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Sen's Conjectures in Yang-Mills : Recombination of Intersecting D-branes

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Sen's Conjectures and Intersecting D-branes

Sen's conjectures on tachyon condensation on $D\overline{D}$:

[1] Tachyon condensation = Pair annihilation of $D\overline{D}$ (Closed string theory)

[2] Tachyon topological defects = Lower dimensional D-branes

These gave new descriptions of D-branes and string theories.

Most of the approaches to verify the conjectures employed String Field Theories, since

- **the tachyon mass squared is of order $1/\alpha'$ so the physics essentially includes all the stringy massive levels.**
- **the tachyon condensation itself involves off-shell phenomena.**

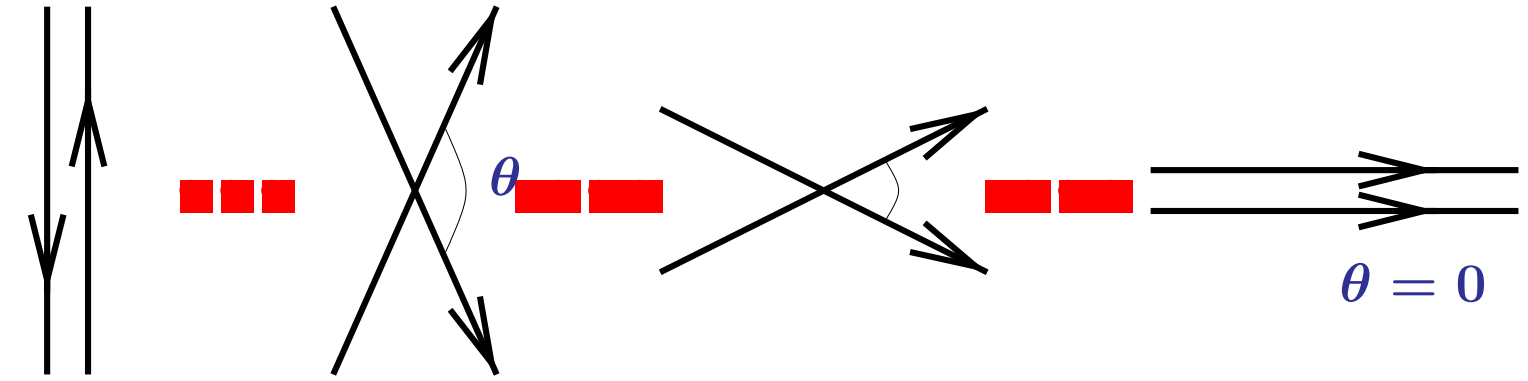
$$\downarrow$$

If we can reduce the tachyon mass squared to nearly massless, we may employ a simpler scheme – low energy effective theories.

How? — By turning on the Intersection angle

$$m^2 = -\theta/2\pi\alpha'$$

[Berkooz-Douglas-Leigh]



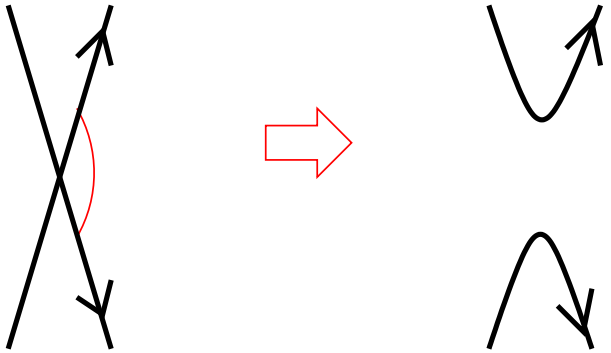
$D\overline{D}$ Intersecting D-branes Parallel D-branes (YM description)

→ If we choose θ very small, we can analyse the conjectures in YM.

Moreover, intersecting D-branes are more generic configurations, so the study of their dynamics is called for.

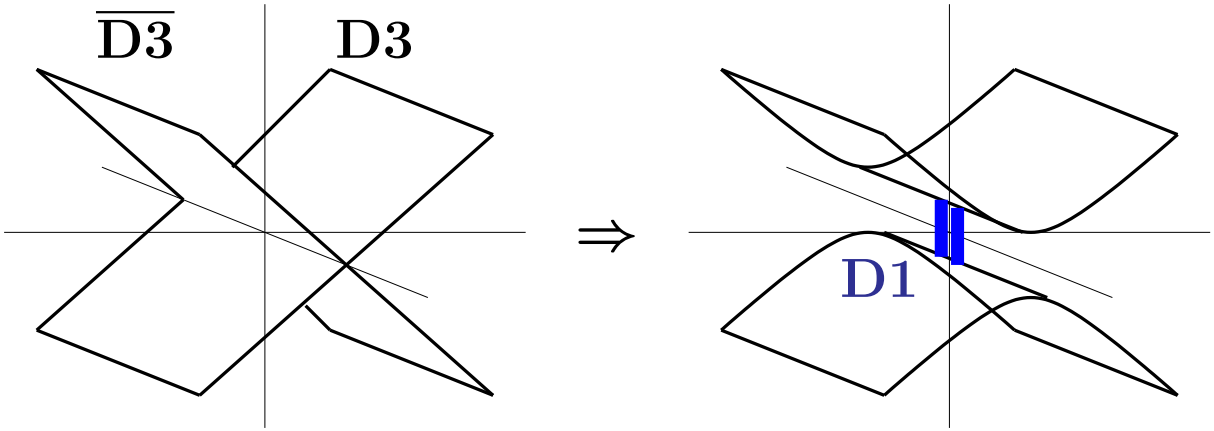
The conjectures in our case

[1] Tachyon condensation = Pair annihilation of $D\overline{D}$?



Tachyon condensation
||
D-brane Recombination

[2] Descent relation?



We will show that this is the case in YM.

Conjecture [1] : Recombination in YM

LEEA of parallel two D-strings (1+1 dim. $SU(2)$ super YM)

$$S = -T_{\rm D1}(2\pi\alpha')^2 {\rm Tr} \int dtdx \left[\frac{1}{2}(F_{\mu\nu})^2 + (D_\mu \Phi)^2 \right]$$

Solution of Intersecting D-strings :

$$\Phi^3 = qx \ , \quad A_\mu = 0 \quad \text{with} \quad q = \frac{1}{\pi\alpha'} \tan(\theta/2)$$

When θ is small, YM is valid.

Two tachyonic fluctuations appear :

- Eigen functions for the fluctuations are

$$\Phi^1 = A^2_x = C_1(t) \exp\left[-\frac{qx^2}{2} \right], \quad \Phi^2 = -A^1_x = C_2(t) \exp\left[-\frac{qx^2}{2} \right]$$

- Those mass squared are ($[(\partial_t)^2 + m^2]C_i(t) = 0$)

$$m^2 = -q = \frac{-\theta}{2\pi\alpha'} + \mathcal{O}(\theta^3)$$

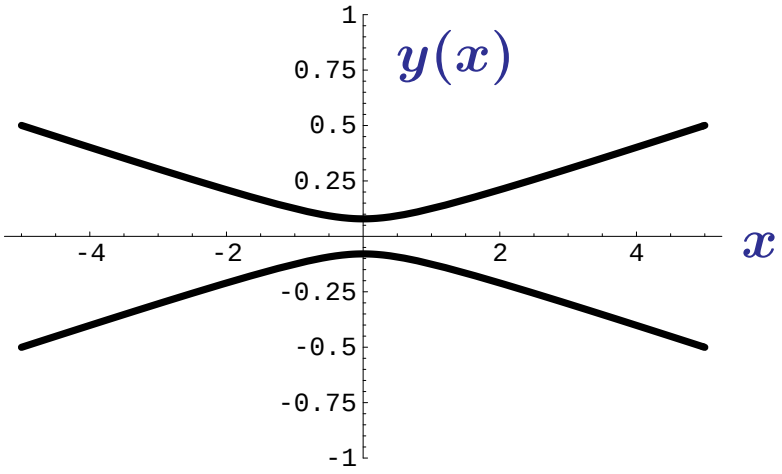
This agrees with the worldsheet result. [A.Hashimoto-Taylor]

Condensation of the tachyonic mode?

$$\Phi(t,x) = \frac{1}{2}\begin{pmatrix} qx & (C_1 - iC_2)e^{-qx^2/2} \\ (C_1 + iC_2)e^{-qx^2/2} & -qx \end{pmatrix}$$

Location of the D-strings are given by the eigenvalues of $Y \equiv 2\pi\alpha'\Phi$

$$y(x) = \pm\pi\alpha'\sqrt{q^2x^2 + C^2e^{-qx^2}}$$



Tachyon condensation
(Condensation of the off-diagonal entries) = Recombination

⇓

Sen's conjecture [1] is qualitatively confirmed,
in the case of the intersecting branes.

Fate of gauge fields and F-strings

There are two $U(1)$ gauge fields on the $D\overline{D}$:

$$A^{(\pm)} = A^{\text{brane}} \pm A^{\text{antibrane}}$$

By the tachyon condensation,

⟨i⟩ $A^{(-)}$ will be Higgsed and become (infinitely) massive.

⟨ii⟩ $A^{(+)}$ flux will be confined to give closed strings after the disappearance of the $D\overline{D}$.

[Yi], [Bergman-Hori-Yi], [Sen]

Correspondence between $D\overline{D}$ picture and YM picture?

$$A_y^{D\overline{D}} \cdot \frac{\partial Y}{\partial x} = A_x^{\text{YM}}$$

$\Rightarrow$ Chan-Paton factor will be exchanged because of $\dfrac{\partial Y}{\partial x} \propto \sigma_3$:

⟨i⟩ $A^{(-)}$: $\sigma_3 \cdot \sigma_3 = 1_{2\times 2} \Rightarrow$ overall trace $U(1)$ in $U(2)$ YM

⟨ii⟩ $A^{(+)}$: $1_{2\times 2} \cdot \sigma_3 = \sigma_3 \Rightarrow$ diagonal $U(1)$ of σ_3 in $SU(2)$ YM

Location of the F-string and D-string sources produced by the matrix configuration is given by the multi-pole coupling in (T-dual of) Matrix theory. [Taylor-vanRaamsdonk], [Myers], [Okawa-Ooguri]

- F-string charge density :

$$\Pi_{xt} = -2\pi\alpha' T_{\rm D1} \int dk\, e^{-iky} \mathrm{Str}[F_{xt} e^{ikY}],$$

$$\Pi_{yt} = -2\pi\alpha' T_{\rm D1} \int dk\, e^{-iky} \mathrm{Str}[F_{xt} D_x Y e^{ikY}],$$

- D-string charge density :

$$I_{xt} = -2\pi\alpha' T_{\rm D1} \int dk\, e^{-iky} \mathrm{Str}[e^{ikY}],$$

$$I_{yt} = -2\pi\alpha' T_{\rm D1} \int dk\, e^{-iky} \mathrm{Str}[D_x Y e^{ikY}].$$

Note : This D-string density provides the eigenvalues of the matrix $Y = 2\pi\alpha'\Phi$.

ex) For the recombined D-strings without electric flux,

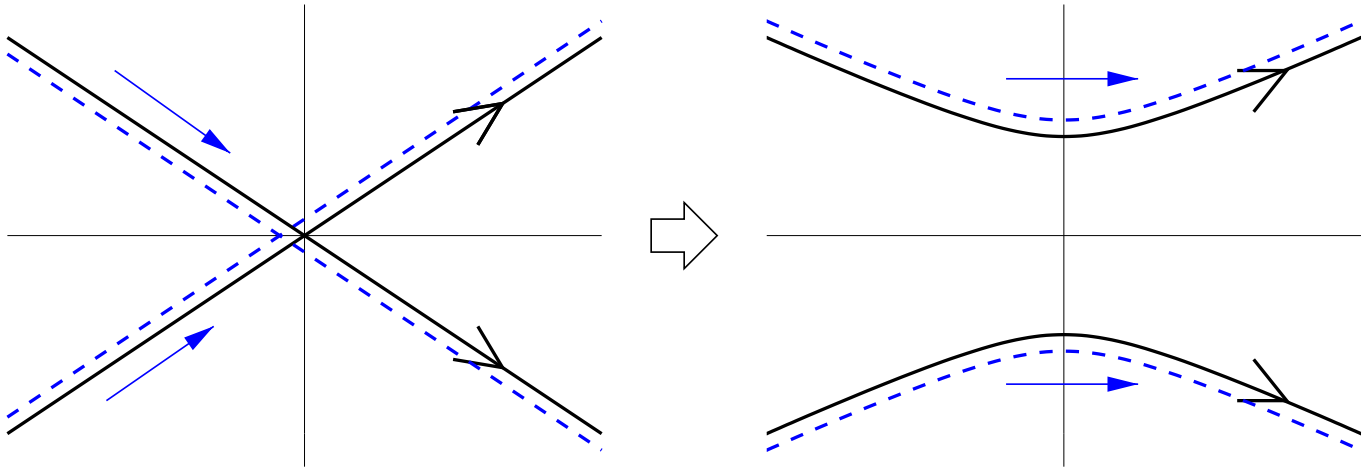
$$I_{xt} = T_{\rm D1} \left[\delta(y - \lambda(x)) + \delta(y + \lambda(x))\right]$$
$$I_{yt} = T_{\rm D1} \lambda'(x) \left[\delta(y - \lambda(x)) - \delta(y + \lambda(x))\right]$$

where $\lambda = \pi\alpha' \sqrt{q^2 x^2 + C^2 e^{-qx^2}}$.

⟨i⟩ Fate of $A^{(-)}$ (overall trace $U(1)$ in $U(2)$ YM)

We turn on this $U(1)$ as $F_{xt} = \begin{pmatrix} q' & 0 \\ 0 & q' \end{pmatrix}$

- This does not affect the fluctuation analysis.
  $\Rightarrow$ Tachyonic modes are present as before.
- The F-string density formula reduces to that of D-string.



$\Downarrow$

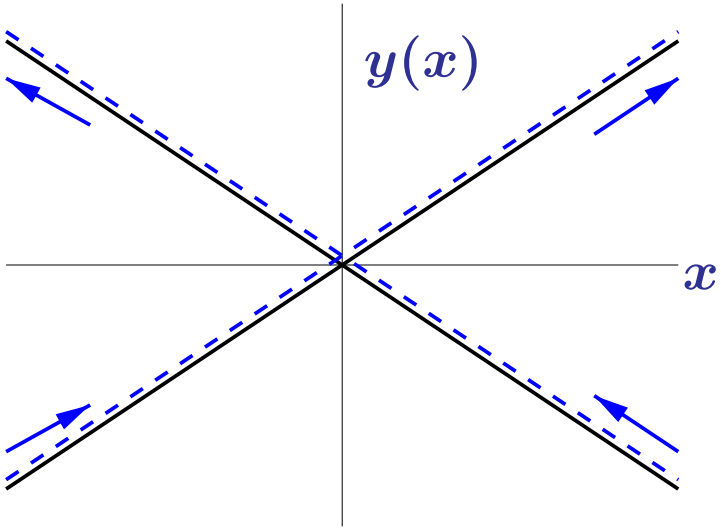
F-string charges are absent between the D-strings.
Consistent with the previous picture that this gauge field disappears due to the Higgs mechanism.

⟨ii⟩ Fate of $A^{(+)}$ (diagonal $U(1)$ of σ_3 in $SU(2)$ YM)

We turn on this $U(1)$ as

$$F_{xt} = \begin{pmatrix} q' & 0 \\ 0 & -q' \end{pmatrix} \quad \text{with} \quad \Phi = \begin{pmatrix} qx & 0 \\ 0 & -qx \end{pmatrix}$$

When $q = q'$, this classical solution is supersymmetric.
= Supersymmetric intersection of a $(q, 1)$ and a $(-q, 1)$ string



- No tachyonic fluctuation appears.
- A massless deformation : $A_t = \Phi = \begin{pmatrix} qx & a \\ a & -qx \end{pmatrix}, \quad A_x = 0$

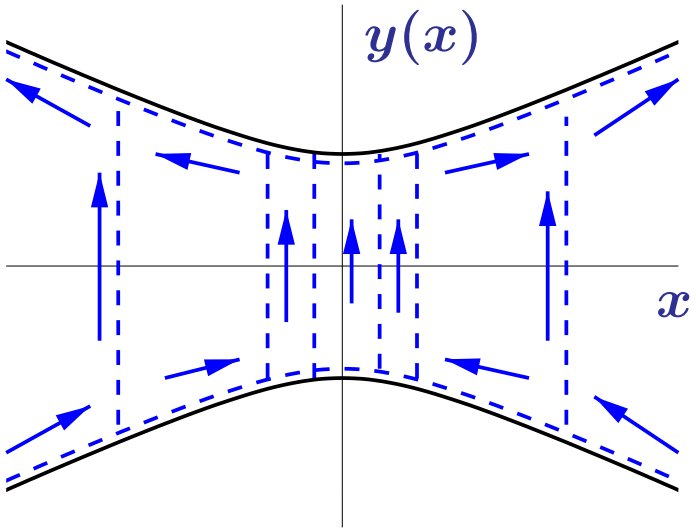
What is the meaning of this deformation?

- D-string density is located on the recombined curves :

$$y(x) = \pm\pi\alpha'\sqrt{q^2x^2 + a^2}$$

- F-strings should "jump" from one D-string to another, to satisfy the current density conservation. In fact,

$$\Pi_{xt} = T_{\mathrm{D1}}\lambda'(x)\left[\delta(y-\lambda(x)) - \delta(y+\lambda(x))\right],$$
$$\Pi_{yt} = T_{\mathrm{D1}}(\lambda'(x))^2\left[\delta(y-\lambda(x)) + \delta(y+\lambda(x))\right] + T_{\mathrm{D1}}\lambda''(x)\left[\theta(y+\lambda(x)) - \theta(y-\lambda(x))\right].$$



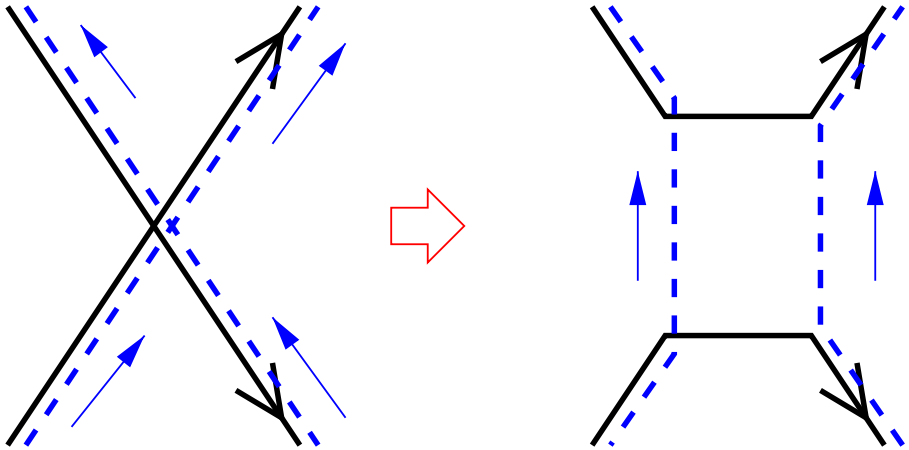
$\Rightarrow$

- Vertical F-strings away from the D-brane worldvolume!
- Locally equivalent to string junctions.
- This provides a representation of F-strings after the disappearance of the $D\overline{D}$.

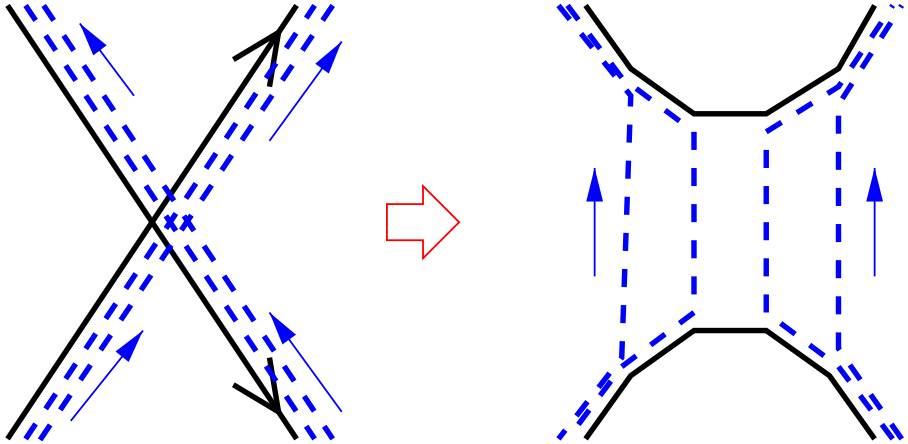
Examples of the supersymmetric deformation.

- Generating a box

[Aharony-Hanany-Kol]
[Sen]



- Multiple F-string fluxes and generalization of the box



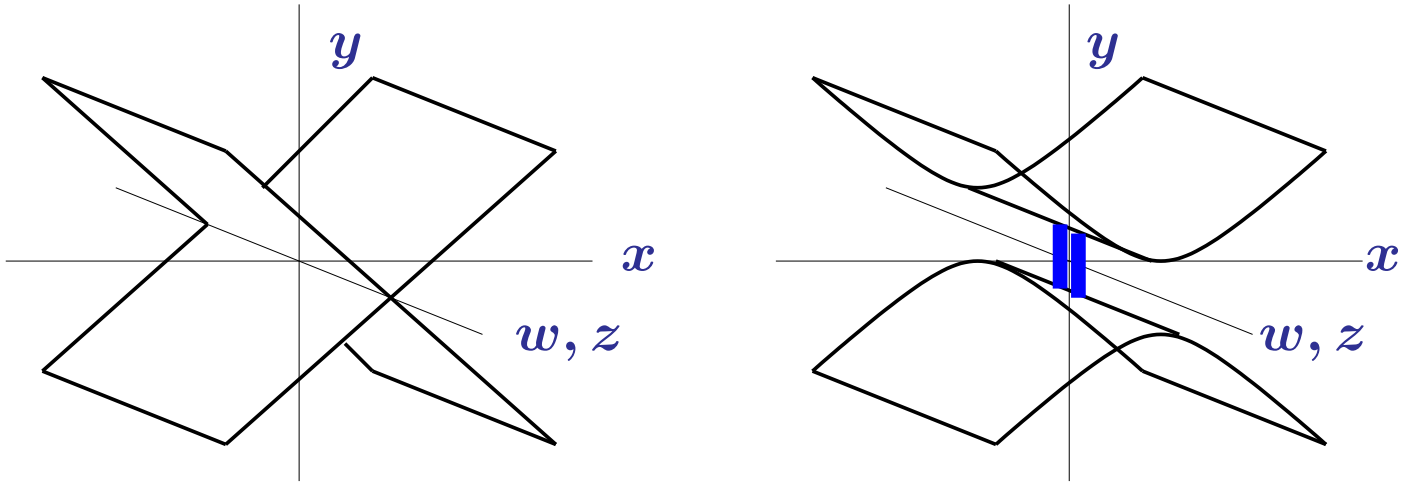
Our density distribution is a smeared version of this.

Conjecture [2] : Descent Relations in YM

Sen's conjecture at $\theta = \pi$: Tachyon vortex in D3 $\overline{\text{D3}}$ = D1

↓

Our configuration expected is a D1 connecting recombined D3s.



We extend our worldvolume with coordinates w, z to have D3s.

→ Turn on a vortex-like tachyon

$$C^{(1)} = cw, \quad C^{(2)} = cz$$

→ The eigenvalues show that the D3-branes touch each other,

$$y = \pm\lambda(x, w, z) = \pm\pi\alpha'\sqrt{q^2x^2 + c^2e^{-qx^2}(w^2 + z^2)}$$

The evaluated D-string charge density :

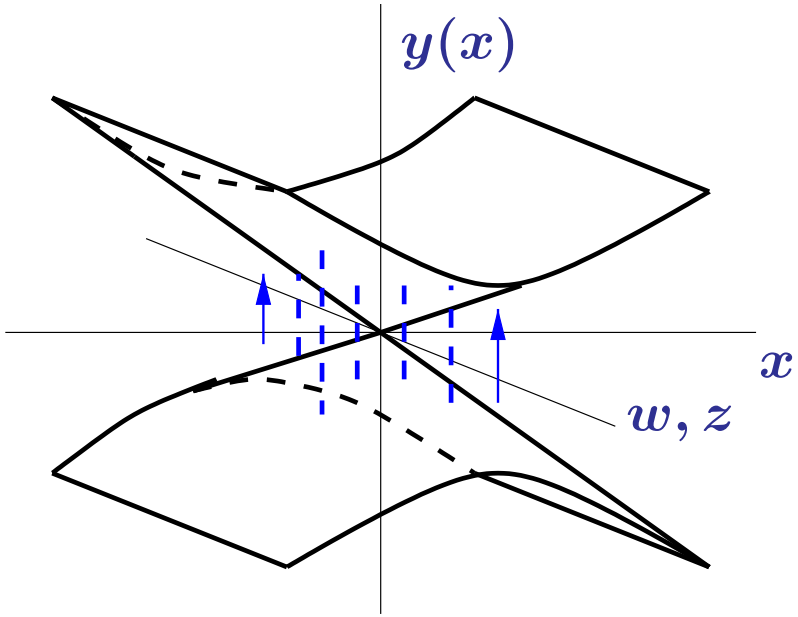
$$j^{yt} = T_{\rm D3}(\pi\alpha')^2 c^2 e^{-2qx^2}\left[\frac{\lambda^2-(\pi\alpha')^2q^2x^2}{\lambda^2}(\delta(y-\lambda)+\delta(y+\lambda))\right.$$

$$\left.+\frac{\lambda^2+(\pi\alpha')^2q^2x^2}{\lambda^3}(\theta(y+\lambda)+\theta(y-\lambda))\right],$$

$$j^{xt} = 0,$$

$$j^{wt} = \pi\alpha' T_{\rm D3} w \frac{c^2 e^{-2qx^2}}{\lambda}\left(\delta(y-\lambda)-\delta(y+\lambda)\right),$$

$$j^{zt} = \pi\alpha' T_{\rm D3} z \frac{c^2 e^{-2qx^2}}{\lambda}\left(\delta(y-\lambda)-\delta(y+\lambda)\right).$$



$\Rightarrow$

- In addition to the D-string charge bound on the D3s, vertical D-strings suspended between the D3-branes are generated.
- The Descent relation is confirmed.

Summary

Sen's conjectures on tachyon condensation are confirmed in YM, by turning on the intersection angle to the $D\overline{D}$ system.

- Recombination process is clarified in YM, and shown to correspond to a local pair-annihilation of $D\overline{D}$.
- Fate of two $U(1)$ gauge fields is studied, and F-strings connecting the D-branes are observed.
- Vortex condensation of the tachyon is shown to give lower-dimensional D-branes, confirming Sen's descent relations.

Future Directions

- F-strings "away from" D-branes, made by the off-diagonal condensation, may have fundamental importance?
- Deeper understanding of the "duality" of x and y?
- Analogue of S-branes and rolling tachyon.
- Application to brane phenomenology.