What is Hol ogra phy in the Plane-Wave Limit of $AdS_5 \times S^5$/SYM Correspondence?

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How to reconcile the BMN conjecture with holographic principle?

In particular, we will discuss
how the GKP-W relation should be realized in the plane-wave limit.

◇ *Puzzles, and Resolution*

◇ *PP-wave Holography for Dp-branes*

Based on

- S. Dobashi, H. Shimada and T. Y., hep-th/0209251
- T. Y, hep-th/0304183 + work in preparation
- M. Asano, Y. Sekino and T. Y., to appear soon, hep-th/0307???

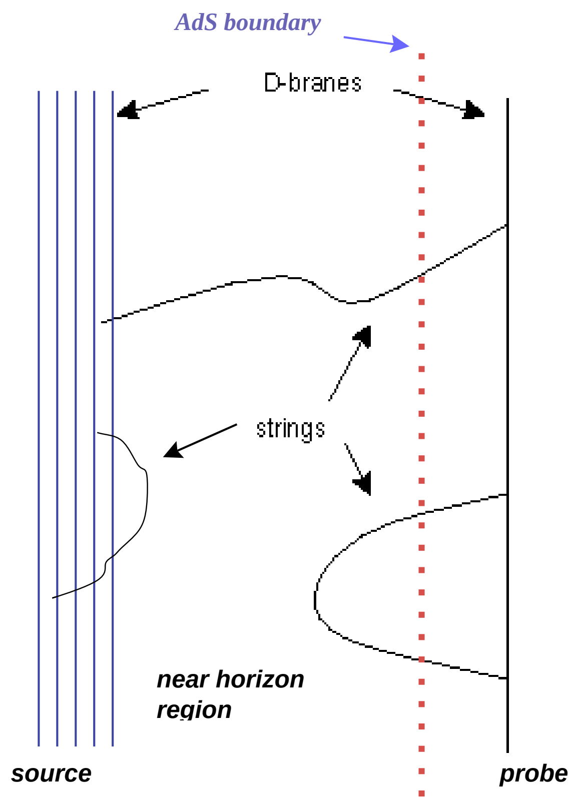
GKP-W relation : Basic "holographic" relation in AdS/CFT correspondence

$$\text{bulk } (\text{AdS}_5) \quad \leftrightarrow \quad \text{boundary } (\text{M}_{3,1})\,(z \to 0)$$

$$Z[\phi_0]_{\text{gravity}} = \langle \exp(\int d^4x \sum_i \phi_0^i(x)\mathcal{O}_i(x))\rangle_{\text{SYM}},$$

$$\diamond \lim_{z\to 0} \phi^i(z,x) \to z^{4-\Delta_i}\phi_0^i(x)$$

Physical basis :
dual descriptions of
source-probe systems
of D3-branes



So far, this relation has been discussed only in the context of supergravity approximation.

◇ *Is it possible to see stringy modes in the large R limit?*

- *Excitation energy for supergravity KK (along S^5) modes*

  $\sim J/R$ (*$J=$ angular momentum along S^5*)

- *Excitation energy for higher stringy modes (string unit)* ~ 1

If J is finite, stringy excitations are decoupled, $J/R \ll 1$.
However, if J is sufficiently large, the stringy excitation energies become comparatively smaller and might be visible.

Berenstein-Maldacena-Nastase (BMN) proposal

identify anomalous dimensions for stringy operators, roughly, as
$\Delta \sim M^2/(J/R^2) \sim J + R^2 N/J + O(1)$
by taking the double limit $J \sim R^2 \to \infty$, keeping $J/R^2 \Leftrightarrow P^+$ finite.
More precisely,

$$\Delta - J = \sum_{\{i\}} N_i \sqrt{1 + \frac{R^4 n_i^2}{J^2}} \Leftrightarrow P^-$$

◇ *Identification of stringy operators*

ϕ_i $(i = 1, \ldots, 6)$: *scalar fields (~ collective coordinates of D3-branes)*

- *large J ground state:*

$$\mathrm{Tr}[Z^J], \quad Z = \phi_5 + i\phi_6$$

- *stringy excitation modes ($n = 0 \to$ SUGRA operators):*

$$a_n^{i+4,\dagger} \leftrightarrow Z^\ell \phi_i \mathrm{e}^{2\pi i n\ell/J} Z^{J-\ell}, \quad a_n^{i,\dagger} \leftrightarrow Z^\ell \underline{(D_i Z)} \mathrm{e}^{2\pi i n\ell/J} Z^{J-\ell} \quad (i = 1, 2, 3, 4)$$

etc

8 transverse string excitations in the bulk

$\Updownarrow$

4 (=$SO(4)$ R-charge directions) + 4 (=$SO(4)$ base space directions of SYM)
at the boundary

◇ $J \to \infty \Rightarrow$ *Particle (or string) picture around particular trajectories (null geomdesics)*

The null geodesic on which the 'PP' wave limit of AdS geometry is based is,
in terms of the Poincaré coordinate

$$z = \frac{1}{\cos\tau}(\geq 1), \quad t = R^2 \tan\tau, \quad \psi = \tau$$

$$ds^2 = \frac{R^2 dz^2}{z^2} + \frac{dx_3^2 - dt^2}{R^2 z^2} + \cdots$$

$\tau \sim$ *the time of the global coordinates for AdS spacetime*

$\longrightarrow$ *time coordinate of light-cone gauge* ($\tau = x^+$)
in which strings can be quantized exactly [Metsaev]
$\longrightarrow$ *periodicity in* τ *(universal covering of hyperboloid)*

This never reaches the boundary ($z \to 0$),
and moreover
goes into horizons ($z \to \infty$) *in a finite interval with respect to this* τ.

Puzzles related to holography

- $z \geq 1 \Rightarrow$ *no direct connection with the AdS boundary where* $z = 0$*?*

  Impossible to apply the GKP-W relation to the BMN operators?

- *If* $\tau \leftrightarrow \tau_r$, $\vec{x} = e^{\tau_r}\vec{x}/|\vec{x}|$, *of radial quantization on the boundary;*

  – *8 transverse directions of bulk string theory involve all of 4 base-space directions (irrespectively of the identification of the global time and the target time on the boundary):*

    $\Rightarrow$ *light-cone time* $x^+ = \tau$ *mixed with transverse directions!?*

  – *The null geodesic requires Minkowski metric, while the boundary theory must be assumed to be Euclidean?*

  – *Periodicity in* τ *from the viewpoint of boundary theory?*

    For instance, integration over τ *from* $-\infty$ *to* $+\infty$ *would lead to divergences in computing various physical amplitudes.*

Our strategy:

Study the BMN limit of GKP-W relation directly.

Resolution: Holography from tunneling

Consider first a scalar wave equation on the AdS background (Minkowski metric, $\omega \sim$ time-like energy with respect to target spacetime)

$$\Big(z^2\partial_z^2 - 3z\partial_z + R^4 z^2\omega^2 - J(J+4)\Big)\phi(z) = 0$$

in the WKB approximation for large $J \sim R^2$

$$\phi(z) \sim N A(z) \exp iS(z)$$

$$\Downarrow$$

$$z^2\Big(\frac{dS}{dz}\Big)^2 - R^4 z^2\omega^2 + J^2 = 0$$

$$A(z) = J^{1/2} z^{3/2}\Big(\frac{dS}{dz}\Big)^{-1/2} \exp\Big[-2iJ\int \frac{dz}{z^2}\Big(\frac{dS}{dz}\Big)^{-1/2}\Big]$$

There is no real solution that reaches the boundary since reality of S requires

$$z^2 \geq J^2/(\omega^2 R^4)$$

Solutions with the holographic boundary condition

$$\lim_{z\to 0}\phi^i(z,x)\to z^{4-\Delta_i}\phi^i_0(x)$$

correspond to tunneling with purely imaginary action S.

$$S\to -iS_E,\ \phi(r)\to NA(z)\exp S_E(z)$$

$$\Downarrow$$

$$z^2\Big(\frac{dS_E}{dz}\Big)^2 = J^2(1-\frac{z^2\omega^2R^4}{J^2})$$
$$S_E(z)\sim \mp J\log z,\quad A(z)\sim z^{2\pm 2}$$

reproducing the well known relation between
the conformal dimension and mass for scalar field

$$m^2 = J(J+4) = \Delta(\Delta-4)\to \Delta = J+4$$

At a formal level, [real picture $\longrightarrow$ tunneling picture] is obtained by 'triple' Wick rotation:

- *affine time :* $\tau \longrightarrow -i\tau$
- *target spacetime :* $t \longrightarrow -ir, \quad \psi \longrightarrow -i\psi$

The last two are necessary to keep ω *and* J *real after* $\tau \longrightarrow -i\tau$ *(*$\frac{J}{R^2} = \omega$*)*

$$\Downarrow$$

Tunneling null geodesic

$$z = \frac{1}{\cosh\tau}, \quad r = R^2 \tanh\tau, \quad \psi = \tau$$

◇ *Reaches the boundary* $z \to 0$ *as* $\tau = \pm T, \quad T \to \infty$

$$z \to 2\mathrm{e}^{-T}$$

◇ UV/IR relation $\Rightarrow e^{-T} \sim$ short-distance cutoff parameter for boundary theory.

Conformal dimensions $(\Delta - J)$ = energy with respect to τ-translation

$$\Updownarrow$$

scaling $z \to \lambda z$ near the conformal boundary

- Affine time direction near the boundary is manifestly orthogonal to the boundary. :

  τ cannot be identified with the radial time

- boundary $\to$ boundary $\Leftrightarrow$ infinite affine time interval:

  $\tau = -T \to +T \quad T \to \infty$ (no periodicity)

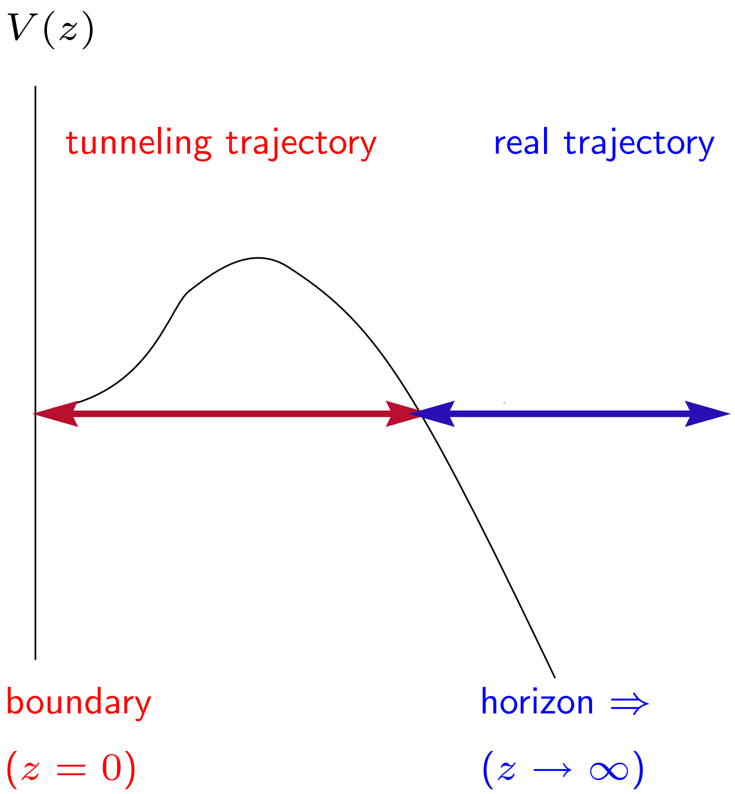
- Boundary theory must be treated as Euclidean,

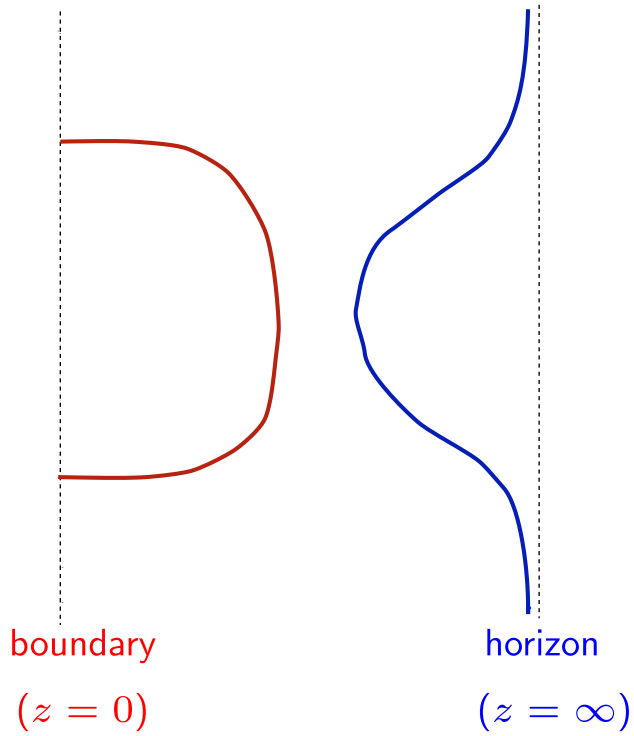
  because we are considering tunneling amplitudes in the semi-classical limit

solves all of the 'puzzles'!

To preserve the GKP-W relation in the PP-wave limit,
we should consider

‘tunneling’ null geodesics





Direct derivation from Witten diagrams

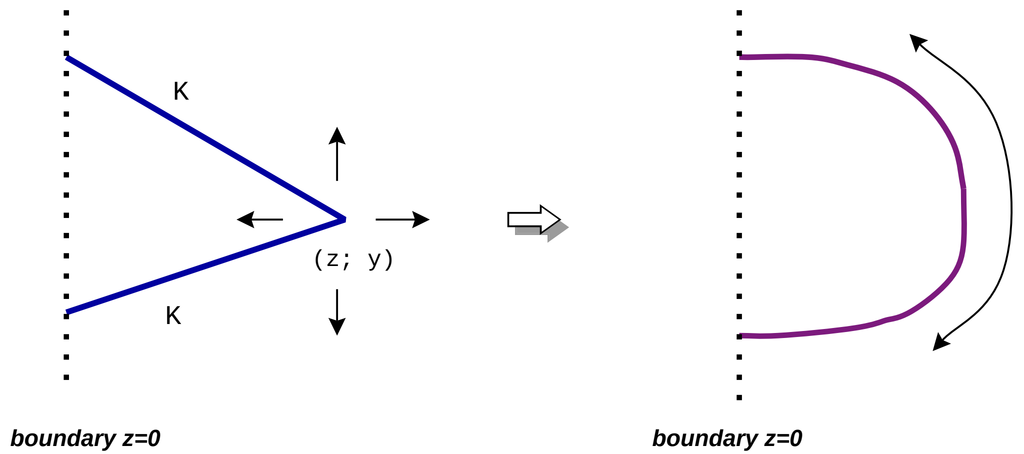
Boundary correlation functions are computed diagramatically by Witten diagrams using the bulk-to-boundary $(z, \vec{y}) \leftrightarrow (0, \vec{x})$ *propagator.*

$$K_\Delta(z, \vec{y}; \vec{x}) = \left(\frac{z}{z^2 + (x-y)^2}\right)^\Delta$$

- *2-pt function:*

$$\frac{1}{|x-x'|^{2\Delta}} = \lim_{\epsilon \to 0} \frac{\epsilon}{N(\Delta)^2} \int \frac{d^4y dz}{z^5} z^\epsilon K_\Delta(z, \vec{y}; \vec{x}) K_\Delta(z, \vec{y}; \vec{x}')$$

In the limit of large J $(\Delta = J + k,\ k\text{=finite})$*, the integral is dominated by a one dimensional integral along the tunneling null trajectory.*



The tunneling trajectory just solves the saddle-point eq.

$$\frac{\partial}{\partial z}\Big[\ln K_\Delta(z,\vec{y};\vec{x}) + \ln K_\Delta(z,\vec{y};\vec{x}')\Big] = 0,$$

$$\frac{\partial}{\partial y^\mu}\Big[\ln K_\Delta(z,\vec{y};\vec{x}) + \ln K_\Delta(z,\vec{y};\vec{x}')\Big] = 0$$

$$\Downarrow$$

$$z(\tau) = \frac{|x-x'|}{2\cosh\tau}, \quad y^\mu(\tau) = \frac{1}{2}(x+x')^\mu - \frac{1}{2}(x-x')^\mu \tanh\tau$$

$$\int \frac{d^4y dz}{z^5} z^\epsilon K_\Delta(z,\vec{y};\vec{x}) K_\Delta(z,\vec{y};\vec{x}') \sim \frac{N(\Delta)^2}{|x-x'|^{2\Delta}} \int_{-T}^{T} d\tau$$

– *consistent with the cutoff suggested from the UV/IR relation:*

$$|\ln z| < \epsilon \to T \sim 1/\epsilon$$

– $\tau \sim$ *collective coordinate with measure*

$$\frac{d^4 ydz}{z^5} \to d\tau, \quad K_\Delta \to \mathrm{e}^{\pm\Delta\tau}/|x-x'|^\Delta$$

- *3-pt function: ($\Delta_i + \Delta_j - \Delta_k \geq 0$)*

$$\frac{C_{123}}{|x_1 - x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1}|x_3 - x_1|^{\Delta_3+\Delta_1-\Delta_2}}$$

$$= \frac{1}{N(\Delta_1, \Delta_2, \Delta_3)} \int \frac{d^4y dz}{z^5} K_{\Delta_1}(z, \vec{y}; \vec{x}_1) K_{\Delta_2}(z, \vec{y}; \vec{x}_2) K_{\Delta_3}(z, \vec{y}; \vec{x}_3) V_{123}(z; \vec{y})$$

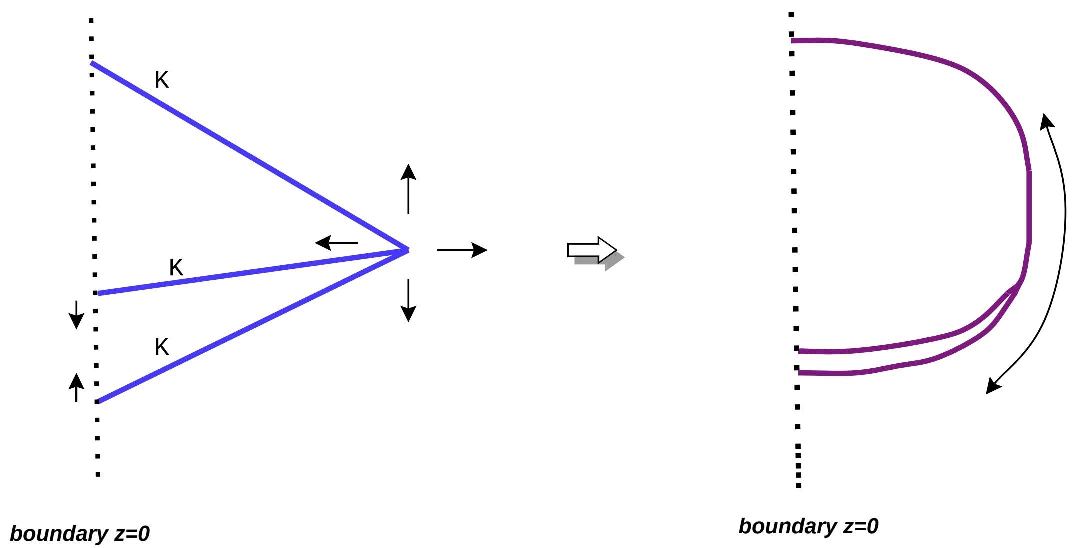
For generic configurations of 3 points x_1, x_2, x_3, there is no smooth saddle point.

However, there are two types of special cases where the integral is dominated by a single tunneling null trajectory parametrized by a single collective coordinate τ.

1. $J_3 = J_1 + J_2$ *in the short-distance limit* $\delta \equiv |x_1 - x_2| \to 0$ *for all* $J_i \to \infty$

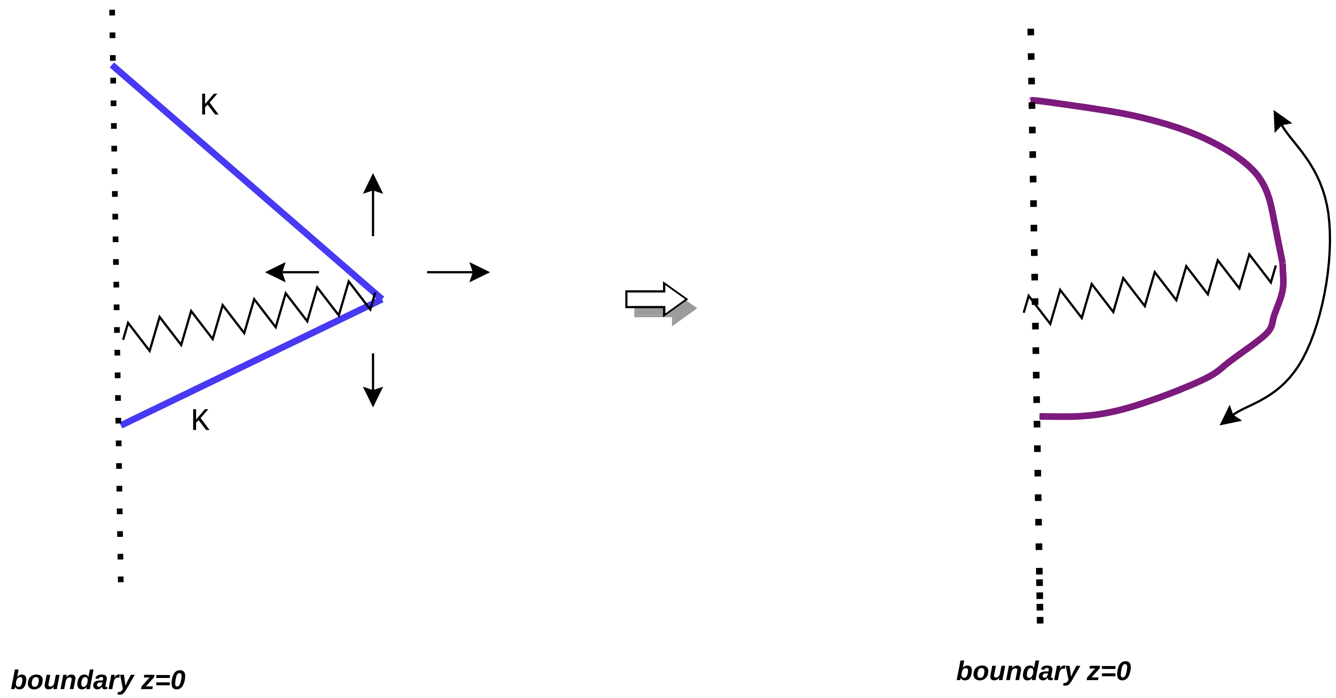
- *3 points* x_i *correspond to BMN operators*
- *valid when*

$$\delta < z \sim |x_3 - x| \mathrm{e}^{-|\tau|} \to \textit{cutoff}: \ |\tau| < T \sim \ln(|x_3 - x|/\delta)$$



2. $J_3 = J_1 = J \to \infty$ and $J_2 = 0$.

- x_1, x_3 correspond to BMN operators
- x_2 to usual BPS operator with finite R-charge.



1. $\Rightarrow$ *Relation between OPE coefficients and light-cone vertices*

$$\diamond \qquad \frac{C_{123}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}|x_3-x_1|^{\Delta_3+\Delta_1-\Delta_2}}$$

$$\Downarrow$$

$$C_{123}\frac{\delta^{-(k_1+k_2-k_3)}}{|x_3-x|^{2\Delta_3}}$$

$$\diamond \qquad \frac{1}{N(\Delta_1,\Delta_2,\Delta_3)}\int\frac{d^4y dz}{z^5}K_{\Delta_1}(z,\vec{y};\vec{x}_1)K_{\Delta_2}(z,\vec{y};\vec{x}_2)K_{\Delta_3}(z,\vec{y};\vec{x}_3)V_{123}$$

$$\Downarrow$$

$$\frac{1}{|x_3-x|^{\Delta_1+\Delta_2+\Delta_3}}\int_{-T}^{T}d\tau \mathrm{e}^{-(k_1+k_2-k_3)\tau}\tilde{V}_{123}\sim\frac{1}{|x_3-x|^{2\Delta_3}}\frac{\delta^{-(k_1+k_2-k_3)T}}{k_1+k_2-k_3}\tilde{V}_{123}$$

$$\rightarrow C_{123}=\frac{\tilde{V}_{123}}{\Delta_1+\Delta_2-\Delta_3}$$

$$\delta^{-(k_1+k_2-k_3)T} C_{123} = \frac{\delta^{-(k_1+k_2-k_3)T}}{k_1+k_2-k_3} \tilde{V}_{123} = \int_{-\infty}^{\infty} d\tau \langle 3 | \mathrm{e}^{H_0 \tau} \tilde{V}_{\mathrm{int}} \mathrm{e}^{-H_0 \tau} | 1, 2 \rangle$$

This is just a Euclidean 3-pt S-matrix in the tree approximation.

It is also well known [e.g. Freedman-Mathur-Matusis-Rastelli] *that*

$C_{123} \neq 0, \;\; V_{123} = 0$ *for* $\quad \Delta_1 + \Delta_2 - \Delta_3 = k_1 + k_2 - k_3 = 0$

'extremal correlators'

See also ansatz used (in ordinary null geodesics approach) by

[Constable-Freedman-Headrick-Minwalla-Motl-Postnikov-Skiba]

2. $\Rightarrow$ *Perturbation to 2-pt correlators*

In the limit $x_3 = -x_1 \equiv x \to \infty$*, the result is simplified to*

$$\frac{C_{123}}{|x|^{\Delta_1+\Delta_2+\Delta_3}} = \frac{\tilde{V}_{123}}{|x|^{\Delta_1+\Delta_2+\Delta_3}} \int_{-\infty}^{\infty} d\tau \frac{\mathrm{e}^{-(k_3-k1)\tau}}{(\cosh\tau)^{k_2}}$$

$$\Downarrow$$

Euclidean 2-pt S-matrix in the presence of an external field

$$C_{123} = \tilde{V}_{123} \int_{-\infty}^{\infty} d\tau \frac{\mathrm{e}^{-(k_3-k_1)\tau}}{(\cosh\tau)^{k_2}} = \tilde{V}_{123} \int_{-\infty}^{\infty} d\tau \langle 3|\mathrm{e}^{H_0\tau}\phi_2(\tau)\mathrm{e}^{-H_0\tau}|1\rangle$$

with the external field $\phi(\tau)$ *produced by the boundary perturbation at* 2*.*

$$\phi_2(\tau) = \phi(z(\tau); y(\tau)) = K_{k_2}(z(\tau); y(\tau))$$

Remarks:

- *Recently, Mann and Polchinski (hep-th/0305230) discussed the case 2 from the viewpoint of the usual Minkowki picture and raised a puzzle: Their question is that the factor corresponding to the non-BMP propagator should be*

$$\int dx^+ \delta(x^+ - \tau) \quad \textit{instead of integrals such as} \quad \int_{-\infty}^{\infty} d\tau \frac{\mathrm{e}^{-(\Delta_3 - \Delta_1)\tau}}{(\cosh \tau)^{\Delta_2}}$$

  since the perturbation is local with respect to radial time τ_r at the boundary. However, this question is based on the identification of global time x^+ with the radial time. We have argued that this identification is not allowed.

  According to our tunneling picture, the time τ along tunneling null trajectory connecting boundary to boundary has no direct relation with boundary coordinates. The integral with respect to our τ should rather be interpreted as a part of integration over physical degrees freedom at various different scales.

- *The integrals over τ cannot be Wick-rotated back to $i\tau$.*
  - *Case 1 would lead to energy-conserving δ-function $\delta(\Delta_3 - \Delta_1 - \Delta_2)$*
  - *Case 2 would lead to divergence, due to the existence of an infinite number of periodic poles in $1/(\cos \tau)^{\Delta_2}$.*

PP-wave holography for Dp-branes

If we follow our tunneling picture, the holographic bulk-boundary correspondence in the PP-wave limit can be straightforwardly extended to general (non conformal) Dp-branes.

Consider the case of D0-brane:

- *Predictions for the large N behavior of Matrix theory.*
- *Typical example with time-dependent mass terms*

◇ *Characterization by scaling property ('generalized conformal symmetry'):*

Jevicki-Yoneya

- *bulk (D0 background)*

$$\vec{x} \to \lambda \vec{x},\, t \to \lambda^{-1} t,\, g_s \to \lambda^3 g_s,$$

- *boundary (SYM quantum mechanics)*

$$X_i \to \lambda X_i,\ t \to \lambda^{-1} t,\ g_s \to \lambda^3 g_s$$

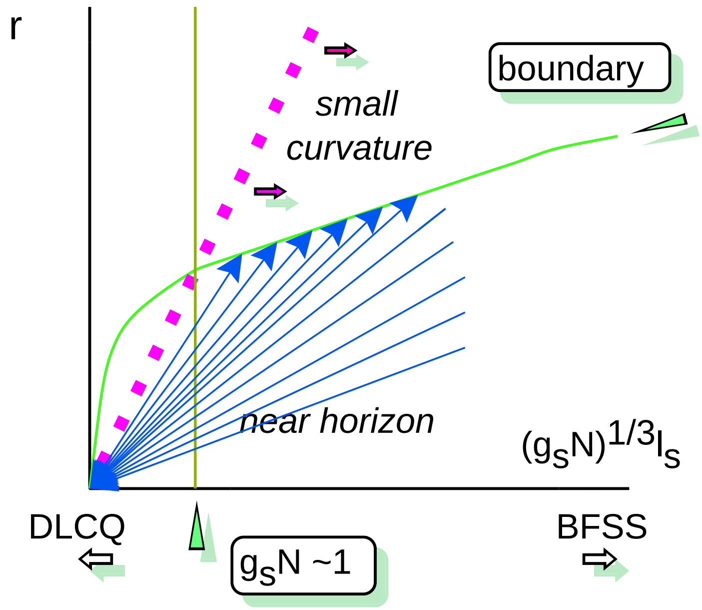


Fig.1 : Oblique AdS/CFT correspondence

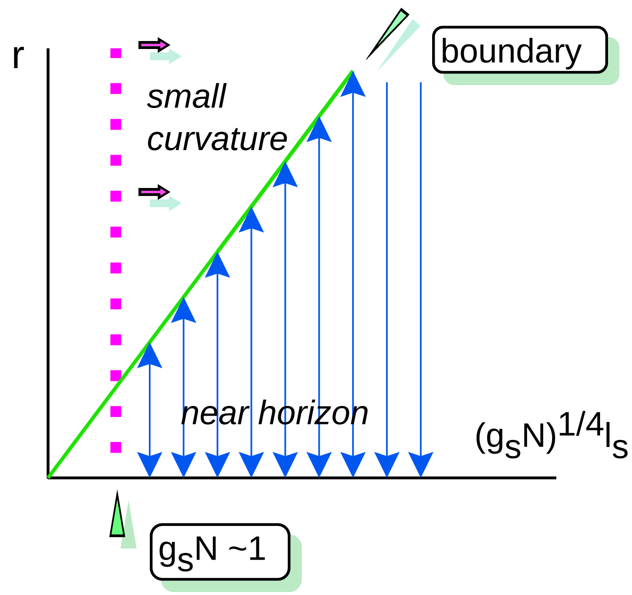


Fig 2 : Ordinary AdS/CFT correspondence

Detailed supergravity analysis predicts the following general form for 2-pt functions

Sekino-Yoneya, Yoneya at Strings'99

$$\langle \mathcal{O}_I(t_1)\mathcal{O}_I(t_2)\rangle \sim \frac{1}{g_s^2\ell_s^8} q^{(\Delta_I+6)/5}|t_1-t_2|^{-(7\Delta_I+12)/5}, \quad q = g_s N \ell_s^7$$

Δ_I = *generalized conformal dimension*

$$\mathcal{O}_I(\tau) \to \mathcal{O}'_I(t') = \lambda^{\Delta_I}\mathcal{O}_I(t), \quad t' = \lambda^{-1}t, \quad g_s \to g'_s = \lambda^3 g_s$$

$$\Delta_I = -1 + 2n_I + \frac{4}{7}\ell_I, \quad n_I = 1 - n_+ + n_-$$

$n_\pm$ =*the number of upper light-cone indices* $\pm$ *in the sense of M-theory interpretation.*

For example, ($\tilde{X}_i = X_i/q^{1/7}$, $A_0 = 0$ *gauge)*

$$\Delta = -3 + \frac{4\ell}{7}: \; T^{++}_{\ell, i_1 i_2 \cdots i_\ell} = \frac{1}{R}\mathrm{STr}(\tilde{X}_{i_1}\tilde{X}_{i_2}\ldots\tilde{X}_{i_\ell} + \cdots), \quad (\ell \geq 2)$$

$$\Delta = -1 + \frac{4\ell}{7}: \; T^{+i}_{\ell, i_1 i_2 \cdots i_\ell} = \frac{1}{R}\mathrm{STr}(\dot{X}_i\tilde{X}_{i_1}\tilde{X}_{i_2}\ldots\tilde{X}_{i_\ell} + \cdots), \quad (\ell \geq 2)$$

$$\Delta = +1 + \frac{4\ell}{7}: \; T^{ij}_{\ell, i_1 i_2 \cdots i_\ell} = \frac{1}{R}\mathrm{STr}(\dot{X}_i\dot{X}_j\tilde{X}_{i_1}\tilde{X}_{i_2}\ldots\tilde{X}_{i_\ell} + \cdots), \quad (\ell \geq 2)$$

$etc.$

This suggests :

For D0 PP-wave background
with J being angular momentum along the 8-9 direction
(global symmetry $\sim$ SO(7)),

ground state	$\mathrm{Tr}(Z^J), \quad Z = X^8 + iX^9$
longitudinal excitation	$\dot{Z}$
transverse excitations	$X^i, \quad (i = 1, \ldots, 7)$

Generalized conformal dimensions corresponding to these excitations are

SO(1) direction $\dot{Z} \quad \rightarrow \quad \Delta_1 = 10/7$

SO(7) directions $X^i \quad \rightarrow \quad \Delta_7 = 4/7$

Consider the tunneling null trajectory for D0 background. In the limit of large $q = g_s N(\ell_s = 1)$, the effective action is quadratic with time dependent (mass)^2.

- *SO(7) direction:*

$$m_7^2 = \frac{7}{16r^2}(-3 + \ell^2 r^5)$$

- *SO(1) direction ($\sim$ 1 dimensional base space of Matrix theory):*

$$m_1^2 = \frac{7}{16r^2}(-3 + 13\ell^2 r^5)$$

Gimon-Zayas-Sonnenschein, ...

where $r = r(\tau)$ is determined by

$$\dot{r} = \pm r_0^{-5/2}\sqrt{r^5 - r_0^5}, \quad r \geq r_0$$

$$r_0 = \ell^{-2/5}, \quad \ell = |t_1 - t_2|$$

Note : Near the near-horizon boundary $r\,(= 1/z^{2/5}) \sim q^{1/7} \to \infty \quad (z \to 0)$, $m(\tau)^2$ are positively large as r^3. In constrast, we would have nagatively infinite mass^2 at the horizon $r = 0$ (singularity) for real null trajectory.

We have to deal with quantum theory of time dependent harmonic oscillator in Euclidean formulation.

$$H(\tau) = \frac{1}{2}\Big(P(\tau))^2 + m(\tau)^2 X(\tau)^2\Big),\ X^\dagger(\tau) = X(-\tau),\ P = i\dot{X}$$

This is exactly (analytically) solvable. *In particular, the boundary($\tau = -T$)-to-boundary($\tau = T$) 2-pt S-matrix is diagonalized near the boundary as*

$$S(T) = \mathcal{T}\exp\Big[-\int_{-T}^{T} d\tau H(\tau)\Big] \to (1+B)^{a^\dagger a + 1/2}$$

$$1 + B = \frac{1}{2}\Big(\frac{f_+(T)}{f_-(T)} - \frac{\dot{f}_+(T)}{\dot{f}_-(T)}\Big)$$

where $f_\pm$ is the solution of the equation of motion with particular boundary conditions, $f_\pm(\tau) \to 0, \quad \tau \to \pm T$:

$$\frac{d^2 X(\tau)}{d\tau^2} = m^2(\tau) X(\tau),\ X(\tau) = f_+(\tau) a + f_-(\tau) a^\dagger, \quad [a, a^\dagger] = 1$$

$$f_+ \frac{df_-}{d\tau} - f_- \frac{df_+}{d\tau} = 1, \quad f_-(\tau) = f_+(-T)$$

Using explicit solutions for $f_{\pm}(\tau)$, the dependence of the S-operator on the target space-distance $|t_1 - t_2|$ are found to be

$$S(T)_7 \sim (|t_1 - t_2|^{-4/5})^{a^\dagger a + 1/2}, \quad S(T)_1 \sim (|t_1 - t_2|^{-2})^{a^\dagger a + 1/2},$$

in agreement with the field theory analysis

- *each SO(7) excitation* $\rightarrow \Delta_7 = 4/7$
- *each SO(1) excitation* $\rightarrow \Delta_1 = 10/7$

There results are readily extended to general Dp-branes ($p < 7$).

Summary

1. Holographic correspondence between bulk and boundary in the PP-wave limit can be understood as a tunneling phenomenon.

2. The familiar identification of global time along the Minkowski null geodesics with the radial time on the boundary does not give reasonable holographic picture in the PP-wave limit.

3. GKP-W relation is meaningfully extended to the BMN operators in two-group short-distance limits

   Correlators on the boundary $\sim$ Euclidean S-Matrix in the bulk

4. Predictions for 2-pt correlators for Matrix theory (D0) in the large N limit, extending the tunneling picture to the time dependent PP-wave background.