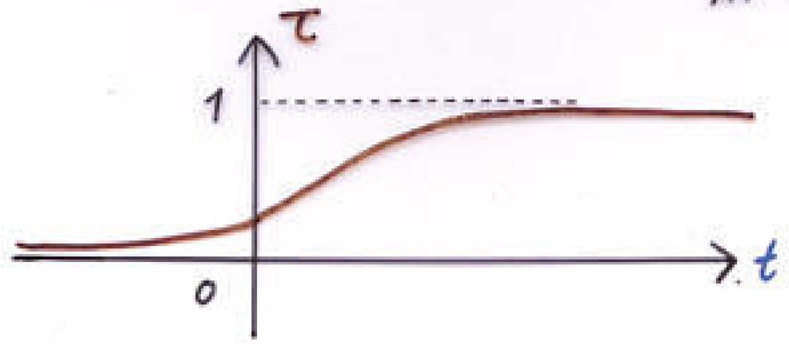
BULK-BOUNDARY PROPAGATOR ON WORLDSHEET

$$\boxed{\tau(t,z) = \frac{1}{2\pi i}\log\left(\frac{t-z}{t-\bar{z}}\right)}$$



$$\langle X^i(z)\, X^j(t)\rangle = -i\,\theta^{ij}\,\tau(t,z)$$

AT $\alpha'=0$



$0 < \tau < 1$ FOR $-\infty < t < +\infty$

STAR PRODUCT

$$\langle e^{i p_1 X(t_1)} \cdots e^{i p_m X(t_m)} \rangle$$

$$= \exp\Big[-\tfrac{i}{2}\sum_{a<b} p_a \theta p_b \underbrace{\epsilon(t_a - t_b)}_{\pm 1}\Big] \cdot \delta(p_1 + \cdots + p_m)$$

$$\Downarrow$$

$$\langle f_1(X(t_1)) \cdots f_m(X(t_m)) \rangle$$

$$= \int dx \; f_1(x) * \cdots * f_m(x)$$

WITH A CLOSED STRING TACHYON

$$\langle e^{ikX(z)} \; f_1(X(t_1)) \cdots f_m(X(t_m)) \rangle$$

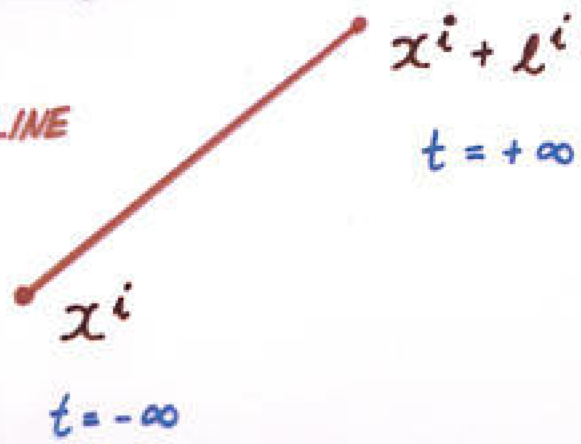
$$= \int dx \; e^{ikx} * f_1(x + \ell \cdot \tau(t_1, z)) * \cdots * f_m(x + \ell \cdot \tau(t_m, z))$$

WHERE $\underline{\ell^i = k_j \theta^{ji}}$

THE OPEN STRINGS ARE LOCATED

AT $x^i + \ell^i \tau(t_a, z)$

ALONG THE STRAIGHT LINE



TACHYON + GAUGE FIELDS

$$(z-\bar{z})^2 \langle e^{ikX(z)} A_{i_1}(X(t_1)) \frac{dX^{i_1}}{dt_1} \cdots A_{i_m}(X(t_m)) \frac{dX^{i_m}}{dt_m} \rangle$$

$$= \int dx\, e^{ikx} * A_{i_1}(x + \ell \cdot \tau(t_1, z))\, \ell^{i_1} \frac{d\tau}{dt_1} * \cdots$$

$$* A_{i_m}(x + \ell \cdot \tau(t_m, z))\, \ell^{i_m} \frac{d\tau}{dt_m}$$

WE CAN EXPONENTIATE THIS.

$$(z-\bar{z})^2 \langle e^{ikX(z)} \exp\left[i \int_{-\infty}^{\infty} A_i(X(t)) \frac{dX^i}{dt} \cdot dt \right] \rangle$$

$$= \int dx\, e^{ikx} * P\exp\left[i \int_0^1 A_i(x + \ell\tau)\, \ell^i\, d\tau \right]$$

STRAIGHT OPEN WILSON LINE

THE OPEN WILSON LINE IS GAUGE INVARIANT IF $\ell^i = k_j \theta^{ji}$.

(ISHIBASHI, ISO, KAWAI + KITAZAWA / 991004
DAS + REY / 0008042
GROSS, HASHIMOTO + ITZAKI / 0008075)
DAS + TRIVEDI / 0011131
LIU / 0011125 ...

WE CAN TURN THIS INTO STRING THEORY S-MATRIX COMPUTATION AND FIND AN OPERATOR IN GAUGE THEORY COUPLED TO THE CLOSED STRING TACHYON.

THE GAUGE THEORY OPERATOR

COUPLED TO THE CLOSED STRING TACHYON IS

$$\boxed{\int dx\, e^{ikx} * P\exp\left[i\int_0^1 d\tau \left(A_i(x+\ell\tau)\ell^i + \Phi^\alpha(x+\ell\tau)y_\alpha\right)\right]}$$

Φ^α : SCALAR FIELD TRANSVERSE TO THE BRANE

$$y_\alpha = 2\pi\alpha' k_\alpha \,, \quad \ell^i = k_j \theta^{ji}$$

TACHYON ON-SHELL CONDITION

$$G_{ij}\ell^i\ell^j + g^{\alpha\beta} y_\alpha y_\beta = 0$$

- k_μ IS ARBITRARY

  $\Rightarrow$ THE OPERATOR CAN BE LOCALIZED IN THE COMMUTATIVE DIRECTIONS ON THE BRANE.

- k_i IS CONSTRAINED BY THE ON-SHELL CONDITION

  $\Rightarrow$ THE INVERSE FOURIER TRANSFORMATION IN k_i IS NOT POSSIBLE

  (FOR THE OPERATORS COUPLED TO CLOSED STRING STATES).

- THE ON-SHELL CONDITION IS INDEPENDENT OF THE AMOUNT OF CLOSED STRING EXCITATION.

Graviton in Superstring

$$T^{ij} = \int dx\, e^{ikx} * \mathcal{P}\exp\left[i\int_0^1 d\tau \left(A_i(x+l\tau)l^i + \Phi^\alpha(x+l\tau)y_\alpha\right)\right]$$

$$* \, \theta^{ii'}\theta^{jj'} \int_0^1 d\tau_1 \int_0^1 d\tau_2$$

$$\Big[\left(F_{i'm}(x+l\tau_1) - \theta^{-1}_{i'm}\right)\left(F_{j'n}(x+l\tau_2) - \theta^{-1}_{j'n}\right) G^{mn}$$

$$+ F_{i'\mu}(x+l\tau_1) F_{j'\nu}(x+l\tau_2) G^{\mu\nu}$$

$$+ D_{i'}\Phi^\alpha(x+l\tau_1) D_{j'}\Phi^\beta(x+l\tau_2) g_{\alpha\beta} \Big]$$

- In the limit of zero momentum,

$$T^{ij}(k_M = 0) \sim \frac{\partial S}{\partial g_{ij}}$$

where

$$S = \frac{-1}{g_{YM}^2} \int dx \sqrt{\det G} \left[\frac{1}{4} G^{ij} G^{kl} (F_{ik} - \theta^{-1}_{ik}) * (F_{jl} - \theta^{-1}_{jl}) + \cdots \right]$$

$$G^{ij} = \frac{1}{(2\pi\alpha')^2} \theta^{im}\theta^{jn} g_{mn}$$

$$g_{YM}^2 = \frac{2\pi g_{string}}{\sqrt{\det(\theta^{im} g_{mj}/2\pi\alpha')}}$$

- THE CONSERVATION LAW $k_M T^{MN} = 0$ CAN BE VERIFIED USING IDENTITIES SUCH AS,

$$k_i \theta^{ii'} \int_0^1 d\tau_1 F_{i'm}(x+\ell\tau) * P\exp\left[i\int_0^1 d\tau A_i(x+\ell\tau)\ell^i\right]$$

$$= i D_m * P\exp\left[i\int_0^1 d\tau A_i(x+\ell\tau)\ell^i\right]$$

AND THE EQUATION OF MOTION $D_m F_{jn} G^{mn} = \cdots$

- IN THE LIMIT $\theta^{ij} \to 0$,

  T^{ij} DOES NOT REDUCE TO THE ENERGY-MOMENTUM TENSOR OF THE THEORY IN COMMUTATIVE SPACE.

$$T^{ij}_{\text{COMMUTATIVE}} \sim G^{ii'}G^{jj'}\left(F_{i'm}F_{j'n}G^{mn} - \frac{1}{4}G_{i'j'}F^2 + \cdots\right)$$

$$\left\{\begin{array}{ll} \bullet\ G^{ii'}G^{jj'} \to \theta^{ii'}\theta^{jj'} & \bullet\ \text{SHIFT BY } \theta^{-1} \\ \bullet\ \text{NO } -\frac{1}{4}G_{i'j'}F^2 & \text{IN } T^{ij}(\theta \to 0). \end{array}\right.$$

NOTE:

THE DISK AMPLITUDES TO COMPUTE T^{ij}

IN NONCOMMUTATIVE CASE $\sim O(\alpha'\theta^2)$

IN COMMUTATIVE CASE $\sim O(\alpha'^3)$

CURIOSITY OF BOSONIC STRING

$$T^{ij}_{\text{BOSONIC}} = \int dx\, e^{ikx} * P\exp\left[i\int_0^1 d\tau \left(A_i(x+\ell\tau)\ell^i + \Phi^a(x+\ell\tau) y_a \right) \right]$$

$$* \left(\theta^{ii'}\theta^{jj'} + \theta^{ij'}\theta^{ji'} \right)$$

$$\Big[\; i\int_0^1 d\tau_1\, e^{2\pi i \tau_1} \left(F_{i'm}(x+\ell\tau_1)\ell^m + D_{i'}\Phi^\alpha(x+\ell\tau_1) y_\alpha \right)$$

$$* \, i\int_0^1 d\tau_2\, e^{-2\pi i \tau_2} \left(F_{j'n}(x+\ell\tau_2)\ell^n + D_{j'}\Phi^\beta(x+\ell\tau_2) y_\beta \right)$$

$$+ \, i\int_0^1 d\tau \left(D_{i'}F_{j'm}(x+\ell\tau)\ell^m + D_{i'}D_{j'}\Phi^\alpha(x+\ell\tau) y_\alpha \right) \Big]$$

- $T^{ij}_{\text{BOSONIC}} \neq T^{ij}_{\text{SUPERSTRING}}$

  THE DISK AMPLITUDES TO COMPUTE T^{ij}

  IN BOSONIC STRING $\sim O(k^2\theta^4)$

  IN SUPERSTRING $\sim O(\alpha'\theta^2)$

- KINEMATICALLY CONSERVED

  $k_M T^{MN} = 0$ WITHOUT USING THE EQUATION OF MOTION.

  WE ONLY NEED $\ell^i \ell^j F_{ij} = 0,$

  $$\int_0^1 d\tau\, \partial_i f(x+\ell\tau)\ell^i = f(x+\ell) - f(x),$$

  etc.

$\Rightarrow$ IS THE COUPLING TO THE BULK METRIC THROUGH THE CURVATURE ?

Comparison with Dual Gravitational Description

$\mathcal{N}=4$ Theory in $d=4$ (Hashimoto + Itzaki / 9907166, Maldacena + Russo / 9908134)

◎ Commutative Case ($AdS_5 \times S^5$)

Minimally coupled scalar field in the bulk.

$$\phi(u,x) \sim u^{2 \pm \sqrt{(j+2)^2 + m^2}} \, e^{ikx}, \quad u \to \infty$$

- Can take the inverse Fourier transformation in k.

  $\longrightarrow$ Source localized in the x-space.

- The exponent of u depends on j ($SO(6)$ spin).

  $\longrightarrow$ Cannot be localized on S^5.

◎ Noncommutative Case ($\mathcal{M}_5 \times S^5$)

$$\phi(u,x) \sim u^{-\frac{5}{2} \pm \frac{m}{2|l|}} \, e^{\pm u \sqrt{g_{YM}^2 N} \cdot |l|} \, e^{ikx}, \quad u \to \infty$$

$$(l^i = k_j \theta^{ji})$$

- Cannot take the inverse Fourier transformation in k_i
- Can be localized in the commutative directions in the x-space.
- The exponents are independent of j.

  $\to$ Can be localized on S^5

  $\Leftrightarrow$ $\boxed{y_\alpha}$ in the open Wilson line.