Strings from Tachyons

with J. McGreevy
J. Teschner

Strings 2003, Kyoto

Consider N D-particles in D dimensional (non-critical) string theory:

$$S = \int dt \, \mathrm{Tr}\left(V(Y) \left(1 - (D_t X^i)^2 + [X^i, X^j]^2 - (D_t Y)^2 + [Y, X^i]^2 + \,.\,.\, \right) \right)$$

i =1, . . , D-1

X are the D-particle positions, Y is the open string tachyon.

Hypothesis: S provides a matrix model for non-critical strings:

$$S_{ws} = \int d^2\sigma \left((\partial_\alpha X^\nu)^2 + (\partial_\alpha \varphi)^2 + QR^{(2)}\varphi + \mu e^{\varphi} \right)$$

ν = 0, . . , D-1

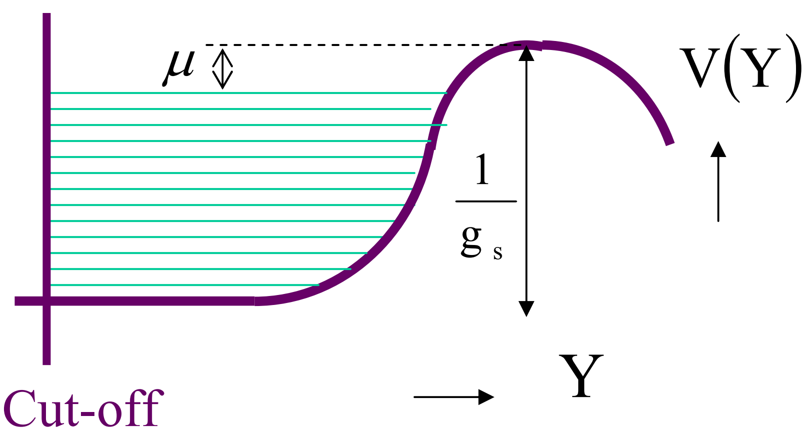
Special case: D=1

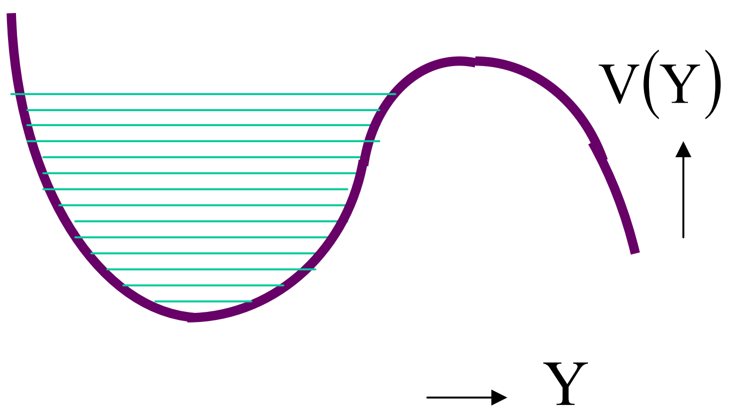
➡ The large N quantum mechanics of the matrix valued open string tachyon looks like the c=1 matrix model:

$$S(Y) = \int dt \; \mathrm{Tr}\left(V(Y) - \frac{1}{g_s}(D_t Y)^2\right)$$

$$D_t = \partial_t - [A_t, \;\;] \qquad\qquad V(Y) = \frac{1}{g_s}\left(1 - Y^2 + \;\ldots\right)$$

Y is a hermitian N x N matrix



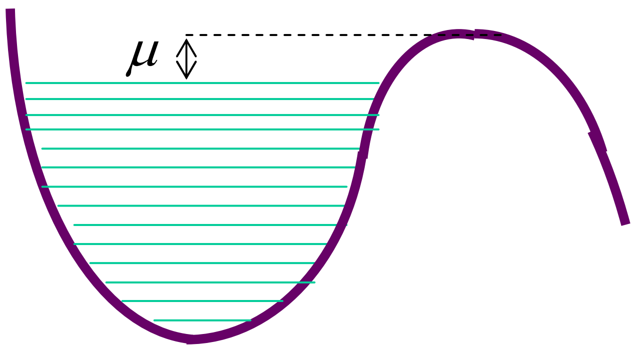


Via this interpretation, we can interpret the double scaling limit

$$N \to \infty, g_s \to 0 \qquad \mu \ \text{fixed}$$

as a decoupling limit. It is a special case of a large class of dualities between 2-D string theories:

$$(N, g_s) \cong (\tilde{N}, \tilde{g}_s) \qquad \text{if} \quad \mu = \tilde{\mu}$$



After diagonalization, the matrix quantum mechanics provides a wavefunction for the eigenvalues given by:

$$\Psi_N(\lambda,\tau)=\int^{\tau}\prod_{i=1,N} D\lambda_i \; e^{\frac{i}{\hbar}\int\sum(\dot{\lambda}_i^2 - V(\lambda_i))}\Delta_N(\lambda(\tau))$$

$$\Delta_N(\lambda)=\prod_{i<j}\left(\lambda_i-\lambda_j\right)$$ **Vandermonde determinant**

ψ satisfies:

$$i\hbar\partial_t\Psi=\hat{H}_N\Psi$$

with:

$$\hat{H}_N=\Delta_N H_N^{(0)}\Delta_N^{-1} \qquad H_N^{(0)}=\sum_{i=1,N}\left(p_i^2+V(\lambda_i)\right)$$

We want to add a probe eigenvalue:

$$(Y)_N \to \quad (Y)_{N+1} = \begin{pmatrix} \boxed{z} & v_1 & v_2 & .. & v_N \\ v_1^+ & \phi_{11} & \phi_{12} & .. & \phi_{1N} \\ v_2^+ & \phi_{21} & \phi_{22} & .. & \phi_{2N} \\ .. & .. & .. & .. & .. \\ v_N^+ & \phi_{N1} & \phi_{N2} & .. & \phi_{NN} \end{pmatrix}$$

We define the adiabatic wave function $\breve{\Psi}_N(\lambda; z(\tau))$ such that:

$$\boxed{\Psi_{N+1}(\lambda_i, z, \tau_0) = \int^{\tau_0} Dz \; e^{\frac{i}{\hbar}\int(\dot{z}^2 - V(z) + \hbar A_B \dot{z})} \; \breve{\Psi}_N(\lambda_i; z(\tau))}$$

A_B is a Berry phase.

The adiabatic wave function satisfies the Schrodinger equation with:

$$H_N^{(0)} \to H_N^{(0)} + H_{int}$$

$$H_{int} = [H_N^{(0)}, \Phi(z)] \qquad \Phi(z) = \mathrm{Tr}\log(Y - z)$$

$\Phi(z)$ = macroscopic loop operator

Liouville $\quad X^0$-CFT

$$\langle \mu + \omega | H_{int} | \mu \rangle = e^{i\delta(\omega) - i\omega t} \frac{\cos \pi s(t)}{\sinh \pi\omega} = \langle v_P | B_s \rangle \langle v_\omega | B_t^D \rangle_{P=\omega}$$

Neumann $\quad$ Dirichlet

$$\cosh \pi s(t) = \frac{z(t)}{\sqrt{\mu}}$$

Fateev, Zamolodchikov[2]; Teschner

Consider the operator that creates an eigenvalue at location (z,t):

$$\Psi(z,t) = \int^{t} Dz\ e^{\frac{i}{\hbar}\int^{t} d\tau\left(\dot{z}^2 - V(z) + \hbar\frac{d\Phi}{d\tau}\right)} = \underbrace{\psi(z,t)}_{\text{Wave function}}\ e^{i\Phi(z,t)}$$

This is the standard bosonization formula!

$$\Phi(z,t) = \int \frac{dp}{2\pi\sqrt{2p}}\ a_p^{+}\ \langle v_p | \underbrace{B_{zz}}_{\text{Dirichlet}} \rangle \langle v_\omega | \underbrace{B_\lambda}_{\text{Roll} = \lambda e^{x^o}} \rangle_{p=\omega}$$

Klebanov, Maldacena, Seiberg

We thus obtain the quantum descent relation:

$$\psi(z,t)\, e^{i\Phi(z,\tau)} = \int^{(z,\tau)} Dz(t)\; e^{\frac{i}{\hbar}S(z(t))}\, e^{-\frac{i}{\hbar}\int dt\; H_{int}(t)}$$

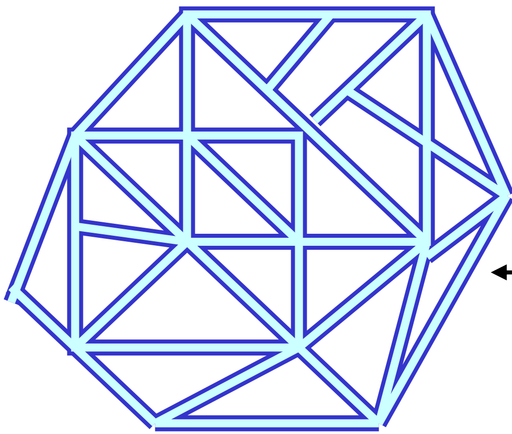
$$\Phi = \left|B_{ZZ}\right\rangle \left|B_{\lambda}\right\rangle \qquad\qquad H_{int}(t) = \left|B_{s}\right\rangle \left|B_{t}^{D}\right\rangle$$

Quantum

Classical

3) After a maximal T-duality, the D-dimensional matrix QM is mapped to D-dimensional YM+Higgs theory:

$$S_D = \int d^D x \;\; \mathrm{Tr}\left(F_{\mu\nu}^2 - \left(D_\nu Y\right)^2 + V\left(Y\right)\right)$$

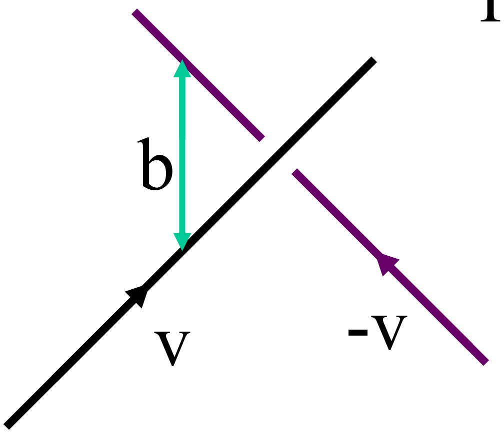


$$\frac{1}{k_\nu^2 + 1} \rightarrow e^{-k_\nu^2} \rightarrow e^{-(x_i^\nu - x_j^\nu)^2}$$

The (UV regularized) planar diagrams of this theory provide a dynamical triangulation+discretization of string worldsheets embedded in D+1 dimensions. (Ambjorn e.a., David, Kazakov e.a.)

Liouville

4) Suppose we use the matrix QM to compute the effective potential between two D-particles. We find:



$r^2 = b^2 + v^2 t^2$

$$\Gamma = \det(\partial_t^2 + r^2)^{D-4} \det(\partial_t^2 + r^2 + 2v)\ \det(\partial_t^2 + r^2 - 2v)$$

$$= \int \frac{ds}{s} \frac{e^{-b^2 s}}{\sinh(vs)} \left(D - 4 + 2\cosh(2vs)\right)$$

$$= \int dt V_{eff}(b^2 + v^2 t^2)$$

Eikonal

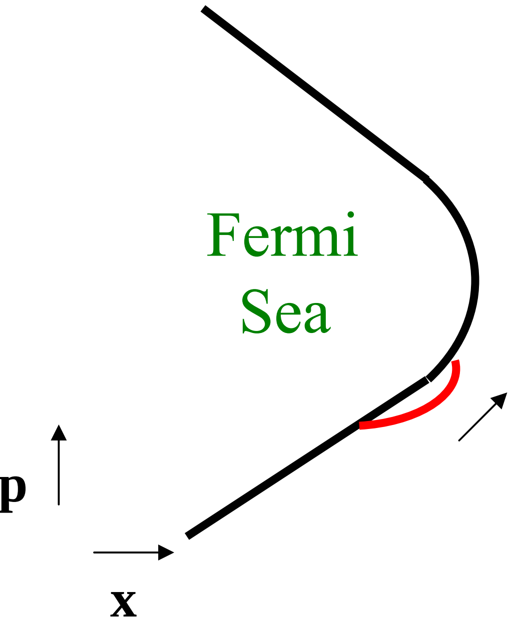
$$\rightarrow \boxed{V_{eff} = \tfrac{1}{2}(D-2)\,|\,b\,| + \tfrac{1}{6}(26-D)\frac{v^2}{b^3} + \ \ldots}$$

Tachyon mass **Dilaton tadpole!**

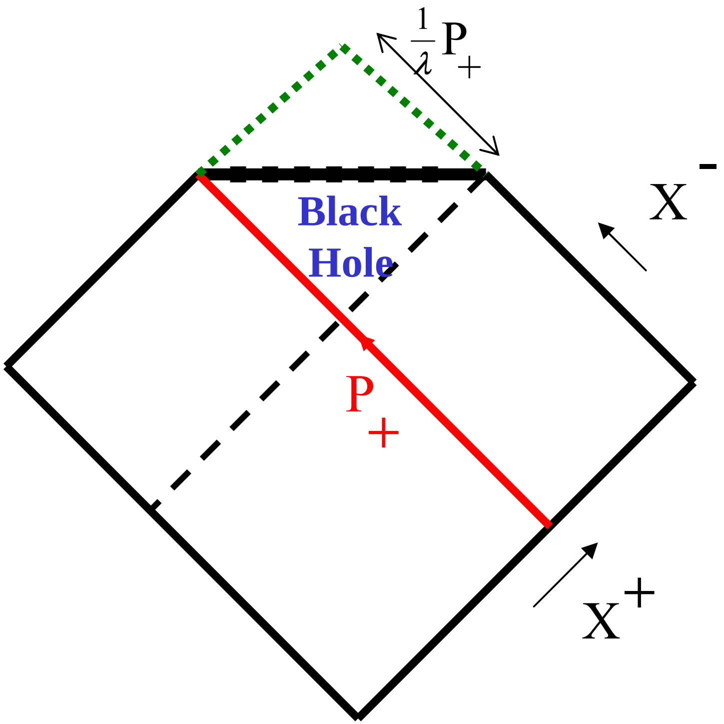
?

The c=1 open/closed string duality has several attractive features:

- It is non-supersymmetric.
- Holography with an S-matrix.
- Is exactly solvable → can perform detailed tests.
- Useful laboratory to study:
    * D-brane recoil.
    * time-dependence.
    * gravitational backreaction.



Matrix model



Target space

The c=1 S-matrix preserves an infinite set of the W_∞ charges:

$$\boxed{W_{mn} = \int du\ e^{(n-m)u} (\partial_u \Phi_{\boldsymbol{in}})^{m+1} = \int dv\ e^{(n-m)v} (\partial_v \Phi_{\boldsymbol{out}})^{n+1}}$$

Special generators:

central element $W_{00} = \hbar N \equiv \lambda$ $W_{11} = H_N$ *total energy*

in-Kruskal energy $W_{10} = P_+$ $W_{01} = P_-$ *out-Kruskal energy*

Special commutator:

$$\boxed{[P_+, P_-] = i\lambda}$$

Consider the in-state with given in-Kruskal momentum p_+ :

$$|\mathrm{in}\rangle = e^{\frac{i}{\lambda}p_+P_-}|0\rangle \qquad = |\mathrm{thermal\ out-state}\rangle$$

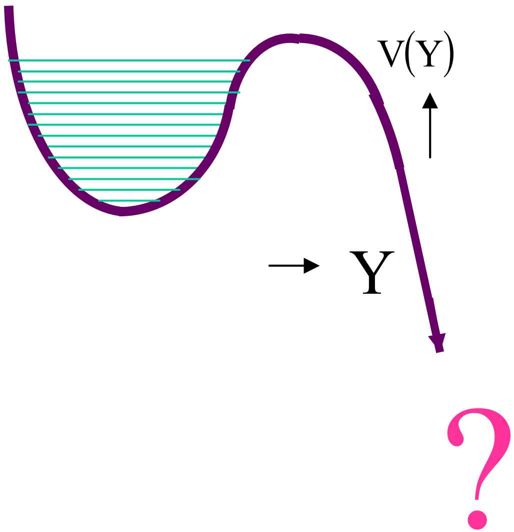
Shifts: $x^- \to x^- + \frac{1}{\lambda}p_+$

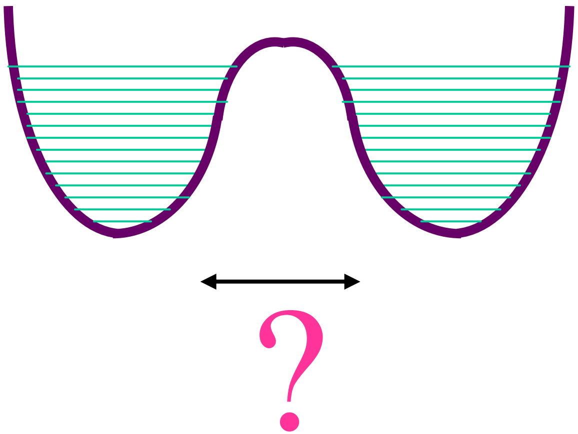
Gravitational shockwave

Crucial point: the non-local leg pole transformation relates P_- to an (approximately) local operator in space-time:

$$T(u) = \frac{\Gamma(-\partial_u)}{\Gamma(\partial_u)}\Phi_{in}(u) \quad \Rightarrow \quad \partial T(u_o) \approx \int_{-\infty}^{u_o} e^u \partial\Phi_{in}(u) \underset{u_o\to\infty}{\approx} P_-$$

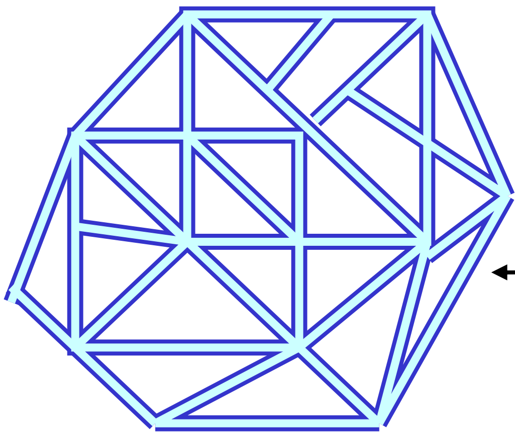
Challenge: Non-perturbative consistency, possibly by embedding in a supersymmetric string theory.





3) After a maximal T-duality, the D-dimensional matrix QM is mapped to D-dimensional YM+Higgs theory:

$$S_D = \int d^D x \ \mathrm{Tr}\left(F_{\mu\nu}^2 - \left(D_\nu Y\right)^2 + V(Y)\right)$$

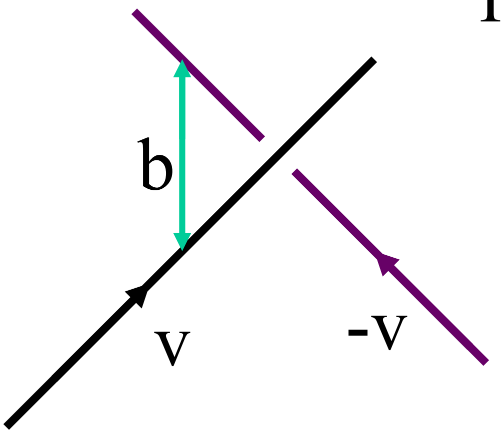


$$\frac{1}{k_\nu^2 + 1} \rightarrow e^{-k_\nu^2} \rightarrow e^{-(x_i^\nu - x_j^\nu)^2}$$

The (UV regularized) planar diagrams of this theory provide a dynamical triangulation+discretization of string worldsheets embedded in D+1 dimensions. (Ambjorn e.a., David, Kazakov e.a.)

Liouville

4) Suppose we use the matrix QM to compute the effective potential between two D-particles. We find:



$$r^2 = b^2 + v^2 t^2$$

$$\Gamma = \det(\partial_t^2 + r^2)^{D-4} \det(\partial_t^2 + r^2 + 2v)\ \det(\partial_t^2 + r^2 - 2v)$$

$$= \int \frac{ds}{s} \frac{e^{-b^2 s}}{\sinh(vs)} \big(D - 4 + 2\cosh(2vs)\big)$$

$$= \int dt V_{eff}(b^2 + v^2 t^2)$$

Eikonal

$$\boxed{V_{eff} = \tfrac{1}{2}(D-2)\,|\,b\,| + \tfrac{1}{6}(26-D)\frac{v^2}{b^3} + \ \ldots}$$

Tachyon mass **Dilaton tadpole!**

?