WORLDSHEET DERIVATION OF A LARGE N DUALITY

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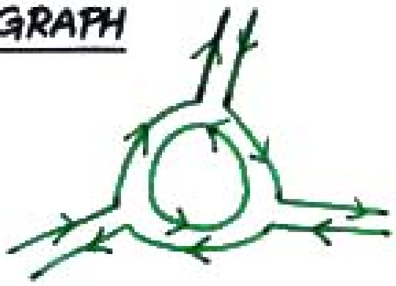
BASED ON A WORK WITH CUMRUN VAFA

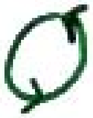
HEP-TH /0205297

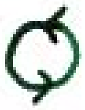
STRING THEORY FROM A LARGE N LIMIT OF A GAUGE THEORY

$$S = \frac{1}{g_{YM}^2} \int \mathcal{L}(A), \quad A_\mu = U(N) \text{ GAUGE FIELD}$$

RIBBON GRAPH



EACH  GIVES A FACTOR OF N

FILL  BY A DISK 

$\Rightarrow$ A CLOSED RIEMANN SURFACE

'T HOOFT COUNTING

$$(g_{YM}^2)^{2g-2} \cdot (g_{YM}^2 N)^h$$

g = GENUS OF THE SURFACE

h = # LOOPS = # HOLES

$$F = \sum_{g=0}^{\infty} (g_{YM}^2)^{2g-2} F_g(t), \qquad t = g_{YM}^2 N$$

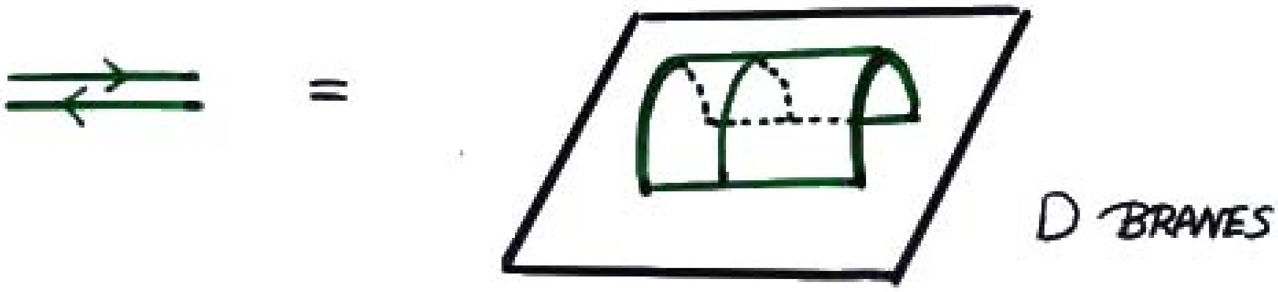
CONJECTURE

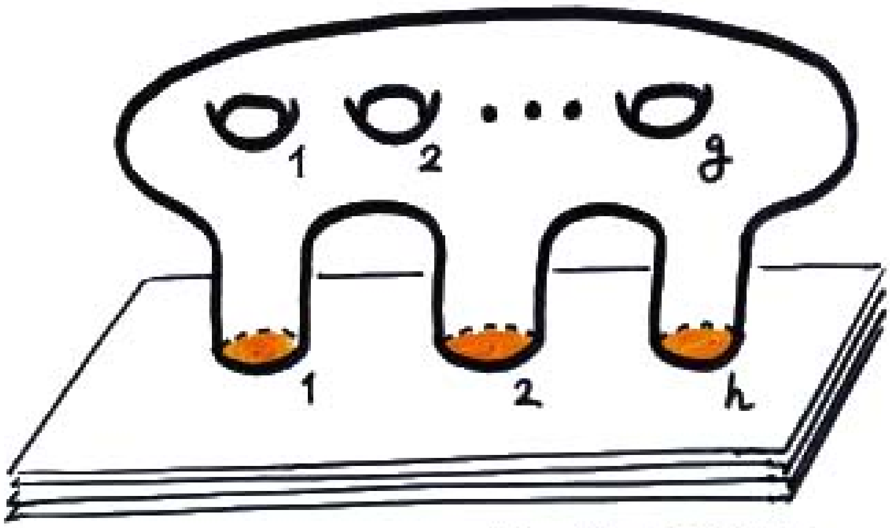
THERE IS A CLOSED STRING THEORY WHOSE g-LOOP AMPLITUDE IS GIVEN BY $F_g(t)$.

$\lambda_s = g_{YM}^2$: STRING COUPLING CONSTANT

$t = g_{YM}^2 N$: PARAMETER OF WORLDSHEET THEORY

RIBBON GRAPHS COME TO LIFE ON D BRANES





N D BRANES

LOW ENERGY LIMIT
$\Downarrow$
GAUGE THEORY

$\times \ \lambda_s^{2g-2} \cdot (N\lambda_s)^h$

$\Updownarrow$ CONJECTURE

$\times \ \lambda_s^{2g-2}$

$t = N\lambda_s$: SOME GEOMETRIC MODULUS

THE 't HOOFT CONJECTURE IS THE LOW ENERGY LIMIT OF A MORE GENERAL CONJECTURE ABOUT THE EQUIVALENCE OF :

D BRANES $\Longleftrightarrow$ CLOSED STRING BACKGROUND

SEVERAL EXAMPLES OF LARGE N DUALITIES HAVE BEEN DISCOVERED ASSUMING:

- D BRANES $\longleftrightarrow$ CLOSED STRING BACKGROUND
- M THEORY DUALITIES

(1) AdS/CFT CORRESPONDENCE

MALDACENA

(2) TOPOLOGICAL STRING DUALITIES

- CHERN-SIMONS GAUGE THEORY $\leftarrow$ OPEN STRING ON A BRANES ON DEFORMED CY_3.

  $\Updownarrow$

  CLOSED STRING WITH A TWIST ON RESOLVED CY_3.

  GOPAKUMAR + VAFA

- HOLOMORPHIC MATRIX MODEL $\leftarrow$ OPEN STRING ON B BRANES ON RESOLVED CY_3.

  $\Updownarrow$

  CLOSED STRING WITH B TWIST ON DEFORMED CY_3.

  "KODAIRA-SPENCER THEORY OF GRAVITY"

  DIJKGRAAF + VAFA

IT IS DESIRABLE TO PROVE THESE CONJECTURES WITHOUT APPEALING TO M THEORY DUALITIES.

STATEMENT OF THE CONJECTURE

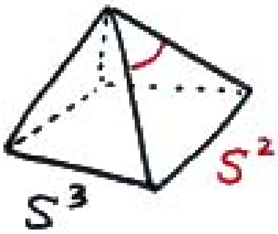
GOPAKUMAR + VAFA / 9811131

U(N) CHERN-SIMONS THEORY ON S^3

$$S = \frac{k}{4\pi} \int \mathrm{tr} \left(A dA + \frac{2}{3} A^3 \right) , \quad g_{YM}^2 = \frac{i}{k+N}$$

$$\Updownarrow$$

TOPOLOGICAL CLOSED STRING ON CONIFOLD



$\lambda_s = g_{YM}^2 = \dfrac{i}{k+N}$: STRING COUPLING CONSTANT

$t = g_{YM}^2 N = \dfrac{iN}{k+N}$: NS-NS 2 FORM B

$$i \int_{S^2} B = t$$

UPLIFTING TO M THEORY

VAFA / 0008142

CHERN-SIMONS THEORY ON S^3 $\Rightarrow$ LOW ENERGY EFFECTIVE ACTION TERMS ON N D_6 BRANES ON $S^3 \times \mathbb{R}^4$

$$S^3 \times \mathbb{R}^4 \subset T^* S^3 \times \mathbb{R}^4$$

TOPOLOGICAL CLOSED STRING ON CONIFOLD $\Rightarrow$ CORRESPONDING TERMS IN IIA STRING ON CONIFOLD WITH RR FLUX

BCOV / 9309140

THESE CONFIGULATIONS IN IIA STRING THEORY ARE SMOOTHLY INTERPOLATED WHEN UPLIFTED TO M THEORY ON G_2 HOLONOMY MANIFOLDS.

ATIYAH, MALDACENA + VAFA / 0011256

PROOF

THE LINEAR SIGMA-MODEL PROVIDES A GOOD DESCRIPTION OF THE CLOSED STRING WORLD SHEET FOR $t = g_{YM}^2 N \to 0$.

WITTEN / 9301042

V : VECTOR MULTIPLET

A_i, B_i $(i=1,2)$: CHIRAL MULTIPLETS OF OPPOSITE CHARGES

$$\text{POTENTIAL} = |\sigma|^2 \cdot (|a_1|^2 + |a_2|^2 + |b_1|^2 + |b_2|^2) + e^2 \cdot (|a_1|^2 + |a_2|^2 - |b_1|^2 - |b_2|^2).$$

$\sigma \in V$, $a_i \in A_i$, $b_i \in B_i$

e : ELECTRIC CHARGE $\to \infty$ IN THE INFRARED.

$t = g_{YM}^2 N$ APPEARS IN THE THETA TERM :

$$\frac{t}{2\pi} F_{12} = \int d\theta \; W$$

$W = t\,\Sigma$: SUPERPOTENTIAL

$\Sigma = \bar{D}_+ D_- V = \sigma + \cdots$; TWISTED CHIRAL SUPERFIELD

THE HIGGS BRANCH THEORY $(|\sigma|^2 \ll e^2)$

FLOWS TO THE SIGMA-MODEL ON CONIFOLD.

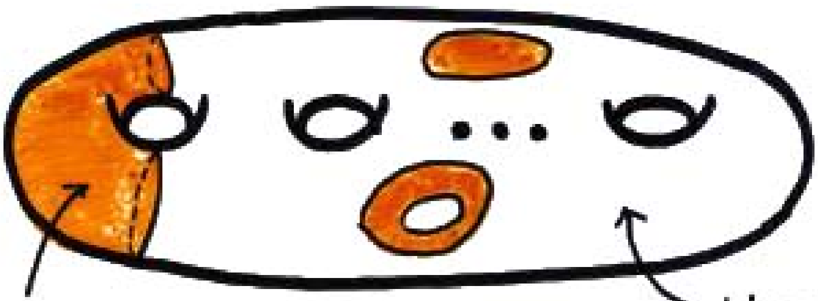
WHEN $t \to 0$, THE THEORY DEVELOPS TWO PHASES :

- H PHASE : $\sigma = 0$; $a_i, b_i \neq 0$

  $\Rightarrow$ TARGET SPACE GEOMETRY

- C PHASE : $\sigma \neq 0$; $a_i, b_i = 0$ NEW PHASE AT $t=0$

FOR EACH CONFIGURATION OF THE σ FIELD,
WE CAN SEPARATE THE WORLDSHEET INTO THE TWO PHASES.



σ_* : CUTOFF

C PHASE : $|\sigma| > \sigma_*$

H PHASE : $|\sigma| < \sigma_*$

A PART OF THE FUNCTIONAL INTEGRAL OVER σ
CAN BE TURNED INTO A SUM OVER
SHAPES AND SIZES OF THE C DOMAIN.

COLLECTIVE COORDINATES OF σ

WE NEED TO SHOW :

- CONTRIBUTIONS FROM C DOMAINS SUCH AS :

VANISH.

NAMELY, EVERY C DOMAIN HAS THE TOPOLOGY OF THE DISK.

- EACH DISK IN C PHASE GIVES THE FACTOR OF t.

 $= t = g_{YM}^2 N$

OUR TASK IS SIMPLIFIED BY THE LOCALIZATION OF THE FUNCTIONAL INTEGRAL OVER σ.

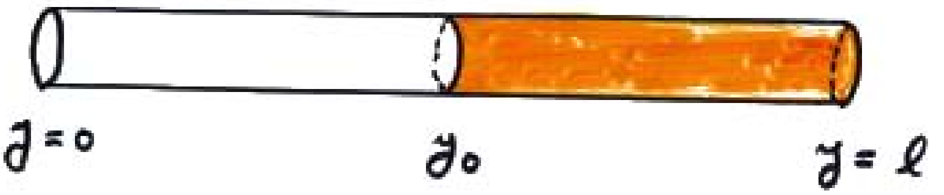
BECAUSE OF THE TOPOLOGICAL TWISTING, σ = CONST ON A FIXED RIEMANN SURFACE.

$\Rightarrow$ THE WORLDSHEET IS EITHER IN PURE C PHASE OR IN PURE H PHASE.

C AND H DO NOT CO-EXIST.

IN THE TOPOLOGICAL STRING, WE INTEGRATE OVER $\mathcal{M}_g$.

σ CAN BE NON-CONSTANT IF THERE IS A LONG CYLINDER.



$\ell \to \infty$

$$\sigma(y_0) = \sigma_0 \ , \quad |\sigma_0| = \sigma_*$$

$\Rightarrow$ REDUCTION TO A ONE-DIMENSIONAL QUANTUM MECHANICS.

CHANGE OF FUNCTIONAL INTEGRAL VARIABLES:

$$\sigma(y_0),\ \bar{\sigma}(y_0) \longrightarrow y_0 \text{ AND THE PHASE OF } \sigma_0$$

- y_0 BECOMES A PART OF THE MODULI OF THE WORLDSHEET WITH THE BOUNDARY.
- EACH C DOMAIN CONTRIBUTES AS

$$\oint d\sigma_0 \frac{\partial}{\partial \sigma_0} \mathcal{F}^{(C)}(\sigma_0)$$

BY THE TOPOLOGICAL BRST SYMMETRY,

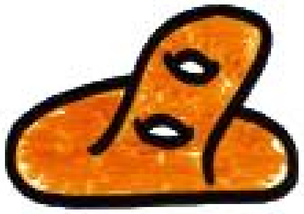
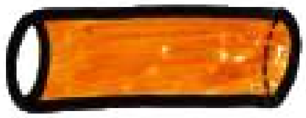
THE AMPLITUDE $\mathcal{F}^{(C)}(\sigma_0)$ OF EACH C DOMAIN

IS A HOLOMORPHIC FUNCTION OF σ_0.

$$\Rightarrow \oint d\sigma_0 \frac{\partial}{\partial \sigma_0} \mathcal{F}^{(C)}(\sigma_0) = 0 \quad \text{IF } \mathcal{F}^{(C)}(\sigma_0)$$

IS SINGLE-VALUED.

THIS IS THE CASE FOR

 ,  , ...

THE ONLY NON-ZERO CONTRIBUTION COMES

FROM THE CASE WHEN THE C DOMAIN IS A DISK,

THE AMPLITUDE CAN BE COMPUTED EXACTLY.

$\oint d\sigma_0 \frac{\partial}{\partial \sigma_0}$  $= \oint \frac{d\sigma_0}{\sigma_0^2} e^{t\sigma_0}$

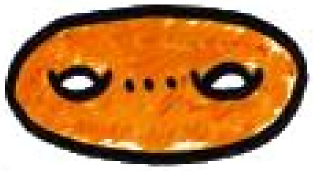
DISK

$$= t = g_{YM}^2 N$$

THIS IS WHAT WE WANTED

TO SHOW.

PURE C PHASE

$$\int_{\mathcal{M}_g}$$  $$= \chi(\mathcal{M}_g) \times \frac{1}{t^{2g-2}}$$

SINGULAR AS $t \to 0$.

THIS DOES NOT CORRESPOND TO ANY TERM IN 't HOOFT EXPANSION.

WHAT IS IT IN THE GAUGE THEORY?

$$\sum_g (g_{YM}^2)^{2g-2} \times \chi(\mathcal{M}_g) \times \frac{1}{(g_{YM}^2 N)^{2g-2}}$$

$$= \sum_g \frac{B_{2g}}{2g(2g-2) N^{2g-2}} \simeq -\log \mathrm{vol}(U(N))$$

$$\mathrm{vol}(U(N)) = \frac{(2\pi)^{\frac{1}{2}N(N+1)}}{(N-1)!(N-2)!\cdots 3!2!1!}$$

$$(\because \; U(N) \sim S^1 \times S^3 \times \cdots \times S^{2N-1})$$

$\mathrm{vol}(U(N))$ IS THE MEASURE FACTOR IN

$$\int [dA] \, e^{\frac{ik}{4\pi}\int \mathrm{tr}(AdA + \frac{2}{3}A^3)}$$

THE CLOSED STRING THEORY KNOWS ABOUT THE GAUGE THEORY BEYOND THE 't HOOFT EXPANSION.