A New Hat For The $c = 1$ Matrix Model

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See also T. Takayanagi and N. Toumbas

Noncritical string theory:

- Worldsheet: Liouville ϕ + matter x
- Spacetime: Dynamics in two dimensions
- Matrix model: Eigenvalues λ are fermions. They depend on time x.

λ is not the same as ϕ. They are related through a nonlocal transform (Moore and N.S.).

What about the type 0A and 0B theories in $d \leq 2$ ($\hat{c} \leq 1$)?

Conjecture: It is similar to $c \leq 1$ matrix model but the two sides of the potential are filled.

It is nonperturbatively stable.

Spectrum of $\widehat{c} = 1$

NS tachyon is massless

$$T_{NS}(k) = \exp\left[ikx + (1 - |k|)\phi\right]$$

(we suppress the superghosts).

The zero momentum tachyon is the worldsheet "cosmological constant"

$$\mu \int d\theta d\widetilde{\theta} e^{\Phi(\theta,\widetilde{\theta})}$$

(the cosmological constant is μ^2).

In the 0B theory another massless particle from the RR sector

$$T_R(k) = \exp\left[-s\frac{i}{2}(H + \overline{H}) + ikx + (1 - |k|)\phi\right]$$

$s = \text{sign}(k)$

Global $\mathbb{Z}_2$ symmetry $(-1)^{F_L}$ ($F_L =$ left moving spacetime fermion number) acts as -1 on all the RR states.

Orbifolding by this symmetry leads to the 0A theory in which the RR scalar is absent. Twisted sector: two gauge fields – no particles.

Sphere correlation functions

Using Liouville theory Di-Francesco and Kutasov computed the tree level S-matrix of these particles. In terms of

$$A_{L,R}(k) = \frac{1}{2}\left(f(k)T_{NS}(k) \pm g(k)T_R(k)\right)$$

($f(k)$ and $g(k)$ are functions of k – "leg factors") the result is

$$\langle \prod_{i=1}^{n} A_L(k_i) \prod_{j=1}^{m} A_R(p_j)\rangle = 0 \;\; (n, m \geq 1)$$
$$\langle \prod_{i=1}^{n} A_L(k_i)\rangle = \langle \prod_{i=1}^{n} A_R(k_i)\rangle = \langle \prod_{i=1}^{n} T(\sqrt{2}k_i)\rangle_B$$

where $\langle\prod_{i=1}^{n} T(\sqrt{2}k_i)\rangle_B$ is the answer in the bosonic $c = 1$ model.

The tree level correlation functions are the same as in two decoupled $c = 1$ systems with α' rescaled by 2.

This is consistent with our conjecture:

A_L (A_R) are perturbations of the left (right) Fermi sea.

T_{NS} (T_R) are even (odd) perturbations of the two components of the Fermi sea.

Ground Ring

Additional physical vertex operators exist because of null vectors in (super) Virasoro representations.

Bosonic $c = 1$

There are operators which are inserted at points on the worldsheet (not integrated). They form a ring which is generated by

$$
\begin{aligned}
a =& (cb + \partial\phi + i\partial x)(\bar{c}\bar{b} + \bar{\partial}\phi + i\bar{\partial}x)e^{ix-\phi} \\
\bar{a} =& (cb + \partial\phi - i\partial x)(\bar{c}\bar{b} + \bar{\partial}\phi - i\bar{\partial}x)e^{-ix-\phi}
\end{aligned}
$$

Witten interpreted their Lorentzian version

$$
\begin{aligned}
a =& (q+p)e^{-t} \\
\bar{a} =& (q-p)e^{t} \\
H =& p^2 - q^2 = -a\bar{a}
\end{aligned}
$$

as the phase space variables and the Hamiltonian of the matrix model.

Using free field theory, $a\bar{a} = 0$, but including the Liouville interaction the ring relation is deformed

$$a\bar{a} = \mu$$

It is interpreted as the Fermi surface of the matrix model.

For simplicity, set $\mu = 0$ (in the paper $\mu \neq 0$).

The tachyon of the bosonic string leads to modules of the ring

$$aT(k) = \begin{cases} k^2 T(k+1) & k > 0 \\ 0 & k < 0 \end{cases}$$
$$\overline{a}T(k) = \begin{cases} k^2 T(k-1) & k < 0 \\ 0 & k > 0 \end{cases}$$

Interpret: the tachyon is a ripple on the Fermi surface.

Type 0B $\hat{c} = 1$

The ring is generated by RR fields

$$a = e^{\frac{i}{2}(H+\overline{H})+\frac{i}{2}x-\frac{1}{2}\phi} + \ldots$$
$$\overline{a} = e^{-\frac{i}{2}(H+\overline{H})-\frac{i}{2}x-\frac{1}{2}\phi} + \ldots$$

(we suppress the superghosts).

Again, we find the ring relation

$$a\overline{a} = \mu$$

The two independent fields $A_L(k)$ and $A_R(k)$ lead to separate modules of the ring.

Interpret: $A_{L,R}$ are deformations of the two separate components of the Fermi surface.

Unstable D-branes

In the bosonic string the open string tachyon potential is bounded from below in one side, but unbounded from below in the other side. This is exactly as in the one cut matrix model.

In type 0 theory the potential is bounded in both sides. This is exactly as in the two cut matrix model.

Matrix model = gauge theory of the unstable D-branes.

One loop partition function

Compactify the type 0 theory on a circle of radius R.

Following Sakai, Tanii, Bershadsky and Klebanov we use free field theory to evaluate the torus amplitude in Liouville theory. The three even spin structures are straightforward

$$\mathcal{Z}_{even} = -\frac{\ln \mu}{8\sqrt{2}} \left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R} \right)$$

The odd spin structure is more subtle: ψ_x, ψ_L, β and γ zero modes.

The γ zero mode cancels the ψ_L zero mode (Killing spinor). The β zero mode leads to an insertion of the supercharge. It absorbs the ψ_x zero mode.

Adding the right movers, the odd spin structure amplitude can be expressed in terms of the bosonic string amplitude

$$\begin{aligned} \mathcal{Z}_{odd} &\sim \langle \partial x \overline{\partial} x \rangle_B \sim R \frac{\partial}{\partial R} \mathcal{Z}_{Bosonic} \\ &\sim R \frac{\partial}{\partial R} \left(\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R} \right) \sim \left(\frac{R}{\sqrt{\alpha'}} - \frac{\sqrt{\alpha'}}{R} \right) \end{aligned}$$

We can fix the normalization by matching with the target space field theory calculation at large R (the coefficient of $\frac{1}{R}$).

We conclude

$$\mathcal{Z}_{0A} = -\frac{\ln\mu}{12\sqrt{2}}\left(2\frac{R}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}}{R}\right)$$

$$\begin{aligned}\mathcal{Z}_{0B} &= -\frac{\ln\mu}{12\sqrt{2}}\left(\frac{R}{\sqrt{\alpha'}} + 2\frac{\sqrt{\alpha'}}{R}\right)\\ &= -\frac{\ln\mu}{12}\left(\frac{R}{\sqrt{2\alpha'}} + \frac{\sqrt{2\alpha'}}{R}\right)\end{aligned}$$

- T-duality: $R \leftrightarrow \frac{\alpha'}{R}$ exchanges $0A \leftrightarrow 0B$.

- $\mathcal{Z}_{0B}(\alpha') = 2\mathcal{Z}_{Bosonic}(2\alpha')$ as expected from the matrix model.

- Using our conjecture and the known exact vacuum amplitude in the two cut matrix model we know the exact vacuum amplitude in the type 0 theory.

$\widehat{c} < 1$

Consider the single matrix model in a two cut phase. Its critical behavior is the same as that of the unitary matrix model (Gross and Witten).

Unlike $\widehat{c} = 1$, here the eigenvalues in the two cuts interact at the leading order in $\frac{1}{N}$ − the sphere.

For even potentials the theory has a series of multicritical points labeled by an integer k and described by the mKdV hierarchy (Periwal and Shevitz). They were conjectured to be related to superminimal models coupled to supergravity (N.S. and E. Witten, unpublished; Crnkovic, Douglas and Moore).

By analogy to the one cut models of the bosonic string:

- They are identified with the $(p = 2, q = 4k)$ superminimal models coupled to supergravity.

- The first critical point $k = 1$ is the Gross-Witten transition. It is identified with 2d supergravity.

- The $\mathbb{Z}_2$ even (odd) matrix model operators correspond to NS (RR) operators in the worldsheet theory.

- The "scale" is set by the lowest dimension NS operator (not the cosmological constant).

- The critical exponents of the matrix model match with those computed in Liouville theory.

- The RR ground state operator is absent because it cannot be "dressed" by Liouville. The corresponding Liouville wave function is normalizable.

- The $\mathbb{Z}_2$ odd operator t_{k-1} is redundant in the matrix model. It is identified with the boundary cosmological constant in the worldsheet.

- The torus amplitude $-\frac{2k+1}{24k}$ is the same in Liouville and mKdV.

Using the matrix model we can solve the theory perturbed by RR deformations. These are not supersymmetric field theories coupled to 2d supergravity.

They flow to new critical points which cannot be described by superconformal field theories coupled to supergravity.

mKdV corresponds to half of the "Hamiltonians" in the ZS hierarchy. These critical points are described by the other half.