# Open strings in electric fields and the Milne universe

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## *Introduction*

Despite obvious physical relevance, time dependent backgrounds in string theory have been hard to come by:

- String perturbation technology is well suited for S-matrix computations around a fixed coherent vacuum state
- Time dependence implies production of squeezed states, and ambiguous choice of vacuum: is string field theory mandatory ?
- First quantized string theory requires analytic continuation of target space and world sheet, often problematic
- String theory is not content on a finite time interval, but often forces us into Big Bang / Big Crunch singularities, CTC, ...

## *Milne Universe, the bright side*

These issues may be easier to address in solvable CFT such as orbifolds of flat space, WZW, ...:

- The Lorentzian orbifold: $R^{1,1}/Z$, $X^{\pm} \equiv X^{\pm} e^{\pm\beta}$ describes the Milne universe, ie a circle linearly contracting and reexpanding in time:

  $$ds^2 = -dT^2 + T^2 dy^2 \ , \quad y \equiv y + \beta$$

  together with Rindler whiskers with CTC:

  $$ds^2 = -r^2 d\eta^2 + dr^2 \ , \quad \eta \equiv \eta + \beta$$

  and non-Hausdorff light cone, all attached at a cosmological singularity

  *Horowitz Steif; Seiberg; Nekrasov*

- This same singularity arises locally in many other examples: $SU(2) \times Sl(2)/U(1) \times U(1)$, $Sl(2)/U(1)$, $AdS/\Gamma$, ... yet it is far from generic: Kasner singularity, BKL oscillatory behavior

  *Nappi Witten, Elitzur Giveon Kutasov Rabinovici*
  *Craps Kutasov Rajesh*

- Variants have been proposed which ought to simplify things even further: electric Melvin universe, null boost, ...

  *Costa Cornalba Kounnas; Liu Moore Seiberg; ...*

## *Milne Universe, the dark side*

Unfortunately, many difficulties lie in the shade:

- Tree level scattering amplitudes of untwisted states are badly divergent, due to large graviton exchange near the singularity, signaling large backreaction

  *Berkooz Craps Kutasov Rajesh, LMS*

- Physical states do not seem to exist in twisted sectors, hence may not condense and resolve the singularity

  *Nekrasov*

- Non perturbatively, interactions of an incoming particle with its orbifold images will lead to black hole formation and gravitational collapse

  *Horowitz, Polchinski*

## *Moving D-branes, electric fields*

Open strings offer a simpler arena to investigate time-dependence without dealing with gravitational instabilities:

- The closest open string analogue of the Milne Universe is a head-on D-brane collision. Analogues of twisted closed strings are open strings stretched between D-branes. *The cosmological singularity issue is replaced by bound state formation...*

  *Bachas; Douglas Kabat Pouliot Shenker*

- Even simpler, the T-dual situation is open strings in a constant electric field:

$$F_{+-} = \begin{pmatrix} E & \\ & -E \end{pmatrix}$$

  11 and 22 strings are like untwisted closed strings, 12 and 21 are like winding strings. *Space-time non-commutativity may cause us some trouble...*

  *Bachas Porrati; Bachas Hull*

## *Particle production and backreaction*

Physically, the fate of these backgrounds is rather clear:

- For open strings in a constant electric field, Schwinger pair production of charged open strings will screen the electric field, or discharge the condensator at infinity.
- For moving D-branes, stretched strings will be pair produced, and will accelerate/slow down the collision. The final outcome depends on the production/recombination rates.
- Q1: *Can backreaction be consistently incorporated in open string theory ?*
- *Q2: Can pair production resolve the Milne singularity ?*

Q1 and Q2 are still too hard, but we'll answer

- Q0: *Does there exist physical states in charged or twisted sectors which are liable to condense in pairs ? What are the rules to compute their scattering amplitudes ?*

## *Electric field vs Milne Universe*

- Eigenmodes of closed strings twisted sector of order $w$ are free fields satisfying

$$X^{\pm}(\sigma+2\pi,\tau)=e^{\pm\nu}X^{\pm}(\sigma,\tau)\ ,\quad \nu=w\beta$$

hence the normal mode expansion:

$$\begin{aligned} X_R^{\pm}(\tau-\sigma) &= \frac{i}{2}\sum_{n=-\infty}^{\infty}(n\pm i\nu)^{-1/2}\alpha_n^{\pm}e^{-i(n\pm i\nu)(\tau-\sigma)} \\ X_L^{\pm}(\tau+\sigma) &= -\frac{i}{2}\sum_{n=-\infty}^{\infty}(-n\mp i\nu)^{-1/2}\tilde{\alpha}_n^{\pm}e^{-i(-n\mp i\nu)(\tau+\sigma)} \end{aligned}$$

with canonical commutation relations

$$\begin{aligned} [\alpha_m^+,\alpha_n^-] = -(m+i\nu)\delta_{m+n} \quad &, \quad [\tilde{\alpha}_m^+,\tilde{\alpha}_n^-]=(m+i\nu)\delta_{m+n} \\ (\alpha_m^{\pm})^* = \alpha_{-m}^{\pm} \quad &, \quad (\tilde{\alpha}_m^{\pm})^*=\tilde{\alpha}_{-m}^{\pm} \end{aligned}$$

- Charged open strings are obtained by identifying

$$\alpha_n^{\pm}=\tilde{\alpha}_{-n}^{\pm}\ ,\quad \nu=\frac{2}{\pi}\text{arctanh}(E/\pi)\ ,$$

and adding a canonical pair of constant zero modes $x_0^{\pm}$ with $[x_0^+,x_0^-]=i\pi/E$

- In particular, zero-modes are isomorphic (after rescaling), and involve two commuting pairs of hermitian conjugate variables,

$$[\alpha_0^+,\alpha_0^-]=-i\nu\ ,\quad [\tilde{\alpha}_0^+,\tilde{\alpha}_0^-]=i\nu\ ,$$

## *Are there twisted physical states ?*

- Representing these oscillators on a Fock space with vacuum $|0\rangle$ annihilated by all $\alpha^{\pm}_{n>0}$ and by $\alpha_0^-$ , the normal ordered worldsheet Hamiltonian reads

$$\begin{aligned} L_0 &= -\sum_{n=1}^{\infty}(\alpha_n^+)^*\alpha_n^- -\sum_{n=1}^{\infty}(\alpha_n^-)^*\alpha_n^+ \\ &\quad -\alpha_0^+\alpha_0^- + \frac{1}{2}i\nu(1-i\nu) - 1 + L_{int} \end{aligned}$$

  with a similar answer for $\tilde{L}_0$.

- Due to the $i\nu/2$ term in the ground state energy, all states obtained by acting on $|0\rangle$ by creation operators $\alpha^{\pm}_{n<0}$ and by $\alpha_0^+$ will have imaginary energy, hence the physical state condition $L_0 = 0$ has no solutions.

*Nekrasov*

- Rk: this does not contradict $L_0$ being hermitian, since these states also have zero norm !

## *One-loop amplitude*

- Irrespective of this, the one-loop vacuum energy for open strings in electric fields can be computed,

$$A_{open} = i\pi V_{26} E \int_0^\infty \frac{dt}{(4\pi^2 t)^{13}} \frac{e^{-\pi\nu^2 t/2}}{\eta^{21}(it/2)\ \theta_1(t\nu/2; it/2)}$$

$$\theta_1(v,\tau) = 2q^{1/8} \sin \pi v \prod_{n=1}^{\infty} (1-e^{2\pi i v} q^n)(1-q^n)(1-e^{-2\pi i v} q^n)$$

  Poles at $t = 2k/\nu, k \in Z$ contribute to the imaginary part, which agrees with the sum of Schwinger production rates for each particle in the spectrum:

$$w = \frac{1}{2(2\pi)^{25}} \frac{2E}{\nu} \sum_{k=1}^{\infty} (-)^{k+1} \left(\frac{|\nu|}{k}\right)^{13} \sum_{N=-1}^{\infty} c_b(N) \exp(-2\pi k \frac{N}{|\nu|} - \pi k |\nu|)$$

  where $\eta^{-24}(q) = \sum_{N=-1}^{\infty} c_b(N) q^N$.

  *Bachas Porrati*

- Similarly, for closed strings on Milne space one obtains a modular invariant integral,

$$A_{closed} = \int_{\mathcal{F}} \sum_{l,w=0}^{\infty} \frac{d\tau d\bar{\tau}}{(2\pi^2\tau_2)^{13}} \left| \frac{e^{-2\pi\lambda^2 w^2 \tau_2}}{(2\pi)^3 \eta^{21}(\tau)\ \theta_1(i\lambda(l+w\tau);\tau)} \right|^2$$

  in agreement with the proposed treatment of zero modes.

  *Nekrasov, Cornalba Costa*

## *Space-time representation of zero-modes*

- Zero-modes can be unitarily represented as covariant derivatives acting on wave functions $f(x^+, x^-)$ of the center of motion of the charged string,

$$\alpha_0^\pm = i\partial_\mp \mp \frac{\nu}{2} x^\pm \ , \quad \tilde{\alpha}_0^\pm = i\partial_\mp \pm \frac{\nu}{2} x^\pm$$

- The zero-mode piece of $L_0$, including the evil $\frac{i\nu}{2}$,

$$L_0^{(0)} = -\alpha_0^+ \alpha_0^- + \frac{i\nu}{2} = -\frac{1}{2}(\alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+)$$

  is just the Klein-Gordon operator of a particle of 2D mass $M^2 = -2L_0^{(0)}$ and charge $\nu$.

- For open strings, only one pair $(\alpha_0^+, \alpha_0^-)$ appears in the worldsheet Hamiltonian $L_0$. The other one $(\tilde{\alpha}_0^+, \tilde{\alpha}_0^-)$ describes the position of center of the hyperbolic trajectory, subject to the uncertainty principle.

- For closed strings, the difference $L_0^{(0)} - \tilde{L}_0^{(0)}$ is the zero-mode boost momentum,

$$\mathcal{M}^2 - \tilde{\mathcal{M}}^2 = -i\nu \left( x^+ \partial_+ - x^- \partial_- \right) := J^{(0)}$$

  The matching condition equates

$$\beta w J = N_L - N_R$$

  In addition, the orbifold projection requires the total boost momentum $J$ to be integer.

## *KG and the inverted harmonic oscillator*

- Defining $\alpha_0^\pm = (P \pm Q)/\sqrt{2}$ and same with tildas, the Klein-Gordon operator just becomes an inverted harmonic oscillator:

$$M^2 = \alpha_0^+ \alpha_0^- + \alpha_0^- \alpha_0^+ = \frac{1}{2}(P^2 - Q^2)$$

- Diagonalizing ($\tilde{P}$, $M^2$) in $P$ representation:

$$f(x^+, x^-) = \int d\tilde{p}\ \psi_{\tilde{p}}(u) e^{-i(\tilde{p}+\frac{1}{2}\nu x)t}$$

  where $u = (\tilde{p} + \nu x)\sqrt{2/\nu} \propto P$. Then $\psi_{\tilde{p}}(u)$ is a eigenmode of the inverted harmonic oscillator,

$$\left(-\partial_u^2 - \frac{1}{4}u^2 + \frac{M^2}{2\nu}\right)\psi_{\tilde{p}}(u) = 0$$

- The latter admits a respectable delta-normalizable spectrum of scattering states, in terms of parabolic cylinder functions.

*Moore*

- These correspond to non-compact trajectories of charged particles in the electric field. Waves on the right side of the potential describe electrons, waves on the left side describe positrons. Tunnelling is just (stimulated) Schwinger pair creation,

$$e^- \to (1 + \eta)\ e^- + \eta\ e^+$$

## *In/out vacua and particle production*

- Combinations which create an incoming electron or positron are

$$\begin{aligned}\phi_{in}^{+} &= D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-\frac{3i\pi}{4}}u)e^{-i\tilde{p}t}e^{i\nu xt/2} \ , \\ \phi_{in}^{-} &= e^{-\frac{3i\pi}{4}}D_{-\frac{1}{2}+i\frac{M^2}{2\nu}}(e^{-i\pi/4}u)e^{i\tilde{p}t}e^{-i\nu xt/2} \ ,\end{aligned}$$

  while outgoing electron or positrons are created by

$$\phi_{out}^{\pm}(t,x) = [\phi_{in}^{\pm}(-t,x)]^*$$

- The in and out vacua are related by a non-trivial Bogolioubov transformation

$$|0,in\rangle = \mathcal{N} \exp\left[-\frac{1}{\sqrt{2\pi}}\Gamma\left(\frac{1}{2}+i\frac{M^2}{2\nu}\right)e^{-\frac{\pi M^2}{2\nu}}\right] a_{\tilde{p}}^* b_{\tilde{p}}^* |0,out\rangle$$

  with overlap

$$|\langle 0,out|0,in\rangle|^2 = \exp\left[-\int d\tilde{p} \ln(1+e^{-\pi M^2/\nu})\right]$$

  which reproduces the Schwinger pair creation rate, after properly interpreting $\int d\tilde{p}$.

*Brout Massar Parentani Spindel*

## *Lorentzian vs Euclidean states*

- Analytic continuation $X^0 \to e^{-i\pi/2} X^0 \;, \nu \to e^{i\pi/2}\nu$ takes us from an electric field in $R^{1,1}$ to a magnetic field in $R^2$. At the same time, one should Wick rotate the worldsheet time.

- Under Wick rotation, discrete states with imaginary energy $i\nu(n+1/2)$ become normalizable states of the $\cup$ harmonic oscillator, ie Landau states in a magnetic field.

- Instead, the continuous spectrum of the $\cap$ harmonic oscillator turns into non-normalizable states of the $\cup$ harmonic oscillator.

- The heat kernel of $\cup$ can be expressed as a sum over discrete eigenmodes

$$\sum_{n=0}^{\infty} e^{i(n+1/2)s} \psi_n^*(u_1)\psi_n(u_2) = \frac{1}{\sqrt{4\pi \sin s}} \exp\left[-\frac{1}{4}\left(\frac{u_1^2+u_2^2}{\tan s} - \frac{2u_1u_2}{\sin s}\right)\right]$$

but as the Schwinger parameter gets continued, becomes an integral over the continuous spectrum:

$$\sum_{\epsilon=\pm} \int_{-\infty}^{\infty} dM^2 e^{iM^2 s} \psi^{\epsilon}(M^2, u_1)\psi^{\epsilon}(M^2, u_2)$$

$$= \frac{1}{\sqrt{4\pi i \sinh s}} \exp\left[\frac{i}{4}\left(\frac{u_1^2+u_2^2}{\tanh s} - \frac{2u_1u_2}{\sinh s}\right)\right]$$

*Moore*

## *Physical spectrum at low level*

- The ground state tachyon

$$|T\rangle = \phi(x^+, x^-)|0_{ex}, k\rangle$$

should satisfy the Virasoro constraint

$$L_0|T\rangle = \left[-\frac{1}{2}\left(\alpha_0^+\alpha_0^- + \alpha_0^-\alpha_0^+\right) + \frac{1}{2}\nu^2 - 1 + \frac{1}{2}k_i^2\right]|T\rangle$$

which is the two-dimensional KG equation, for a particle with mass $k_i^2 + \nu^2 - 2$ and charge $\nu$.

- Level 1 states consist of

$$|A\rangle = \left(-f^+\ \alpha_{-1}^- - f^-\ \alpha_{-1}^+ + f^i\ \alpha_{-1}^i\right)|0_{ex}, k\rangle$$

with the mass shell conditions

$$[M^2 - k_i^2 - \nu^2]f^i = 0\ , \quad [M^2 - k_i^2 - \nu^2 \mp 2i\nu]f^\pm = 0$$

The $L_1$ Virasoro constraint eliminates one polarization

- Despite the non-vanishing two-dimensional mass $k_i^2 - \nu^2$, the spurious state $L_{-1}\phi|0\rangle$ is still physical, eliminating an extra polarization. One thus has $D-2$ transverse degrees of freedom, ie a massless gauge boson in $D$ dimensions.

- For closed strings, the same goes through, with a further projection to $J = 0$: there is a zero-momentum transverse graviton in each twisted sector.

## *Quantizing charged KG - 2) light-cone*

- In $\alpha^-$ representation, then $\alpha^+ = i\nu\partial/\partial\alpha^-$, hence $M^2 = i\nu\alpha^-\partial/\partial\alpha^-$ is just a rescaling operator. Eigenmodes are power law

$$f_{k_+}(x^+, x^-) = (2\nu)^{-\frac{1}{4}+\frac{iM^2}{4\nu}}\,\Gamma\left(-\frac{1}{4} - \frac{iM^2}{4\nu}\right)(k_+ + \nu x^-)^{-\frac{1}{2}+\frac{iM^2}{2\nu}} e^{ix^+(k_+ + \frac{1}{2}\nu x^-)}$$

  This basis of functions is most appropriate to expand the modes at a fixed (early) $x^-$ time, i.e. for incoming electrons.

- Alternatively, one may diagonalize $\tilde{\alpha}^+$: this is appropriate to relate outgoing electrons. The S-matrix relating the two basis is just Fourier transform.

- Canonical commutation relations read

$$\begin{aligned}
\left[a_{k_+}, a^*_{k'_+}\right] &= \text{sgn}\,(k_+ + \nu x^-)(2\nu)^{-1/2}|\Gamma(-\frac{1}{4} - \frac{iM^2}{4\nu})|^2 \delta(k_+ - k'_+) \\
\left[a_{k_-}, a^*_{k'_-}\right] &= \text{sgn}\,(k_- - \nu x^+)(2\nu)^{-1/2}|\Gamma(-\frac{1}{4} + \frac{iM^2}{4\nu})|^2 \delta(k_- - k'_-)
\end{aligned}$$

  hence at time $k_+ + \nu x^- = 0$, creation operator becomes annihilation: this is pair creation.

  *Tomaras Tsamis Woodard*

- In order to maintain charge conservation, one should also include the incoming and out-going positron modes on the other side of the wedge.

# *Charged KG in Rindler space*

- For applications to the Milne universe, one should rather diagonalize the boost momentum $J$, ie consider an accelerated observer.

  *Gabriel Spindel; Mottola Cooper*

- In the Rindler patch R, letting $f(r,\eta) = e^{-iJ\eta} f_J(r)$ and $r = e^y$, one gets a Schrodinger equation for a particle in a potential

$$\left[ -r\partial_r r \partial_r + M^2 r^2 - (J + \frac{1}{2}\nu\ r^2)^2 \right] f_J(r) = 0$$

- For $\nu = 0$, this reduces to a Liouville wall. For $\nu \neq 0$ the wall has finite height, and the potential is unbounded from below at large $r$.

- For $\nu J/M^2 > 1/2$, the energy is bigger than the barrier, the electron comes from infinity $I_R^-$ into the horizon $H_R^+$. For $\nu J/M^2 < 1/2$, the electron bounces off the barrier back to $I_R^+$. Again, tunnelling corresponds to stimulated pair emission from or into the horizon.

- Incoming and outgoing Rindler/Unruh modes can be defined as usual. The Bogolioubov transformation shows Schwinger pair creation in the bulk, as well as thermal particle production from the horizon at $T_R = a/(2\pi) = \nu/(2\pi M)$.

## *Vertex operators and correlation functions*

- Tree level correlators with at most 2 twisted sector states can be obtained on the Minkovskian strip, otherwise analytic continuation is mandatory.

- Upon Wick rotation to the Euclidean magnetic problem, physical states in the charged/twisted sectors become non-normalizable modes of the harmonic oscillator. Don't panic, this eliminates the localized tachyon !

- Vertex operators for the usual magnetic problem are constructed out of (boundary) twist fields, which generate cuts in $(Z, \bar{Z})$,

$$\partial Z(w)\sigma_\theta(0) = w^\theta \sigma_\theta^{(1)} + \dots \ , \quad \Delta_{\sigma_\theta} = \theta(1-\theta)/2$$

*Dixon Friedan Martinec Shenker; Hamidi Vafa*

- These standard twist fields correspond to the harmonic oscillator ground state and its excitations. Instead, we need a new continuous family of twist fields with irrational angle.

- For $\beta \in \mathbb{Q}$, there should still be an infinite number of twisted sectors: spectral flow / long strings.

## *Twisted state production in Milne*

- By analogy with electric field, one expects Schwinger creation of correlated pairs of twisted states on the Milne orbifold. Due to orbifold projection, lightest states will be produced in $J=0$ states, hence breaking spatial homogeneity.

- For a given state of mass $M$, the density of pair creation can be evaluated semi-classically be computing the Jacobian

$$w(X^+,X^-) = \int dx_0^+ dx_0^- \delta[(X^+ - x_0^+)(X^- - x_0^-) + \frac{M^2}{2\epsilon^2}]$$
$$\delta[\epsilon x_0^+ x_0^- + \frac{M^2}{2\epsilon^2} - J] = \frac{1}{\sqrt{X^+X^-(\epsilon^2 + 2M^2 X^+ X^-)}}$$

  hence pair production is peaked on the light-cone.

- Unfortunately, the total pair creation number diverges, due to summing over winding sectors. Electric Melvin universe seems better off. . .

- A more tractable situation perhaps is to send in squeezed twisted states from $-\infty$, along with states one is interested to scatter. Can one use non-local worldsheet deformation ?

*Aharony Berkooz Silverstein*

## *Conclusions - speculations*

- Open strings in an electric field are a simple arena to address conceptual issues of Time and string theory.

- We have laid some of the foundations to study this, but the hard work is ahead of us: S-matrix elements, recombination rate, backreaction...

- Despite strong formal analogies, there is no duality between open strings in electric fields and the Milne universe.

- Time-space non-commutativity however may still be important on black hole horizons, as 't Hooft taught us.

- Analogies with $c = 1$ strings are tantalizing, although open strings do not obey Fermi statistics. *Can one define some double scaling limit towards the top of the potential ? can squeezed states be dealt with by some bosonization technique ?*

*Gose cho domo arigato gozaimashita*
(Thank you very much for your attention)

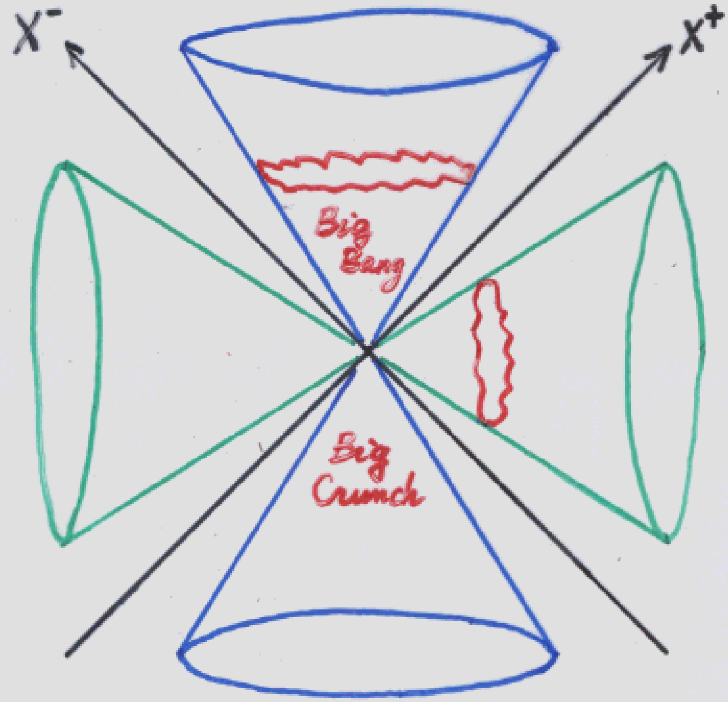
*Pari ni mata rainen*
(See you in Paris next year)

## Lorentzian orbifold

$X^\pm \sim X^\pm e^{\pm\beta}$

### Milne Universe

$$ds^2 = -dT^2 + T^2 d\theta^2 \,, \quad \theta \sim \theta + \beta$$




### Rindler whisker

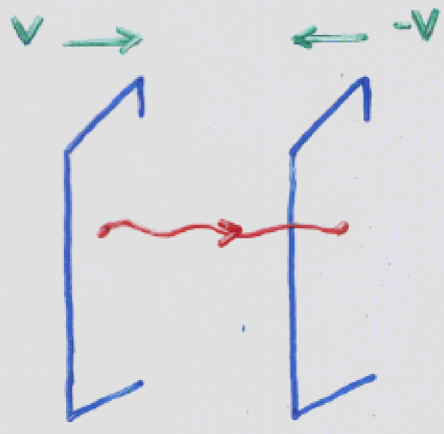
$$ds^2 = -r^2 d\eta^2 + dr^2$$

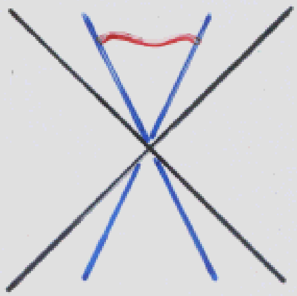
$$\eta \to \eta + \beta$$

CTC

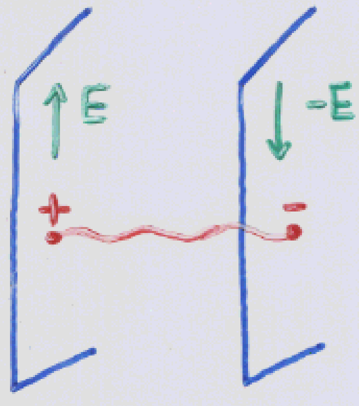
$$X^\pm = T e^{\pm\theta}/\sqrt{2} = \pm r e^{\pm\eta}/\sqrt{2}$$

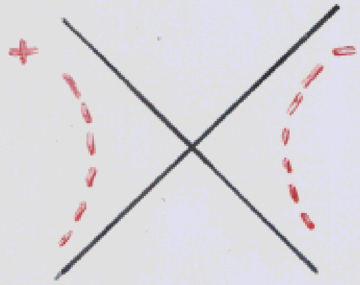
## D-brane collision:

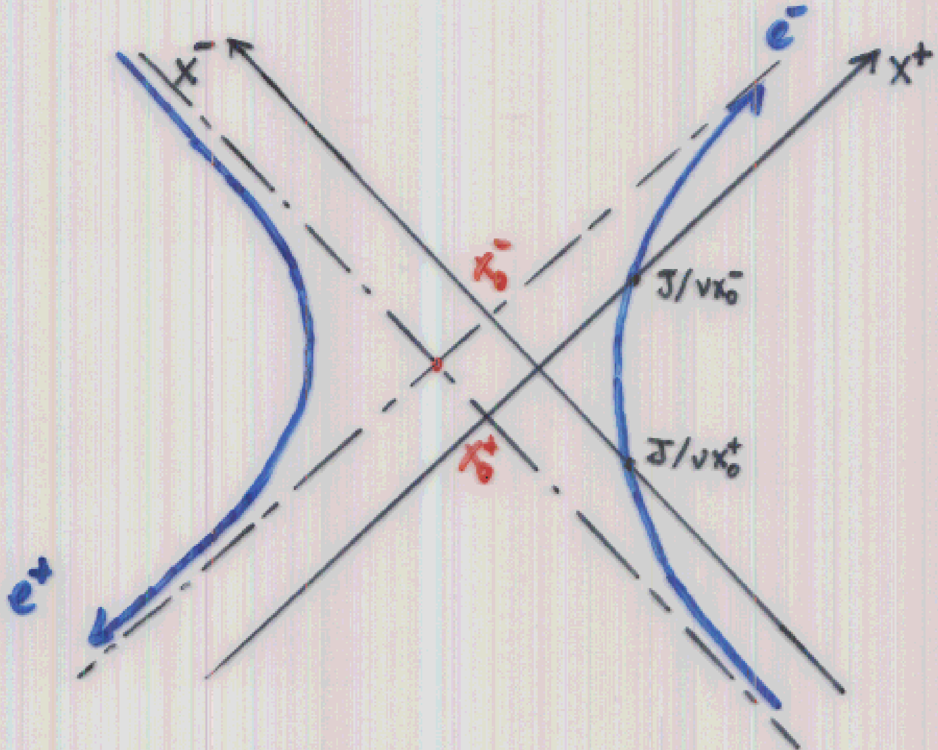


## Electric field:



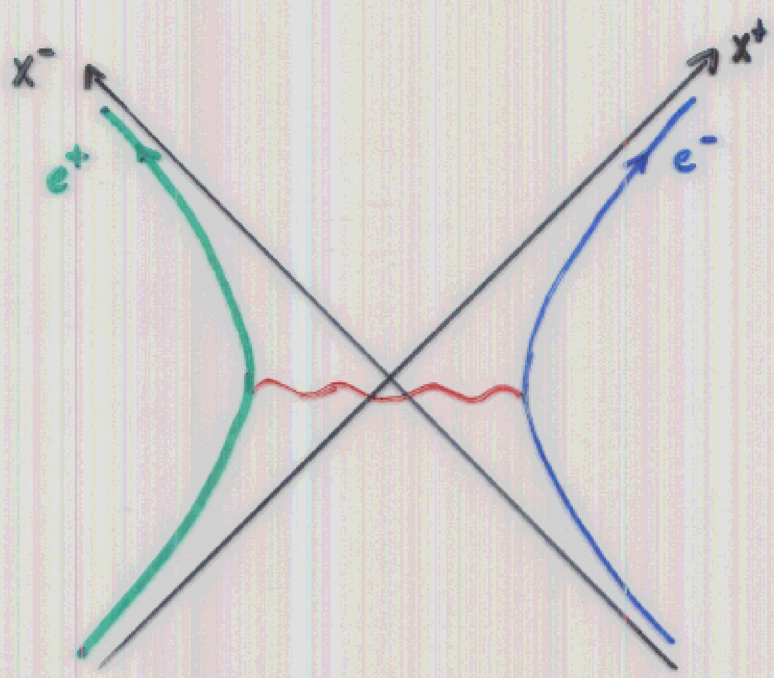



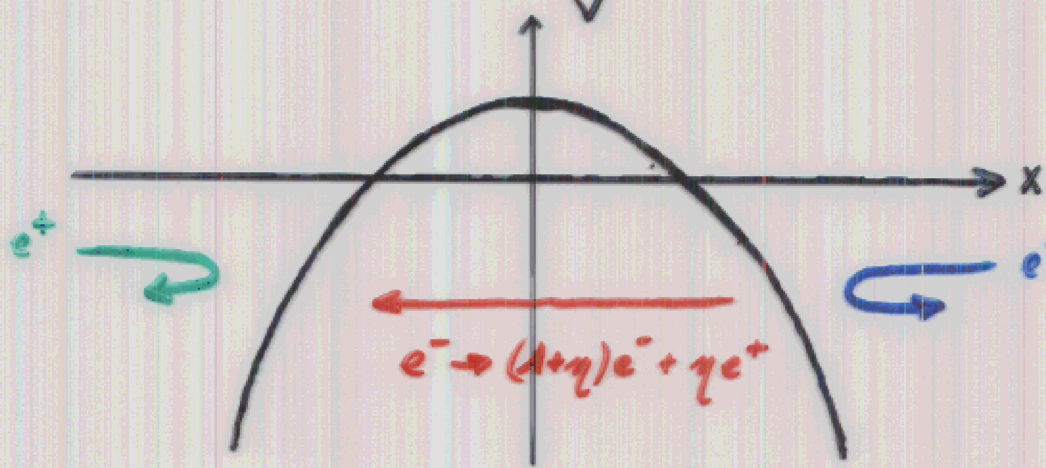



$$(X^+ - x_0^+)(X^- - x_0^-) + \frac{M^2}{2\nu^2} = 0$$

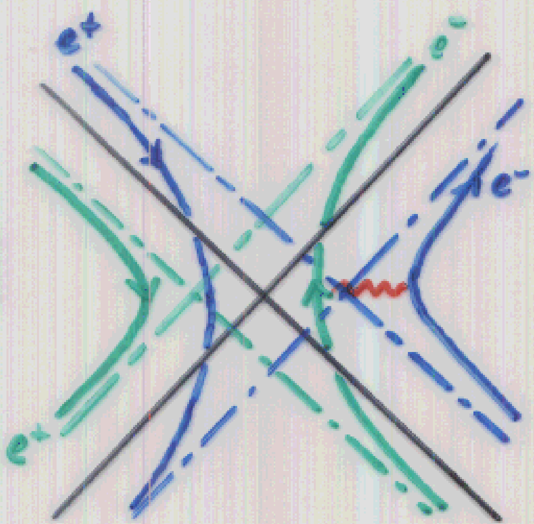
$$P^\pm = \mp \frac{\varepsilon}{2} x_0^\pm \qquad [P^+, P^-] = i\nu$$

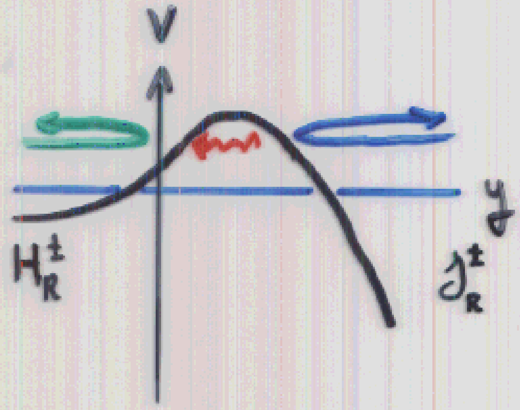
$$J = X^+\pi^- - X^-\pi^+ = \nu\, x_0^+ x_0^- + \frac{M^2}{2\nu}$$


x⁻
x⁺
e⁺
e⁻


V
x
e⁺
e⁻
e⁻ → (1+η)e⁻ + ηe⁺

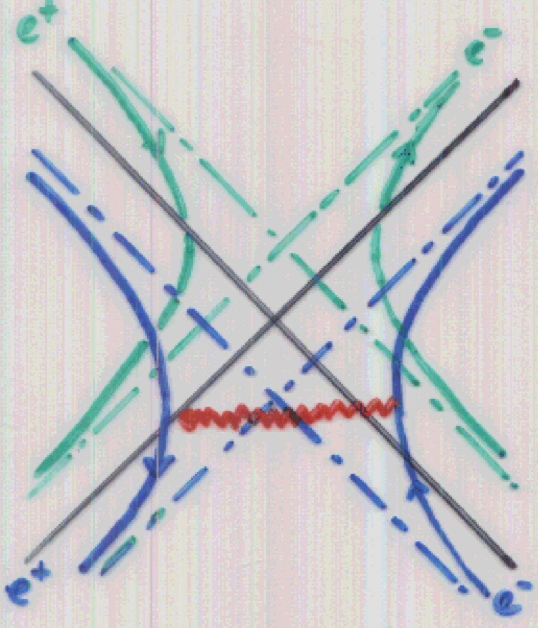
$0 < \frac{\nu J}{M^2} < \frac{1}{2}$

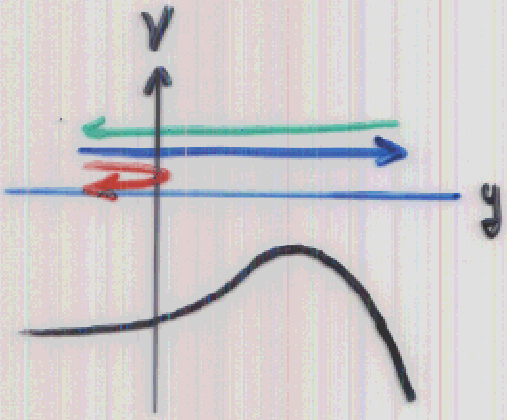



$r = e^y$

$\frac{1}{2} < \frac{\nu J}{M^2}$




$V = M^2 e^{2y} - (J + \frac{1}{2}\nu e^{2y})$

R patch:

$J < 0$